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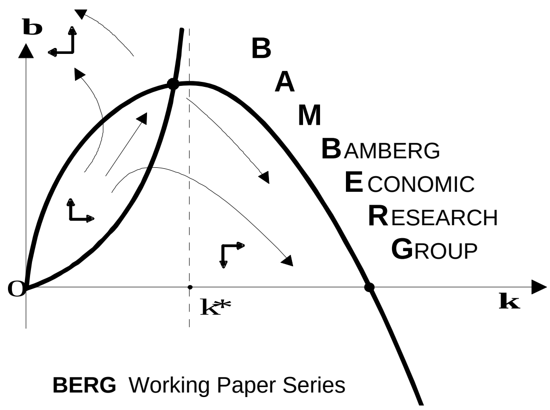

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Macroeconomic and Stock Market Interactions with Endogenous Aggregate Sentiment Dynamics

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Abstract

This paper studies the implications of heterogeneous capital gain expectations on output and asset prices. We consider a disequilibrium macroeconomic model where agents' expectations on future capital gains affect aggregate demand. Agents' beliefs take two forms – fundamentalist and chartist – and the relative weight of the two types of agents is endogenously determined. We show that there are two sources of instability arising from the interaction of the financial with the real part of the economy, and from the heterogeneous opinion dynamics. Two main conclusions are derived. On the one hand, perhaps surprisingly, the non-linearity embedded in the opinion dynamics far from the steady state can play a stabilizing role by preventing the economy from moving towards an explosive path. On the other hand, however, real-financial interactions and sentiment dynamics do amplify exogenous shocks and tend to generate persistent fluctuations and the associated welfare losses. We consider alternative policies to mitigate these effects.

Keywords: Real-financial interactions, heterogeneous expectations, aggregate sentiment dynamics, macro-financial instability

JEL classifications: E12, E24, E32, E44.

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1 Introduction

The way in which the dynamic interaction between stock markets and the macroeconomy has been understood by the economics profession has evolved significantly over the last thirty years. As Shiller (2003) has argued, while the rational representative agent framework and the related Efficient Market Hypothesis represented the dominant theoretical modeling paradigm in financial economics during the 1970s, the behavioral finance approach has gained increasing ground within the economics community over the last two decades. The main reason for this significant paradigm shift is well known: following Shiller (1981) and LeRoy and Porter (1981), a large number of studies have documented various empirical regularities of financial markets – such as the excess volatility of stock prices – which are clearly inconsistent with the Efficient Market Hypothesis, see e.g. Frankel and Froot (1987, 1990), Shiller (1989), Allen and Taylor (1990), and Brock et al. (1992), among many others. During the 1990s several researchers like Day and Huang (1990), Chiarella (1992), Kirman (1993), Lux (1995) and Brock and Hommes (1998) have developed models of financial markets with heterogenous agents following the seminal work by Beja and Goldman (1980) in order to explain such empirical regularities. Ever since, financial market models with heterogeneous agents using rule-of-thumb strategies have become central in the behavioral finance literature, see e.g. Chiarella and He (2001, 2003), De Grauwe and Grimaldi (2005), Chiarella et al. (2006), and Dieci and Westerhoff (2010).

The importance of different types of heterogeneity (regarding preferences, risk aversion or available information) and boundedly rational behavior at the micro level for the dynamics of the macroeconomy has also been increasingly acknowledged in macroeconomics (Akerlof, 2002, 2007). In this context, a particularly fruitful new strand of the literature has focused on the consequences of heterogeneous boundedly rational expectations for the dynamics of the macroeconomy and the conduct of economic policy, see e.g. Branch and McGough (2010), Branch and Evans (2011), De Grauwe (2011, 2012), Proaño (2011, 2013), among others. In these studies, the Brock and Hommes (1997) (BH) approach has been the preferred specification for the endogenous switch between alternative heuristics. In contrast, the development of macroeconomic models using the Weidlich-Haag-Lux (WHL) approach (see Weidlich and Haag, 1983 and Lux, 1995) is still in a nascient stage, with Franke (2012), Franke and Ghonghadze (2014), Flaschel et al. (2015), Chiarella et al. (2015) and Lojak (2016) as notable exceptions.

While the WHL and the BH approaches are quite similar in spirit – and similarly close to Keynes' (1936) and Simon's (1957) views on expectations under bounded rationality (see also Kahneman and Tversky, 1973 and Kahneman, 2003) – there is a fundamental difference between them: In the BH approach the variation in the share of agents using a particular heuristic depends on a measure of utility, or forecast accuracy, related to that particular rule of thumb which is thought to be relevant at the microeconomic level. In contrast, in the WHL approach the switch between different heuristics or attitudes, such as optimism or pessimism, is determined by an aggregate sentiment index composed e.g. by macroeconomic variables describing the state of the economy in the business cycle, see also Franke (2014). The WHL approach thus incorporates an additional link from the macroeconomic environment to microeconomic decision-making based on psychological grounds and on Keynes' notion that "Knowing that our own individual judgment is worthless, we endeavor to fall back on the judgment of the rest of the world which is perhaps better informed. That is, we endeavor to conform with the behavior of the majority or the average. The psychology of a society of individuals each of whom is endeavoring to copy the others leads to what we may strictly term a *conventional* judgment." (Keynes, 1937, p. 114; his emphasis).[1]

In this latter line of research the main contribution of this paper is to study the effects of aggregate sentiments in stock markets on the real economy using the WHL approach to model the expectations formation process in stock markets. More specifically, we incorporate aggregate sentiment dynamics in a stock market populated by heterogeneous agents, and examine the effects of herding and speculative behavior in combination with real-financial market interactions. We adopt the distinction between *chartists* and *fundamentalists* which may be a key ingredient to explain bubbles as argued by Brunnermeier (2008). Ceteris paribus, chartists tend to exert a destabilizing influence on the price of financial assets, whereas the presence of fundamentalists is stabilizing.

In spite of its simplicity, our model features a variety of interesting aspects. The presence of self-reinforcing mechanisms in the aggregate dynamics allows for the existence of nontrivial multiple equilibria. In the economy, there are two sources of instability deriving from the feedback effects between real and financial markets via Tobin's q (as in Blanchard's 1981 seminal model) *and* from the endogenous aggregate sentiment dynamics produced by the interaction of heterogeneous agents in the stock markets. We prove that the dynamical system describing the evolution of the economy always has either a single steady state (with uniformly distributed agents) or three steady states (the equilibrium with uniformly distributed agents, one with a dominance of chartists and one where fundamentalists dominate), but even though various subdynamics of the model can be stable (at either the uniform or the fundamentalist of the three steady states), the complete system may be repelling around all of its equilibria. Given the complexity of the 4D nonlinear system, we use numerical simulations to explore the properties of the economy. Our results show that the dynamical system describing the economy is generally bounded: all trajectories remain in an economically meaningful subset of the state space. In this sense, unfettered markets with possibly accelerating real-financial feedback mechanisms may have some in-built stabilizing mechanism (based on aggregate sentiment dynamics) that prevent the economy from moving along an infeasible path. Nonetheless, real-financial interactions and sentiment dynamics *do* amplify exogenous shocks and may generate persistent fluctuations and the associated welfare losses. Indeed, despite the relatively simple behavior of the subsystem describing the evolution

[1] Indeed, the central equation of the WHL approach which describes the dynamics of population shares might be provided from game theoretic foundations along the lines of Brock and Durlauf (2001), Blume and Durlauf (2003) and He et al. (2016). We are grateful to Tony He for pointing this link out to us.

of output without heterogeneous beliefs, the dynamics of the complete system can exhibit somewhat irregular fluctuations.

Finally, it is worth stressing that, unlike in most of the current macroeconomic literature, our model is based on a dynamic disequilibrium approach in which the evolution of the variables over time is described by gradual adjustment processes, and no equilibrium condition is imposed a priori. This dynamic disequilibrium approach – discussed in detail in Chiarella and Flaschel (2000) and Chiarella et al. (2005) – seems like a natural complement to the behavioral WHL approach to expectation formation, see also Chiarella et al. (2009).

The remainder of the paper is organized as follows. In section 2 we lay out the macroeconomic framework. Section 3 derives the main analytical results concerning the dynamics of the economy. Section 4 illustrates the properties of the model by means of numerical simulations. Section 5 analyzes some policy measures. Section 6 concludes, and the proofs of all Propositions are in the Appendix.

2 The Model

2.1 Core Real-Financial Interactions

We consider a closed economy consisting of households, firms and a monetary authority. We assume that households are the sole owners of the firms' stocks or equities E which represent claims on the firms' physical capital stock K.

Unlike in Chiarella and Flaschel (2000) and Chiarella et al. (2005), we abstract from the "Metzlerian" inventory accelerator mechanism in the modeling of goods market dynamics[2] in order to focus on the interaction emerging from a stock market driven by aggregate sentiment dynamics and the macroeconomy. We assume instead that aggregate production evolves according to a dynamic multiplier specification[3]

$$\dot{Y} = \beta_y (Y^d - Y), \tag{1}$$

where Y represents aggregate output, Y^d aggregate demand and $\beta_y > 0$ the speed of adjustment of output to market disequilibrium as in the seminal paper by Blanchard (1981).

Let p_e denote the equity price, and p the price of capital goods. The Brainard and Tobin (1968) q ratio is then given by

$$q = p_e E / pK. \tag{2}$$

Without loss of generality, we normalize the price of output to one, $p = 1$, and assume further that the horizon of our analysis is sufficiently short as to guarantee that both E and K are constant

[2] These potentially destabilizing macroeconomic channels arising from the real side of the economy could be however reincorporated in the present framework in a straightforward manner.

[3] For any dynamic variable z, $\dot{z}$ denotes its time derivative, $\hat{z}$ its growth rate and z_o its steady state value.

magnitudes. We normalize K assuming $K = 1$. As a result, changes in q are determined solely by changes in p_e. Further, we assume that financial markets dynamics affect the real economy via the impact of Tobin's q on aggregate demand. Hence, aggregate demand is given by:

$$Y^d = a_y Y + A + a_q(p_e - p_{eo})E, \tag{3}$$

where $a_y \in (0, 1)$ is the propensity to spend, A is autonomous expenditure, and $a_q > 0$ measures the responsiveness of output demand to the difference between the actual value of stocks and their steady state value p_{eo}. Inserting equation (3) into equation (1) yields

$$\dot{Y} = \beta_y[(a_y - 1)Y + a_q(p_e - p_{eo})E + A]. \tag{4}$$

In addition to E, we assume that there are two more financial assets, namely, as is customary, money M and short-term fix-price bonds B.[4] For simplicity we assume that the monetary authorities fix the interest rate on the bonds B at the level r, accommodating the households' excess demand for money. This allows us to abstract from the traditional interest rate effect on aggregate output so central in New Neoclassical Consensus models (see e.g. Woodford, 2003) and focus in isolation on the stock price effects under aggregate sentiment dynamics, as discussed below.

Since in our economy profits are assumed to be entirely redistributed to firms' owners (households) as dividends, the expected return on equity ρ_e^e is

$$\rho_e^e = \frac{bY}{p_e E} + \pi_e^e. \tag{5}$$

where $b \geq 0$ is the profit share, $bY/(p_e E)$ is the dividend rate, and π_e^e represents the *average*, or *market* expectation of future capital gains $\pi_e = \dot{p}_e/p_e$, i.e., the growth rate of equity prices.

Finally, we assume that the equity market is imperfect due to information asymmetries, adjustment costs, and/or institutional restrictions, so that the equity price p_e does not move instantaneously to clear the market. More specifically, we assume that

$$\hat{p}_e = \beta_e(\rho_e^e - \rho_{eo}^e) = \beta_e\left(\frac{bY}{p_e E} + \pi_e^e - \rho_{eo}^e\right), \tag{6}$$

where β_e describes the adjustment speed at which the equity price reacts to discrepancies between the expected rate of return on equity and its steady state value, ρ_{eo}^e, which is assumed to be a given and strictly positive parameter in the model. As we will discuss below, while equation (6) seems rather stylized at first sight, it actually describes a complex mechanism due to the intrinsic nonlinearity of the dynamics of the capital gain expectations π_e^e.

[4]See Charpe et al. (2011) for an explicit analysis and also for a critique of allowing governments to issue a perfectly liquid asset B, with a given unit price.

2.2 Aggregate Sentiment Dynamics

Based on the empirical findings of Frankel and Froot (1987, 1990) and Allen and Taylor (1990), and the extensive literature they sparked, we assume that traders in financial markets use various types of heuristics when forming their expectations about future asset price developments. To be specific, we assume that traders in the stock market use either a *fundamentalist* rule (denoted by the superscript f) according to which they expect capital gains to converge back to their long-run-steady state value (assumed to be zero), i.e.

$$\dot{\pi}_e^{e,f} = \beta_{\pi_e^{e,f}}(0 - \pi_e^e), \tag{7}$$

or a *chartist* rule (denoted by c) given by

$$\dot{\pi}_e^{e,c} = \beta_{\pi_e^{e,c}}(\hat{p}_e - \pi_e^e), \tag{8}$$

where $\beta_{\pi_e^{e,f}}$ and $\beta_{\pi_e^{e,c}}$ are the speed of adjustment parameters of the two heuristics-based forecasting rules, respectively.

Suppose that at any given time a share $\nu_c \in [0, 1]$ of the population consists of financial market participants using the chartist rule and a share $\nu_f = 1 - \nu_c$ consists of traders using the fundamentalist rule. The law of motion of aggregate capital gain expectations can then be expressed as

$$\begin{aligned}\dot{\pi}_e^e &= \nu_c(\beta_{\pi_e^{e,c}}(\hat{p}_e - \pi_e^e)) + (1 - \nu_c)(\beta_{\pi_e^{e,f}}(0 - \pi_e^e)) \\ &= \nu_c\beta_{\pi_e^{e,c}}\hat{p}_e - (\nu_c\beta_{\pi_e^{e,c}} + (1 - \nu_c)\beta_{\pi_e^{e,f}})\pi_e^e.\end{aligned} \tag{9}$$

According to this equation the evolution of *aggregate*, *market-wide* expectations of future capital gains is given by the weighted average of the *change* of the expectations, or forecasts, resulting from the use of the fundamentalist or chartist forecasting rule. Further, as the interplay between fundamentalists and chartists is well understood in the literature (see e.g. Hommes, 2006), we assume in the following that $\beta_{\pi_e^{e,c}} = \beta_{\pi_e^{e,f}} = \beta_{\pi_e^e}$ for simplicity and in order to focus on other rather new channels which emerge from the aggregate sentiments dynamics.[5] Then, the above equation becomes

$$\dot{\pi}_e^e = \beta_{\pi_e^e}(\nu_c\hat{p}_e - \pi_e^e). \tag{10}$$

Observe that in equations (7) and (8), both fundamentalists and chartists are assumed to use aggregate expectations π_e^e as the reference value for the updating of their own expectations. This specification is meant to reflect Keynes' (1936, p.156) famous view of the stock market as a process of choosing the most beautiful model in a beauty contest, where the winner is the one who has selected

[5] Further, by assuming that the two heuristics are updated with the same speed or frequency we are able to focus on the implications of the use of the different heuristics *per se*. We think that the latter are more relevant behaviorally and capture the most relevant part of heterogeneity in the stock market.

the model who is chosen as the most beautiful by the (relative) majority of players. Winning requires guessing the views of the other players.

We endogenize the variable ν_c by adopting the aggregate sentiment dynamics approach by Weidlich and Haag (1983) and Lux (1995) as recently reformulated in Franke (2012, 2014), which provides behavioral microfoundations to agents' attitudes in financial markets. Accordingly, agents decide whether to take either a chartist, or a fundamentalist stance depending on the current status of the economy (captured by the key variables Y, p_e), on expectations on the evolution of financial gains (π_e^e), and – crucially – on the current composition of the market (captured by the variable x, defined below).

Formally, suppose that there are $2N$ agents in the economy. Of these, N_c use the chartist forecasting rule and N_f use the fundamentalist rule, so that $N_c + N_f = 2N$. Following Franke (2012) we describe the distribution of chartists and fundamentalists in the market by focusing on the *difference* in the size of the two groups (normalized by $2N$). To be precise, we define

$$x \equiv \frac{N_c - N_f}{2N}. \tag{11}$$

Therefore $x \in [-1, +1]$, $\nu_c = N_c/N = \frac{1+x}{2}$ and $\nu_f = N_f/N = \frac{1-x}{2}$, and $x > 0$ indicates a dominance of chartists, while $x < 0$ implies a majority of fundamentalists at any given point in time.

Let $p^{f\to c}$ be the transition probability that a fundamentalist becomes a chartist, and likewise for $p^{c\to f}$. The change in x depends on the relative size of each population multiplied by the relevant transition probability. Given the continuous time setting of the present framework, we take the limit of $\dot{x}$ as the population N becomes very large as in Franke (2012), so that the intrinsic noise from different realizations at the individual level can be neglected. Then:

$$\dot{x} = (1-x)p^{f\to c} - (1+x)p^{c\to f}. \tag{12}$$

The key behavioral assumption concerns the determinants of transition probabilities: we suppose that they are determined by a *switching index*, s, which captures the expectations of traders on market performance. An increase in s raises the probability of a fundamentalist becoming a chartist, and decreases the probability of a fundamentalist becoming a chartist. More precisely, assuming that the relative changes of $p^{c\to f}$ and $p^{f\to c}$ in response to changes in s are linear and symmetric:

$$p^{f\to c} = \beta_x \exp(a_x s), \tag{13}$$

$$p^{c\to f} = \beta_x \exp(-a_x s). \tag{14}$$

The switching index depends positively on market composition (capturing the herding component of agents' behavior) and on economic activity; and negatively on deviation of the market value of the capital stock and of the average capital gain expectations from their respective steady state values. As in Franke and Westerhoff (2014), this can be written as:[6]

$$s = s_x x + s_y (Y - Y_o) - s_{p_e} (p_e - p_{eo})^2 - s_{\pi_e^e} (\pi_e^e)^2. \tag{15}$$

Deviations of share prices and capital gain expectations from their steady state values tend to favor fundamentalist behavior as doubts concerning the macroeconomic situation become widespread. This can be interpreted as a change in the state of confidence, whereby agents believe that increasing deviations from the steady state eventually become unsustainable.

The economy is described by the 4D dynamical system consisting of equations (4), (6), (10), and (12), where ν_c results from equation (11) and $p^{f \to c}$ and $p^{c \to f}$ are given by equations (13) and (14), i.e.

$$\dot{Y} = \beta_y [(a_y - 1)Y + a_q (p_e - p_{eo})E + A], \tag{16}$$

$$\dot{p}_e = \beta_e \left(\frac{bY}{p_e E} + \pi_e^e - \rho_{eo}^e \right) p_e, \tag{17}$$

$$\dot{\pi}_e^e = \beta_{\pi_e^e} \left(\frac{1+x}{2} \beta_e \left(\frac{bY}{p_e E} + \pi_e^e - \rho_{eo}^e \right) - \pi_e^e \right), \tag{18}$$

$$\dot{x} = (1-x)\beta_x \exp(a_x s) - (1+x)\beta_x \exp(-a_x s). \tag{19}$$

and s is given by equation (15).

The model provides a simple but general framework to capture some key real-financial interactions, and the feedback between economic variables and agents' attitudes and expectations.

3 Local Stability Analysis

Let $\mathbf{z} = (z_1, z_2, \ldots, z_n)$. For any dynamical system $\dot{\mathbf{z}} = g(\mathbf{z})$, a steady state is defined as the state in which $\dot{\mathbf{z}} = \mathbf{0}$. Then, it is straightforward to prove the following Lemma:[7]

[6] We adopt a quadratic specification only for the sake of simplicity and expositional clarity. All of our results can be extended to more general switching index functions $s = s(x, Y, p_e, \pi_e^e)$, with $s'_x > 0, s'_y > 0, s'_{p_e} < 0$, and $s'_{\pi_e^e} < 0$, where s'_i is the derivative of the function $s(\cdot)$ with respect to i.

[7] Recall that the steady state value of the expected return on equity, ρ_{eo}^e, is assumed to be a parameter of the model. Therefore Lemma 1 can be interpreted as identifying a one-parameter *family* of steady states.

Lemma 1 *The dynamical system formed by of equations (16), (17), (18), and (19) always has the following steady state solution:*

$$Y_o = \frac{A}{1-a_y}, \quad (20)$$

$$p_{eo} = \frac{bA}{(1-a_y)\rho^e_{eo}E}, \quad (21)$$

$$\pi^e_{eo} = 0, \quad (22)$$

$$x_o = 0. \quad (23)$$

While Lemma 1 defines the unique steady state values of the variables Y, p_e and π^e_e, which will always exist independently of the steady state values of x, it does not rule out the existence of further steady states which however may arise solely due to the nonlinearity of the population dynamics.

In the following, we shall analyze the local stability of various subparts of the model separately. This exercise allows us to understand the sources of instability (and the stabilizing forces) in the economy before exploring the complete model by means of numerical simulations.

3.1 Core Real-Financial Interactions

We begin by analyzing the interaction between the macroeconomy and the stock market under the assumption of constant capital gains expectations $\pi^e_e = \bar{\pi}^e_e = 0$. This assumption reduces our macroeconomic model to a 2D core system formed by equations (16) and (17).[8]

Proposition 1 *The dynamical system formed by equations (16) and (17) has a unique steady state:* $Y_o = \dfrac{A}{1-a_y}$ *and* $p_{eo} = \dfrac{bA}{(1-a_y)\rho^e_{eo}E}$ *with the following stability conditions:*[9]

(i) if $\dfrac{a_q b}{1-a_y} < \rho^e_{eo}$*, then the steady state is (asymptotically) stable;*

(ii) if $\dfrac{a_q b}{1-a_y} > \rho^e_{eo}$*, then the steady state is an (unstable) saddle point.*

In this model, Tobin's q plays a key role in breaking down the dichotomy between the real and financial components of the economy. An increase in p_e has a positive effect on the rate of change of output, but a negative effect on the expected return on equity. Similarly, real markets influence asset markets via the role of output as the main determinant of the rate of profit of firms, and thus of the

[8] The proofs of all Propositions can be found in Appendix A.

[9] Given the fact that this dynamical subsystem is linear, local stability implies also global stability.

rate of return on real capital. A higher output level has a positive effect on $\hat{p}_e$, but a negative effect on the rate of change of output.[10]

Proposition 1 concerns the interaction of real and financial adjustment processes and does not depend on the presence of capital gain expectations, which are introduced next.

3.2 Real-Financial Interactions with Constant Heterogeneous Beliefs

As a next step, we introduce heterogeneous expectations in the basic 2D macroeconomic model while assuming agents' attitudes, and thus ν_c, to be exogenously given. This allows us to analyze the effect of expectations on the dynamics of real financial interactions. Not surprisingly, introducing heterogeneity in agents' expectations, may play a destabilizing role in the economy.

The next Proposition characterizes the dynamics of the 3D model when $\beta_e < 1$.

Proposition 2 *Consider the dynamical system formed by equations (16), (17) and (18) and let* $\beta_e < 1$*. For any* $\nu_c \in [0, 1]$*, at the steady state given by equations* (20)-(22)*:*

(i) if $a_q b/(1 - a_y) < \rho^e_{eo}$ *then the system is locally (asymptotically) stable,*

(ii) if $a_q b/(1 - a_y) > \rho^e_{eo}$ *then the system is unstable.*

Observe that Proposition 2 holds for *any* $\nu_c \in [0, 1]$, and so it provides some important insights on the dynamics of the system formed by equations (16), (17) and (18). Interestingly, as in the 2D system, the stability of the steady state depends on the relation between a_q, $b/(1 - a_y)$ and ρ^e_{eo}. In the case where $\beta_e < 1$ the introduction of heterogeneous expectations (chartist and fundamentalist) changes neither the number of steady states, nor their stability properties.

The validity of Proposition 2 (the irrelevance of the *exogenous* share of chartists and fundamentalists in the markets for the stability of the system) depends of course on $\beta_e < 1$. The following Proposition applies for the case where $\beta_e > 1$:

Proposition 3 *Consider the dynamical system formed by equations (16), (17) and (18). Further, let*

$$\nu_c^* = \frac{\beta_y(1 - a_y) + \beta_e \rho^e_{eo} + \beta_{\pi^e_e}}{\beta_{\pi^e_e}\beta_e} = \frac{\beta_y(1 - a_y)}{\beta_{\pi^e_e}\beta_e} + \frac{\rho^e_{eo}}{\beta_{\pi^e_e}} + \frac{1}{\beta_e}.$$

[10] It is also interesting to consider briefly the dynamics of the model under perfect foresight i.e. $\pi^e_e = \hat{p}_e$, see e.g. Turnovsky (1995). In this case, the population dynamics and a separate law of motion for share price expectations are redundant, and the law of motion of share prices is:

$$\hat{p}_e = \beta_e \left(\frac{bY}{p_e E} + \hat{p}_e - \rho^e_{eo} \right) \iff \hat{p}_e = \frac{\beta_e}{1 - \beta_e} \left(\frac{bY}{p_e E} - \rho^e_{eo} \right).$$

It is straightforward to confirm by a standard local stability analysis that if $\beta_e < 1$, the conditions for local stability of the steady state are the same as those postulated in Proposition 1.

Under the assumption that $\beta_e > 1$*, if* $\nu_c^* \in [0,1]$ *and* $\nu_c > \nu_c^*$*, then the steady state given by equations* (20)-(22) *is unstable.*

According to Proposition 3, if $\beta_e > 1$ and the share of chartists in the market ν_c is beyond the endogenously determined threshold value ν_c^*, the destabilizing influence of the chartists will lead to macroeconomic instability, as higher capital gains expectations will lead to higher share prices and higher output which will in turn translate into higher capital gain expectations. Accordingly, ν_c^* represents an endogenous upper bound on ν_c above which the system loses stability to exogenous shocks. Higher values for $\beta_{\pi_e^e}$ and/or β_e lower ν_c^*, making the whole system more prone to overall instability.

The previous analysis has only described the dynamics of the economy in a neighborhood of the steady state characterized by equations (20), (21) and (22). The introduction of aggregate sentiments, and by extension of a varying influence of chartist expectations, is likely to lead to explosive dynamics, for instance if either the speed of adjustment in financial markets β_e or the coefficient $\beta_{\pi_e^e}$ are sufficiently high. This explosiveness may be tamed far off the steady state through the activation of nonlinear policy measures or, as we will discuss below, by intrinsic nonlinear changes in behavior, thus ensuring that all trajectories remain within an economically meaningful bounded domain.

We will explore the global dynamics of the system with aggregate sentiment dynamics by numerical simulations in section 4 below. In the next section, we explore the possibility that endogenous changes in the agents' populations, ν_c, reduce the influence of chartists far off the steady state and thereby create turning points in the evolution of capital gain expectations.

3.3 Real-Financial Interactions with Endogenous Aggregate Sentiments

As previously mentioned, while Lemma 1 characterizes a particular steady state solution that always exists, other steady states may also exist for particular parameter constellations. The following proposition focuses on the role of the parameters s_x and a_x for the emergence of multiple steady states.

Proposition 4 *Consider the dynamical system formed by equations* (16)-(19)*. If* $s_x \leq 1/a_x$ *then the steady state given by equations (20)-(23) is unique. If* $s_x > 1/a_x$*, then there are two additional steady state values for* x_o*: one characterized by a dominance of fundamentalists,* e_f*, and one where chartists dominate,* e_c*.*

The intuition behind Proposition 4 is captured in Figure 1, which illustrates the number of steady states of x for different values of a_x and s_x. While the steady state is unique if $s_x \leq 1/a_x$, there are multiple steady states if $s_x > 1/a_x$. For example, for $s_x = 2/a_x$, there are three steady states: one with a large prevalence of fundamentalists ($x \approx -1$), one with populations of equal size ($x = 0$), and one with a large prevalence of chartists ($x \approx 1$).

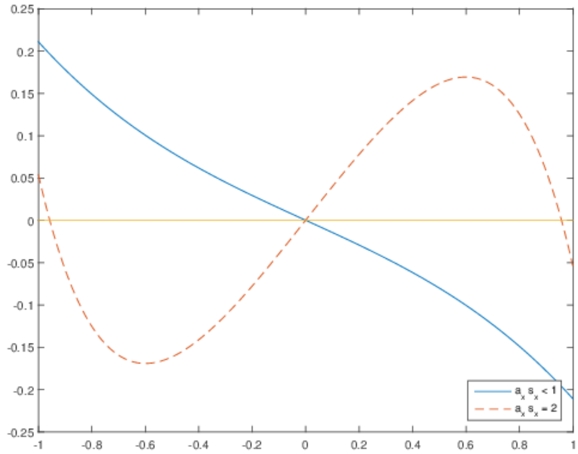

Figure 1: Steady states of population dynamics for different values of a_x and s_x

Before analyzing the dynamics of the complete system numerically in the next section, it is interesting to consider the properties of the opinion dynamics and the expectations part of the model in isolation. We thus assume that output and dividend payments are fixed at their steady state values Y_o and p_{eo} in the rest of this section. By inserting equations (20) and (21) into (18) we get

$$\dot{\pi}_e^e = \beta_{\pi_e^e} \left[\beta_e \frac{1+x}{2} - 1 \right] \pi_e^e, \tag{24}$$

and from equation (15),

$$s = s_x x - s_{\pi_e^e} (\pi_e^e)^2. \tag{25}$$

Inserting this expression in equation (19) yields

$$\dot{x} = \beta_x \left[(1-x) \exp(a_x(s_x x - s_{\pi_e^e}(\pi_e^e)^2)) - (1+x) \exp(-a_x(s_x x - s_{\pi_e^e}(\pi_e^e)^2)) \right]. \tag{26}$$

A quick glance at equation (24) makes clear that the condition $\dot{\pi}_e^e = 0$ can be fulfilled either when $\pi_e^e = 0$, or when $\pi_e^e \neq 0$. This means that the multiplicity of steady states arises here not only through the nonlinear equation (26), as discussed in Proposition 4, but also through equation (24). The next two Propositions deal with the case with $\pi_{eo}^e = 0$.

Proposition 5 *Consider the dynamical system formed by equations (24) and (26). Then:*

(i) if $s_x \in (0, 1/a_x)$, $e_o = (\pi_{eo}^e, x_o) = (0, 0)$ is the only steady state with $\pi_{eo}^e = 0$;

(ii) if $s_x > 1/a_x$, then two additional steady states exist, $e_f = (0, x_o^f)$ and $e_c = (0, x_o^c)$ with $x_o^f < 0$ and $x_o^c > 0$, respectively.

In other words, if the aggregate sentiment dynamics display a strong self-reinforcing behavior, multiple equilibria emerge in which either fundamentalists or chartists dominate. The next Proposition describes some stability properties of the steady states identified in Proposition 5.

Proposition 6 *Consider the dynamical system formed by equations (24) and (26). Then:*

(i) Let $s_x \in (0, 1/a_x)$. If $\beta_e > 2$, then $e_o = (\pi_{eo}^e, x_o) = (0, 0)$ is an unstable saddle point. If $\beta_e < 2$, then e_o is locally asymptotically stable.

(ii) Let $s_x > 1/a_x$. The steady state $e_o = (0, 0)$ is unstable. The steady states $e_c = (0, x_o^c)$ and $e_f = (0, x_o^f)$ are locally asymptotically stable if and only if $(1 + x_o^c)\beta_e < 2$ and $(1 + x_o^f)\beta_e < 2$, respectively.

By Proposition 6, it follows that sentiment dynamics may lead to local instability. This raises the issue of the global viability of the dynamical system formed by equations (24) and (26). It is difficult to draw any definite analytical conclusions on this issue and we shall analyze it in detail by means of numerical methods in the next section. To be sure, opinion dynamics do incorporate a stabilizing mechanism far off the steady state(s), as x always points inwards at the border of the $x-$domain $[-1, 1]$. Yet the global viability of the system will ultimately depend on the properties of the *interaction* between market expectations and opinion dynamics.

Consider, for example, case (i) of Proposition 6 and suppose that $\beta_e > 2$, so that $e_o = (0, 0)$ is unstable. It can be shown that there must be an upper and a lower turning point for π_e^e in the economically relevant phase space $[-1, 1] \times [-\infty, +\infty]$. For suppose, by way of contradiction, that π_e^e tends to infinity. By equation (26) it follows that $\dot{x}$ becomes negative and approaches $-\infty$. But then as x approaches -1, by equation (24) it follows that $\dot{\pi}_e^e$ becomes negative, which contradicts the starting assumption. A similar argument rules out the possibility that π_e^e becomes infinitely negative and therefore there must always be an upper or lower turning point for capital gain inflation or deflation. This implies that all trajectories stay within a compact subset of the phase space and the interaction between expectation dynamics and herding mechanism would thus be bounded, if taken by itself.[11]

It is also worth noting that the dynamical system formed by equations (24) and (26) features two additional steady states for the case where $\pi_{eo}^e \neq 0$, $e_+ = (\pi_{eo}^+, x_o^+)$ and $e_- = (\pi_{eo}^-, x_o^-)$, with

$$x_o = \frac{2}{\beta_e} - 1, \quad \text{and} \quad \pi_{eo}^e = \pm\sqrt{\frac{s_x\left(\frac{2}{\beta_e} - 1\right) - \ln\left(\frac{1}{\beta_e - 1}\right)/2a_x}{s_{\pi_e^e}}}.$$

[11] Given the instability of the steady state, this suggests the existence of a limit cycle.

These steady states[12] are locally asymptotically stable if

$$a_x s_x < \frac{1}{1 - x_o^2}.$$

4 Numerical Simulations

This section examines the properties of the model using numerical simulations.[13] We first illustrate the effects of capital gain expectations on the dynamics of Tobin's q using the 3D model comprising the output equation (16), the share price equation (17) and the capital gains equation (18) and then, in a second step, investigate the complete 4D dynamical system including the endogenous dynamics of aggregate sentiments.

Table 1: Baseline Parameter Calibration of the 2D model

Autonomous spending	A	0.128
Profit share	b	0.35
Elasticity of aggregate demand to income	a_y	0.8
Elasticity of aggregate demand to Tobin's q	a_q	0.05
Adjustment speed of Tobin's q	β_e	2
Adjustment speed of output	β_y	2
Parameter in population dynamics	a_x	0.8
Steady state capital stock	K_o	1
Steady state equity stock	E_o	1
Steady state population	x_o	0
Steady state expectations	π_{eo}^e	0
Steady state expected capital return	ρ_{eo}^e	0.14
Steady state output capital ratio	$\frac{Y_o}{K_o}$	0.64
Steady state share price	p_{eo}	1.6

The calibration of the 2D model is shown in Table 1. The profit share b is set at 0.35, in line with the long term average in Karabarbounis and Neiman (2014). Based on Bloomberg data from 2000 to 2013, the return on equity (adjusted for R&D spending) is on average 14 percent in the United States, so we set $\rho_{eo}^e = 0.14$. Brooks and Ueda (2011) argue that Tobin's q has been fluctuating between 1.4 and 1.7 over the period 1990 to 2013. We set its steady state value within this range at 1.6. It follows that the steady state output capital ratio is $\frac{Y_o}{K_o}$ is 0.64. Mukherjee and Bhattacharya (2010) estimate that, in 18 OECD countries, the propensity to spend out of income fluctuates between 0.6 and 1.2. We set a_y equal to 0.8. Therefore by equation (20) the autonomous spending component $A = Y_o(1 - a_y)$ equals 0.128.

[12] For these steady states to be economically meaningful the following conditions must hold: $x_o = \left[\frac{2}{\beta_e} - 1\right] \in [-1, 1]$ and $2a_x s_x \left(\frac{2}{\beta_e} - 1\right) \geq \ln\left(\frac{1}{\beta_e - 1}\right)$.

[13] The numerical simulation are performed using the SND package (Chiarella et al., 2002).

The elasticity of aggregate demand to Tobin's q, a_q, is set equal to 0.05. The dynamic output multiplier, β_y, and the speed of adjustment of Tobin's q, β_e, are both set equal to 2. Unless otherwise stated, the experiment considered in this section is a 1 percent shock on output with no auto-regressive component. All diagrams reporting simulation results display the deviation of variables from their steady state value in percent, unless otherwise stated.

Figure 2 illustrates the dynamic adjustments of the 3D model consisting of the output equation (16), the share price equation (17) and the capital gains expectations equation (18) for $\beta_{\pi_e^e} = 0$, $\beta_{\pi_e^e} = 0.2$ and $\beta_{\pi_e^e} = 4$.[14] In all cases, the parameter a_q is small enough (0.05) to ensure that the determinant is positive, and $\nu_c = 0.5$, which corresponds to $\nu_c = \frac{1+x}{2}$ with $x_o = 0$ in line with the 4D model calibration presented below.

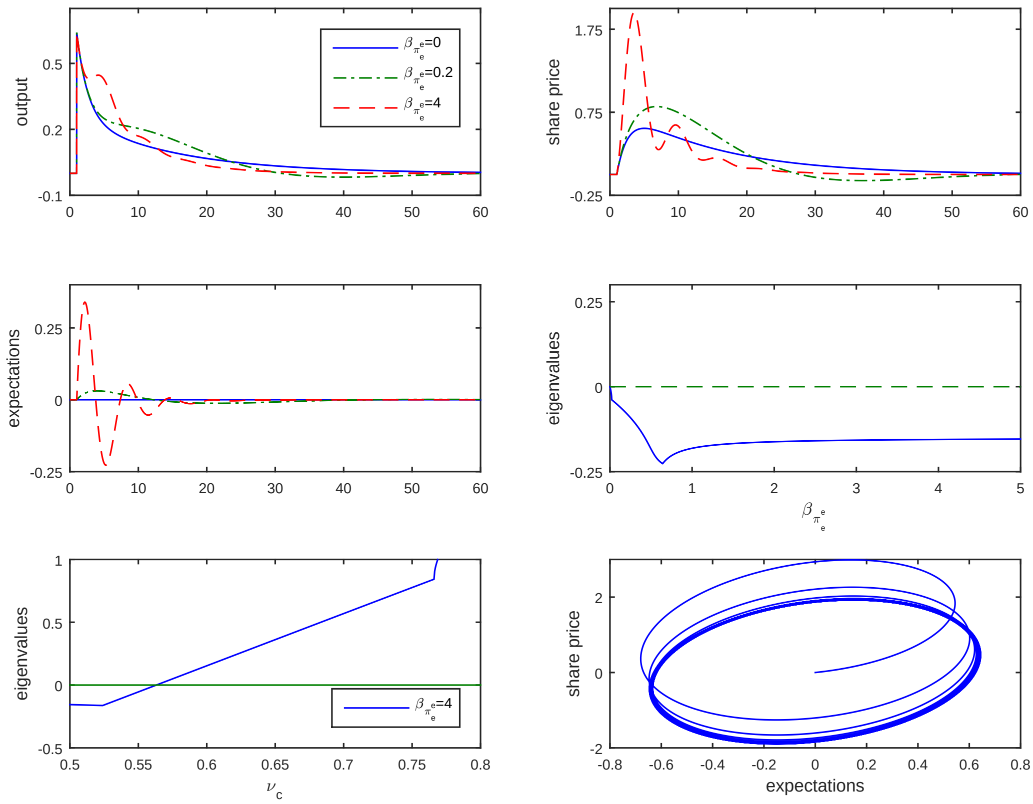

Figure 2: Dynamic responses following a positive one-percent output shock and maximum eigenvalues for the 3D model (Y, p_e, π_e^e).

If $\beta_{\pi_e^e} = 0$ the dynamics of the system is rather simple: the positive shock on output is followed by an increase in share price p_e as the expected return on the capital stock ρ_e^e rises. The dynamics of p_e is hump-shaped as the increase in the share price is modest at the beginning and does not

[14] It is worth noting that the simulations based on $\beta_{\pi_e^e} = 0$ represent the dynamics of the 2D model and are thus related to the analytical stability conditions described in Proposition 1.

immediately reduce the return on capital. When the equity price rises enough to lower the return on equity, the economy converges back to its steady state. If $\beta_{\pi_e^e} = 0.2$ the model displays an oscillatory behavior after the aggregate demand shock due to the activated feedback channel between π_e^e and p_e, as capital gains expectations amplify *both* the increase in the price of equity initiated by a higher return on capital *and* the decline in the price of equity when the rate of return diminishes due to a fall in the price of equities. As the share price p_e undershoots its steady state value it generates further oscillations in aggregate output. These fluctuations are not, however, self-sustaining and the economy returns to the steady state.

The dashed red line in Figure 2 corresponds to the case where the speed of adjustment in capital gains expectations $\beta_{\pi_e^e}$ is increased from 0.2 to 4 with $a_q = 0.05$, which implies that the stability conditions in Proposition 2 continue to hold. As the (negative) trace of the corresponding Jacobian matrix declines with $\beta_{\pi_e^e}$, the model is stable but displays oscillations around the trajectory converging back to the steady state. As shown by the solid blue line in the second row, second column graph, the maximum real part of the eigenvalues is always negative for all values of the speed of adjustment of expectations, $\beta_{\pi_e^e}$. Raising $\beta_{\pi_e^e}$ increases the amplitude of the fluctuations of the expectations but $\beta_{\pi_e^e}$ has a stabilizing effect on output. Adaptive expectations are inherently stable given the influence of the equity price on the real return on equity. In contrast, the graphs in the third row of Figure 2 highlight the importance of the parameter ν_c for the stability of the 3D model (Y, p_e, π_e^e) as discussed in Proposition 3. In the left panel of the third row, the maximum real part of the eigenvalues turns positive for values of ν_c strictly larger than 0.56. Increasing the value of ν_c at 0.56 while keeping $\beta_{\pi_e^e} = 4$ produces self-sustaining oscillations of the model, as shown in the right panel of this figure.[15]

Figure 3 illustrates the case of multiple steady states described at the end of section 3 for the subsystem (π_e^e, x) where the steady state for expectations and population are different from zero. In the upper two panels we set $\beta_e = 1.15$, $s_x = 1.5$ and $a_x = 1$ (so that $s_x > 1/a_x$), which implies $x_o = \dfrac{2}{\beta_e} - 1 = 0.74$ and $\pi_{eo}^e = 0.57$. Following a positive shock on the population variable x, the population dynamics fluctuates around its steady state value following dampening oscillations. In this case, the prevalence of chartist expectations (as $x_o = 0.74 > 0$) does not lead to explosive dynamics due to the relatively slow adjustment in the price of shares. On the contrary, as illustrated in the two lower panels in Figure 3, increasing the speed at which the price of shares adjusts, $\beta_e = 1.5$, makes the steady state $e_+ = (\pi_{eo}^+, x_o^+)$ locally unstable. Following the shock, the population features an explosive oscillatory dynamic response until the excess volatility in the financial markets leads agents to switch towards fundamentalist expectations. The economy then converges towards a stable equilibrium dominated by fundamentalists where capital gains expectations are zero.

The next simulation in Figure 4 considers the influence of the aggregate sentiment dynamics on the price of capital and the financial multiplier by setting $\beta_x = 0.75$. The choice of $a_x = 0.8$ and

[15] Given the parametrization of the model, while the value of ν_c^* is 0.585, the cut-off value for instability is 0.5635. These values corroborate Proposition 3 as identifying a *sufficient* condition for local instability.

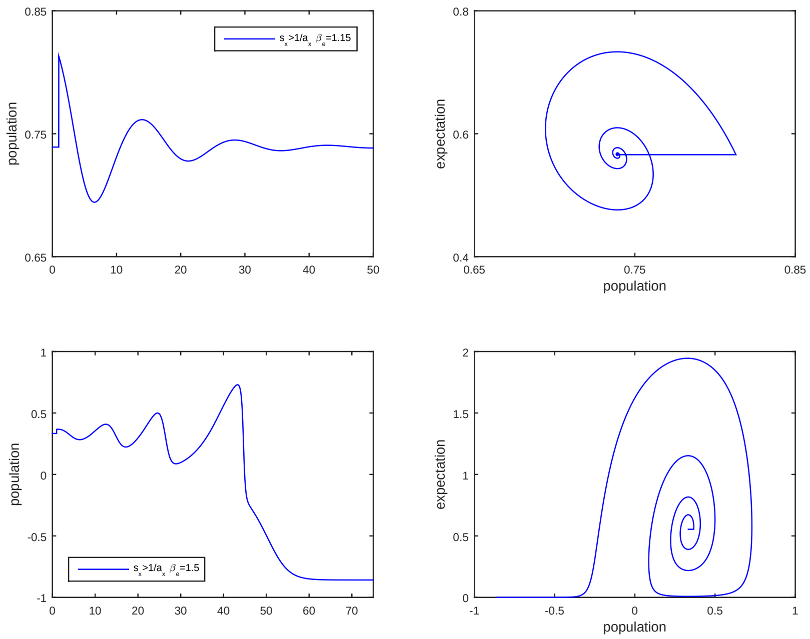

Figure 3: Dynamic response for the 2D model (π_e^e, x) following a positive shock on the population dynamics in the multiple (non-zero) steady state case.

$s_x = 0.8$ corresponds to the case of a unique steady state with $x_o = 0$ for the relative population of fundamentalists and chartists. We now set $s_y = 20$ in order to incorporate the impact of real economic activity on the aggregate sentiments of the agents. As a first step, we focus on a linear version of the opinion switching index abstracting from the influence of price and capital gains volatility by setting $s_{p_e} = s_{\pi_e^e} = 0$ (we analyze the general case with $s_{p_e} \neq 0$ and $s_{\pi_e^e} \neq 0$ in Figure 7 below). The rest of the parameters are similar to those of the dashed green line in Figure 2 ($\beta_{\pi_e^e} = 4$). Figure 4 compares the 3D model just discussed (solid blue line) with the 4D model (green line).

As Figure 4 clearly shows, the addition of the population dynamics generates larger fluctuations in output and equity prices. Following a positive output shock, the increase in chartist population further raises capital gain expectations, which further increases the expected returns on equity and the demand for equity. The dashed-dotted red line corresponds to the 4D model where the self-reference parameter s_x in the aggregate sentiment index is increased from 0.8 to 1. This value of s_x still generates a unique steady state ($x_o = 0$) of the population variable. But the population dynamics now exhibits larger fluctuations between -0.2 and 0.3. These larger fluctuations translate into wider oscillations in capital gains expectations, share prices, and economic activity, with the reversal of expectations towards fundamentalism generating a decline in output by 6 percent.

Given that the stability conditions cannot be derived analytically for the 4D model, the interpretation of the numerical simulations is indicative only. In order to interpret them recall that Proposition 6

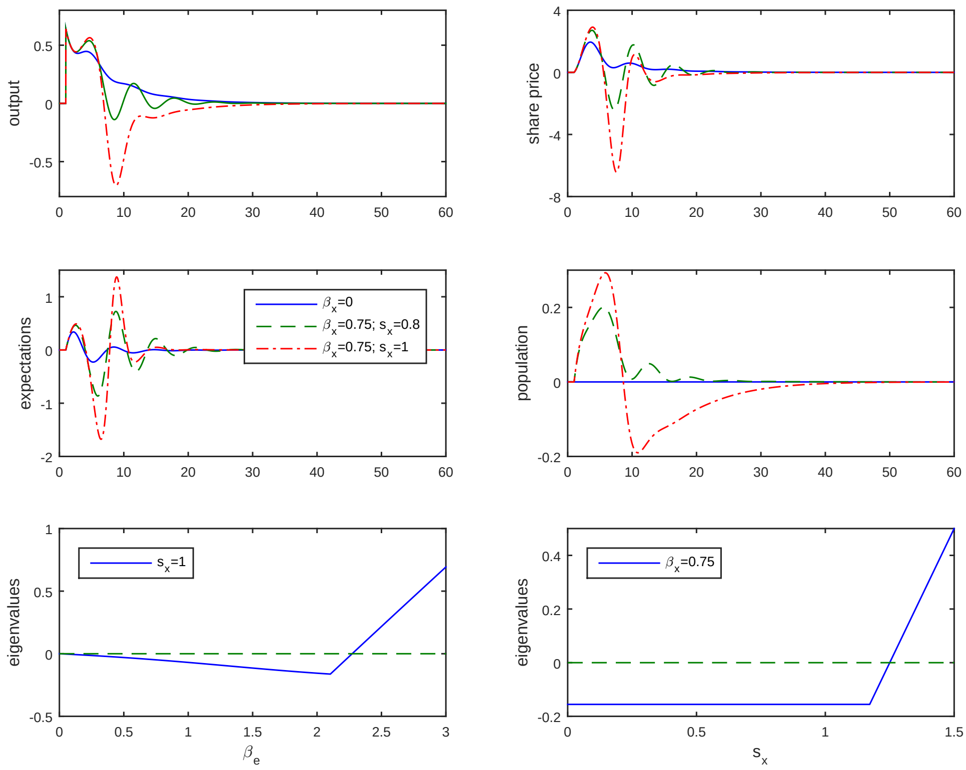

Figure 4: Dynamic adjustments to a one percent output shock in the 3D model (Y, p_e, π_e^e) and the 4D model (Y, p_e, π_e^e, x) (first two rows) and maximum eigenvalue diagrams (last row)

stated that the 2D model formed by equations (24) and (26) has a unique steady state if $s_x \in (0, 1/a_x)$ and is stable if $\beta_e < 2$. Similarly, as shown in section 3.2 above, the value of β_e affects the stability of the 3D dynamical system formed by equations (16)-(18). This suggests that the parameter β_e may play a key role in determining the stability properties of the whole system. The left figure of the third panel in Figure 4 confirms this intuition: it plots the maximum real part of the eigenvalues of the system around the steady state with $x_o = 0$ with respect to different values of β_e. The maximum real part of the eigenvalues turns positive for β_e larger than 2.3, indicating that the 4D model loses stability for large values of β_e. Comparably, the right panel of the third row displays the maximum real part of the eigenvalues of the system around the steady state with $x_o = 0$ for s_x varying between 0 and 1.5. In line with the previous simulation, the system is stable when s_x is smaller than 1.25. The system of equations has a unique steady state towards which the economy converges.

Next we analyze the dynamics of the 4D model assuming $s_{p_e} = s_{\pi_e^e} = 0$ with $s_x = 1.5$. Given $a_x = 0.8$, these parameter values lead to the existence of three steady states, as discussed in Proposition 4. In this case, a negative shock on output steers the population dynamics towards a steady state dominated by fundamentalists at $x_o = -0.65$ as illustrated in Figure 5. Given the parametrization

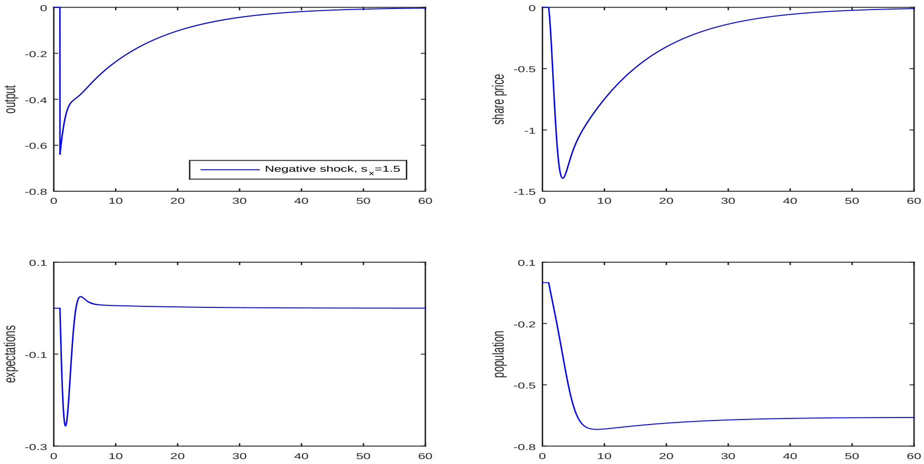

Figure 5: Dynamic adjustments to a negative one percent output shock in the 4D model.

of this simulation, output and share prices converge back to their corresponding steady states in a monotonic manner.

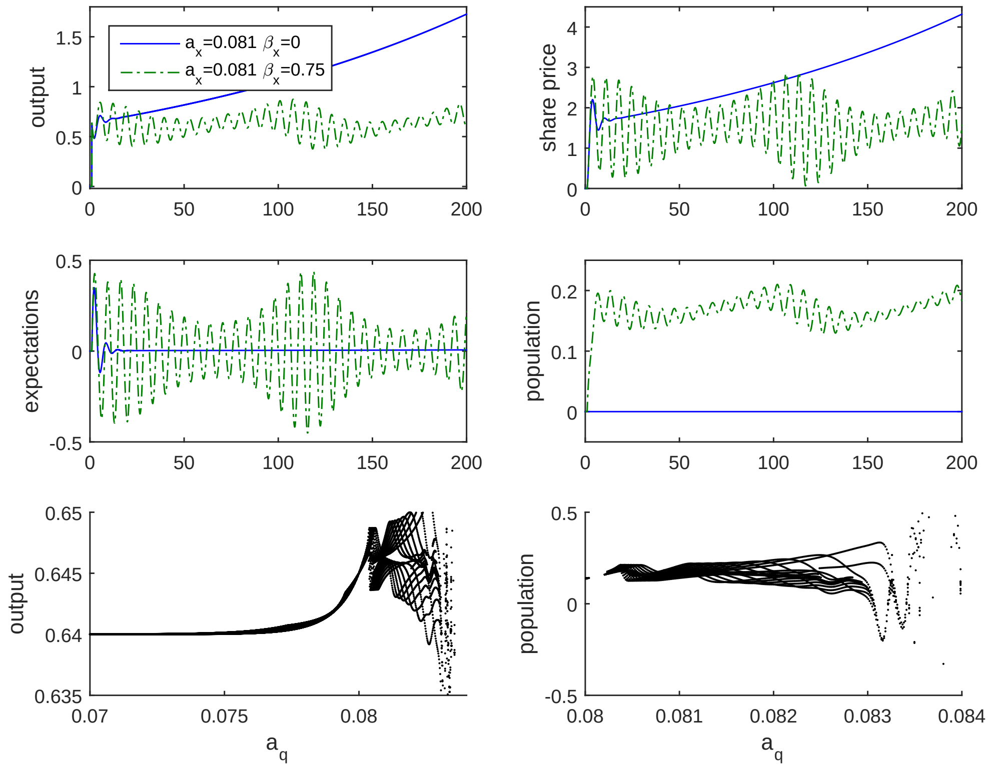

Figure 6: Explosive dynamics in the 3D model (Y, p_e, π_e^e) versus bounded dynamics in the 4D model (Y, p_e, π_e^e, x).

While the aggregate sentiment dynamics tends to amplify financial instability in the proximity of the steady state, the non-linearity embedded in the population dynamics generates forces that keep the aggregate fluctuations within viable boundaries. Figure 6 illustrates how global stability is generated by the sentiment dynamics. The solid blue line corresponds to the 3D model presented in Figure 2 with the parameter a_q (which represents the sensitivity of output to Tobin's q) increased from 0.05 to 0.081. For a value of $a_q = 0.081$, the 3D model is unstable as illustrated by the monotonically explosive trajectory of output and of the price of equities in the top row, and of the capital gain expectations in the left panel in the second row.[16] The instability is located in the financial sector and arises because of a positive feedback between the rate of return on equity, the price of equity, and its accelerator effect on the real economy. The dashed line corresponds to the case where the 3D model is augmented by aggregate sentiment dynamics with $\beta_x = 0.75$, $s_x = 0.8$, $s_y = 12.5$ and $s_{p_e} = s_{\pi_e^e} = 0$. The economy does not display an explosive behavior now, being characterized instead by bounded cycles with high frequency oscillations taking place around lower frequency fluctuations. The non-linearity embedded in the sentiment dynamics sets an upper and a lower bound to the amplitude of the cycles. The lower two panels plot the bifurcation diagrams for output and the relative size of the two populations for $a_q \in [0.07; 0.084]$. The diagram shows the Hopf bifurcation for $a_q = 0.08$, beyond which the model displays oscillations.

As already mentioned, the simulations of the 4D model shown in Figures 4 through 6 have all considered a linear version of the sentiment switching index with s_{p_e} and $s_{\pi_e^e}$ equal to zero in equation (15). In Figure 7, we consider the case where the opinion switching index depends negatively on the volatility of capital gain expectations and of the share price. As the graphs in Figure 7 show, the activation of these nonlinear terms does modify the dynamics of the model. When the sentiment switching index also depends on these two volatility terms, there is a coordination in the expectations of financial market agents towards fundamentalism. We illustrate this emergent feature by the following two examples.

The first example corresponds to the case where $\beta_e = 0.75$ and $s_x = 1$ and is illustrated in the upper panels of Figure 7. Therein the blue line corresponds to the 4D model of Figure 4 with a linear switching index specification ($s_{p_e} = s_{\pi_e^e} = 0$), while the green line corresponds to the case where the switching index contains also nonlinear terms ($s_{p_e} = s_{\pi_e^e} = 20$), both with $\beta_e = 0.75$ and $s_x = 1$. As it can be clearly observed, the extent of the dynamic reaction of the full nonlinear 4D model following a positive output shock is smaller than the reaction of the 4D model with a linear switching index, as the volatility in share price and capital gain expectations reduces the fluctuations in the population dynamics.

The second example corresponds to the dynamically explosive case discussed for the 3D model in Figure 6 and is illustrated in the lower panels of Figure 7. Therein, the blue line corresponds

[16] The scale of the graph gives the impression that π_e^e returns to its initial steady state value, but in fact it diverges, too, albeit very slowly.

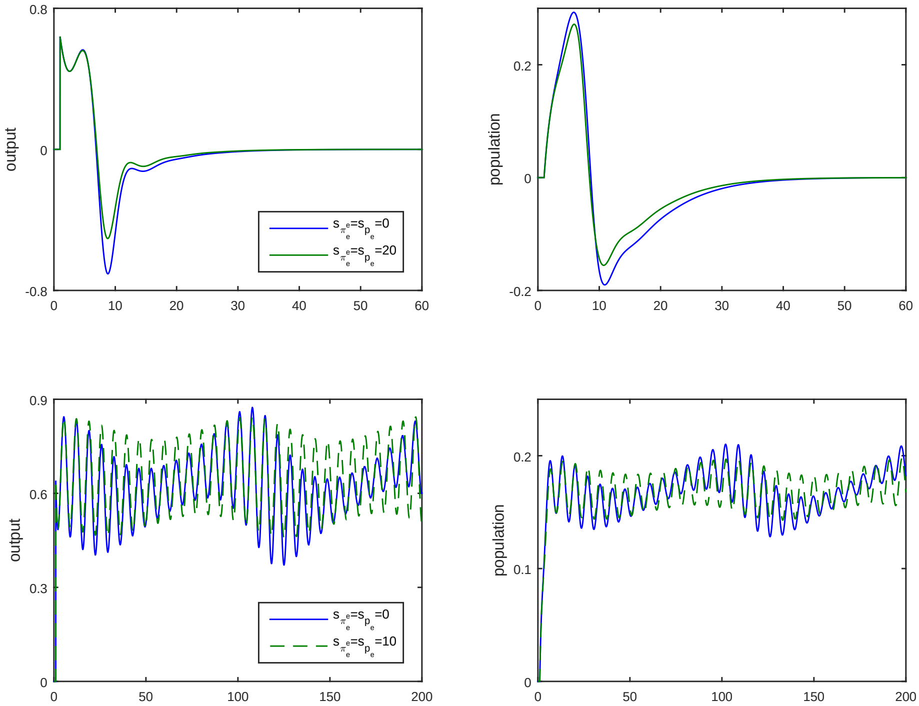

Figure 7: Dynamic adjustments of the full 4D model (Y, p_e, π_e^e, x) for different values of s_{p_e} and $s_{\pi_e^e}$ for the dynamically stable case (upper panels) and the explosive case (lower panels).

to Figure 6 where the nonlinearity in the population dynamic stabilizes an otherwise explosive 3D model. More precisely, what characterized the dynamics of the 4D model shown in Figure 6 was that fluctuations took place along both high and low frequencies. Adding a second type of nonlinearity in the 4D model via the volatility terms in the sentiment switching index seems to reduce in particular the amplitude of the low frequency population fluctuations.[17]

5 Dynamics under Unconventional Monetary Policies

The previous numerical analysis showed the ambivalent effects of the interaction between capital gains expectations and the composition of the population of financial agents on the stability of our model economy. In this section, we briefly outline some policies that could stabilize both real *and* financial markets. Two policy proposals immediately come to mind, in the light of the current financial crisis and the measures adopted to tackle it.

Given the economic debate of the last years about a renewed regulation of international financial markets, it is natural to consider the impact of a tax on capital gains. Taxing finance either via a

[17] Appendix B contains additional simulations illustrating the properties of the full model highlighting in particular the possibility of complex dynamics and performing various robustness checks by means of bifurcation diagrams.

"Tobin Tax" or by increasing the marginal tax rate on capital is often suggested by policy makers as a way of curbing financial market instability, see e.g. Admati and Hellwig (2013). A second policy focuses on the ability of the Central Bank to reduce the pro-cyclicality of the sentiment switching index by convincing agents that it will act vigorously to prevent bubbles in financial markets. Indeed, as central banks greatly influence financial markets sentiments beyond the conventional interest rate policy via their communication policies, the ability of a central banker to coordinate financial traders' expectations on a stable equilibrium may be crucial in times of financial distress, see e.g. Siklos and Sturm (2013).

In Figure 8, the first two policies are assessed with respect to the dashed-dotted red line which corresponds to the green line in the top row of Figure 7 generated with $\beta_x = 0.75$ and $s_x = 1$. Further, we assume $s_{p_e} = s_{\pi_e^e} = 20$ as in Figure 7 of the previous section. In the following we thus simulate the impact of various policies in the full 4D model. Taxing capital gains is taken into account by introducing the tax rate τ_{p_e} in the equation for capital gain expectations (equation (18)).

$$\dot{\pi}_e^e = \beta_{\pi_e^e} \left[(1 - \tau_{p_e}) \left(\frac{1+x}{2} \right) \hat{p}_e - \pi_e^e \right]. \tag{27}$$

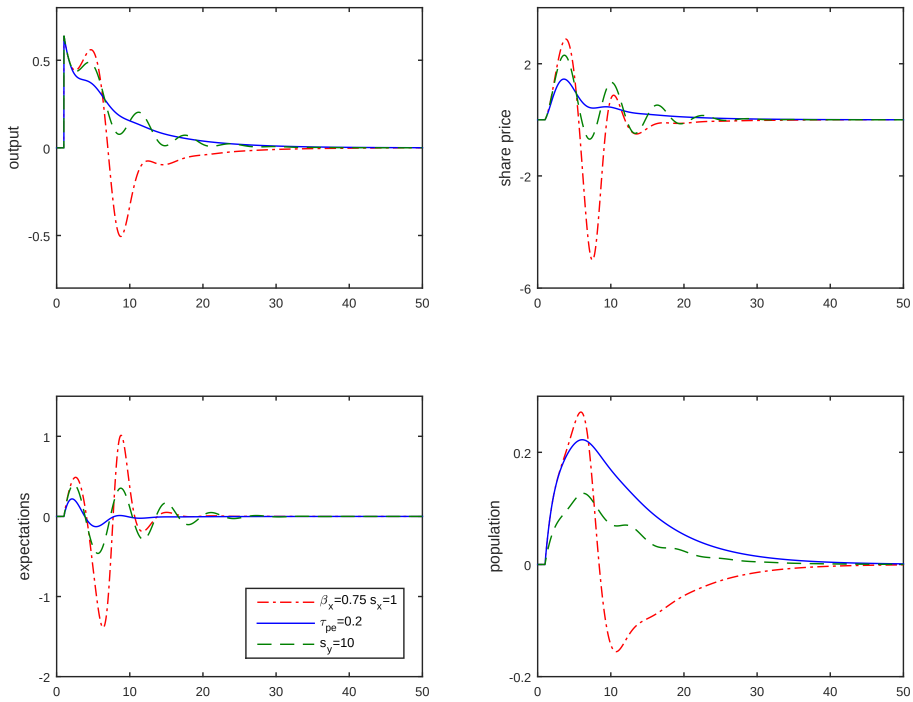

Figure 8: Dynamics under capital gains taxation and central bank communication policy in the full 4D model (Y, p_e, π_e^e, x).

The dynamics illustrated by the continuous blue line was generated assuming a tax rate of 20%. As it can be clearly observed, taxing capital gains has a strong impact on the output dynamics as it almost entirely smooths out output fluctuations, and it also reduces the amplitude of the fluctuations in expectations. A side effect is that the sentiment dynamics now follows a humped-shaped trajectory, rather than an oscillating pattern. As a result, the fluctuations in share prices are much more limited than in the case illustrated in the top row of Figure 7.[18]

The dashed green lines describe the dynamics of the 4D model under a successful central bank communication policy which modifies the perceptions of financial market participants. We specify this scenario in our stylized framework by a reduction of the sentiment index parameter s_y from 20 to 10. This type of policy has a direct impact on the volatility of financial markets and the real sector, and the reduction in s_y translates into a sharp reduction in output fluctuations.

6 Conclusions

We have studied in this paper a stylized dynamic macroeconomic model of real-financial market interactions with endogenous aggregate sentiment dynamics and heterogenous expectations in the tradition of the Weidlich-Haag-Lux approach as recently reformulated by Franke (2012). Following Blanchard (1981), we focused on the impact of equity prices on macroeconomic activity through the Brainard-Tobin q, leaving the nominal interest rate fixed for the sake of simplicity, and also because goods prices were assumed to be constant.

Using this extremely stylized but – due to the intrinsic nonlinear nature of the Weidlich-Haag-Lux approach – complex theoretical framework, we showed that the interaction between real and financial markets need not be necessarily stable, and might well be characterized by multiple equilibria (and even complex dynamics, see Appendix B below). The crucial theoretical, empirical, and policy question, then, is whether unregulated market economies contain some mechanisms ensuring the stability or global boundedness of the economy, or whether centrifugal forces may prevail, making some equilibria locally unstable and, potentially, the whole system globally unstable.

Our numerical simulations show that global stability can obtain if, far off the steady state, aggregate sentiment dynamics favor fundamentalist behavior during booms and busts which ensures that there are upper and lower turning points. Yet, both the local analysis and the simulations suggest that market economies can be plagued by severe business fluctuations and recurrent crises. We showed that two policy measures often advocated in the Keynesian literature, namely Tobin-type taxes (here on capital gains), and Central Bank intervention, can mitigate these problems.

[18] Actually, the tax τ_{p_e} is not restricted to apply to actual transactions and is imposed on *both* actual *and* notional capital gains. Therefore, rather than a Tobin tax, it may be more appropriately interpreted as a wealth tax of the kind advocated by Piketty (2014). It is therefore quite interesting to note that, in addition to any redistributive effects, such a wealth tax may also help to mitigate business cycles and financial turbulence. We are grateful to Bruce Greenwald for pointing this out to us.

Our theoretical framework can be extended in a variety of directions. First, through the incorporation of a varying goods price level and an active conventional interest rate policy, the interaction between macroprudential and conventional policies could be investigated. Also, given the central role of aggregate sentiments and bounded rationality, we may use the model to investigate the efficiency of these policies near or at the zero-lower bound of interest rates. Finally, we could analyze the dynamics of the model under alternative heuristics than the traditional chartist and fundamentalist rules. We intend to pursue some of these alternatives in future research.

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Appendix A

For any matrix J, let $\text{tr}(J)$ be the trace of J and let $|J|$ be its determinant.

Proof of Proposition 1

At a steady state, the Jacobian matrix J of equations (16) and (17) is:

$$J = \begin{pmatrix} -\beta_y(1-a_y) & \beta_y a_q E \\ \frac{\beta_e b}{E} & -\beta_e \rho_{eo}^e \end{pmatrix}.$$

It is easy to see that $\text{tr}(J) < 0$. Furthermore, the determinant of J is

$$|J| = \beta_y(1-a_y)\beta_e\rho_{eo}^e - \frac{\beta_y a_q E \beta_e b}{E}.$$

Therefore $|J| > 0$ if and only if

$$(1-a_y)\rho_{eo}^e > a_q b.$$

Thus, $|J| > 0$ if and only if

$$\rho_{eo}^e > \frac{a_q b}{1-a_y}. \qquad \text{(Q.E.D.)}$$

Proof of Proposition 2

For any $\nu_c \in [0,1]$, at the steady state given by equations (20)-(22), the Jacobian of the 3D system formed of equations (16), (17) and (18) is

$$J = \begin{pmatrix} -\beta_y(1-a_y) & \beta_y a_q E & 0 \\ \frac{\beta_e b}{E} & -\beta_e \rho_{eo}^e & \beta_e p_{eo} \\ \frac{\beta_{\pi_e^e}\beta_e \nu_c b}{p_{eo}E} & -\frac{\beta_{\pi_e^e}\beta_e \nu_c \rho_{eo}^e}{p_{eo}} & \beta_{\pi_e^e}(\nu_c\beta_e - 1) \end{pmatrix}. \qquad (28)$$

According to the Routh-Hurwitz theorem, the necessary and sufficient conditions for stability of the system are:

(C1) $\text{tr}\,(J) < 0$;

(C2) $J_1 + J_2 + J_3 > 0$, where J_i represents the principal minor of order i of the matrix J;

(C3) $|J| < 0$; and

(C4) $B = -\text{tr}\,(J)\,(J_1 + J_2 + J_3) + |J| > 0$.

Condition (C1) clearly holds. If $a_q < (1-a_y)\rho^e_{eo}$, then (C2) and, since it can be proved that $|J| = -\beta_{\pi^e_e} J_3$, (C3) also hold. As for (C4):

$$\begin{aligned} -\mathrm{tr}(J)\ (J_1+J_2+J_3) &= \big(\beta_y(1-a_y)+\beta_e\rho^e_{eo}+\beta_{\pi^e_e}(\nu_c\beta_e-1)\big) \\ &\quad \cdot \big(\beta_e\rho^e_{eo}\beta_{\pi^e_e} - \beta_y(1-a_y)\beta_{\pi^e_e}(\nu_c\beta_e-1)+\beta_y(1-a_y)\beta_e\rho^e_{eo}-\beta_y a_q\beta_e b\big)\,, \end{aligned}$$

and

$$|J| = -\beta_{\pi^e_e}\left(\beta_y(1-a_y)\beta_e\rho^e_{eo} - \frac{\beta_y a_q E_o \beta_e b}{E_o}\right).$$

Therefore, simplifying terms, $B > 0$ if and only if

$$\begin{aligned} &\big[\beta_y\left(1-a_y\right)+\beta_e\rho^e_{eo}-\beta_{\pi^e_e}\left(\nu_c\beta_e-1\right)\big]\left\{\beta_e\beta_{\pi^e_e}\rho^e_{eo}-\beta_y\left(1-a_y\right)\left(\nu_c\beta_e-1\right)+\beta_y\beta_e\left[\left(1-a_y\right)\rho^e_{eo}-a_q b\right]\right\} \\ &\qquad +\beta_e\beta_{\pi^e_e}\beta_y\left[a_q b-\left(1-a_y\right)\rho^e_{eo}\right]>0 \end{aligned}$$

or, equivalently, after some straightforward algebra,

$$\begin{aligned} &\left[\beta_y\left(1-a_y\right)+\beta_e\rho^e_{eo}\right]\left\{\beta_e\beta_{\pi^e_e}\rho^e_{eo}+\beta_y\left(1-a_y\right)\left(1-\nu_c\beta_e\right)+\beta_y\beta_e\left[\left(1-a_y\right)\rho^e_{eo}-a_q b\right]\right\}+\beta_{\pi^e_e}\left(1-\nu_c\beta_e\right) \\ &\cdot\big[\beta_e\beta_{\pi^e_e}\rho^e_{eo}+\beta_y\left(1-a_y\right)\left(1-\nu_c\beta_e\right)\big]+\nu_c\beta_e\beta_e\beta_{\pi^e_e}\beta_y a_q b-\nu_c\beta_e\beta_e\beta_{\pi^e_e}\beta_y\left(1-a_y\right)\rho^e_{eo}>0 \end{aligned}$$

Note that if $1 > \beta_e$ and $(1-a_y)\,\rho^e_{eo} > a_q b$ then all terms in the previous expression except for the last one are strictly positive. Then in order to prove that the desired inequality holds it suffices to note that

$$\beta_y\left(1-a_y\right)\beta_e\beta_{\pi^e_e}\rho^e_{eo}-\nu_c\beta_e\beta_e\beta_{\pi^e_e}\beta_y\left(1-a_y\right)\rho^e_{eo}=\beta_y\left(1-a_y\right)\beta_e\beta_{\pi^e_e}\rho^e_{eo}\left(1-\nu_c\beta_e\right)>0. \qquad \text{(Q.E.D.)}$$

Proof of Proposition 3

Since condition (C1) does not hold for $\nu_c > \frac{\beta_y(1-a_y)+\beta_e\rho^e_{eo}+\beta_{\pi^e_e}}{\beta_{\pi^e_e}\beta_e}$, the steady state of the 3D system is locally unstable. (Q.E.D.)

Proof of Proposition 4

Note that the steady state value of Y, p_e and π_e are uniquely determined independently of x by conditions (20)-(22) in Lemma 1. Given this, we focus on equation (19) where the probabilities and switching index are given by equations (13), (14) and (15), respectively. Let Y, p_e and π_e be equal to their steady state values so that $s = s_x x$.

Define then the following real valued function $g : (-1, +1) \to \Re$

$$g(x) := s_x x - \frac{1}{2a_x}[\ln(1+x) - \ln(1-x)] \tag{29}$$

This function has the property that $g(x) = 0$ if and only if $\dot{x} = 0$ as can be seen from (19) setting $\dot{x} = 0$ and taking the logs. The equation $g(x) = 0$ always has a solution for $x = 0$ and thus there is always a steady state with $x_o = 0$.

(i) Observe that

$$\lim_{x\to 1} g(x) = -\infty, \tag{30}$$

$$\lim_{x\to -1} g(x) = +\infty, \tag{31}$$

and the derivative of $g(x)$ is

$$g'(x) = s_x - \frac{1}{a_x(1-x^2)}. \tag{32}$$

Then if $s_x \leq \frac{1}{a_x}$, $g'(x) < 0$ and $g(x)$ is strictly decreasing for all $x \in (-1, 1)$. So, if $s_x \in (0, 1/a_x]$, $x_o = 0$ is the only value of x such that $g(x) = 0$ and so $\dot{x} = 0$.

(ii) By equation (32), $g(x)$ is increasing if and only if

$$g'(x) = s_x - \frac{1}{a_x(1-x^2)} \geq 0 \Leftrightarrow x^2 \leq \frac{s_x a_x - 1}{s_x a_x}.$$

Because $s_x a_x > 1$, it follows that $g(x)$ is strictly increasing for $x \in \left(-\sqrt{\frac{s_x a_x - 1}{s_x a_x}}, \sqrt{\frac{s_x a_x - 1}{s_x a_x}}\right)$ and strictly decreasing for $x \in \left(-1, -\sqrt{\frac{s_x a_x - 1}{s_x a_x}}\right) \cup \left(\sqrt{\frac{s_x a_x - 1}{s_x a_x}}, 1\right)$. Then, noting that $g(0) = 0$ and $g'(0) > 0$, by equations (30) and (31), and the continuity of $g(x)$, there exist three steady states: one with equal populations ($x_o = 0$), one where fundamentalists dominate ($x_o < 0$) and one where chartists dominate ($x_o > 0$). (Q.E.D.)

Proof of Proposition 4

The proof of Proposition 4 is a trivial modification of the proof of Proposition 3. (Q.E.D.)

Proof of Proposition 5

At any steady state (x_o, π^e_{eo}) with $\pi^e_{eo} = 0$, the Jacobian of the system formed by equations (24)-(26) is:

$$J = \begin{pmatrix} \beta_{\pi^e_e}\left[\dfrac{1+x_o}{2}\beta_e - 1\right] & 0 \\ 0 & 2\beta_x \exp(a_x s_x x_o)\left[(1-x_o)a_x s_x - \frac{1}{1+x_o}\right] \end{pmatrix}. \tag{33}$$

(i) At the steady state with $x_o = 0$ and $\pi^e_{eo} = 0$, the Jacobian becomes

$$J = \begin{pmatrix} \beta_{\pi^e_e}\left(\frac{\beta_e}{2} - 1\right) & 0 \\ 0 & 2\beta_x(a_x s_x - 1) \end{pmatrix}. \tag{34}$$

Because $s_x \in (0, 1/a_x)$, if $\beta_e > 2$ then $|J| < 0$, and the steady state is an unstable saddle point. Conversely, if $\beta_e < 2$ then tr$J < 0$ and $|J| > 0$, and the steady state is stable.

(ii) The stability properties of the steady state with $x_o = 0$ and $\pi^e_{eo} = 0$ can be derived with a straightforward modification of the argument in part (i) noting that $s_x > 1/a_x$.

In order to derive the stability properties of $e_f = (0, x^f_o)$ and $e_c = (0, x^c_o)$, note that $J_{22} \lesseqgtr 0$ if and only if $(1 - x_o)a_x s_x \lesseqgtr \frac{1}{1+x_o}$ or equivalently

$$x^2_o \gtreqless \frac{a_x s_x - 1}{a_x s_x}. \tag{35}$$

By the argument in part (ii) of Proposition 3, it follows that both at e_c and at e_f, $x^2_o > \frac{a_x s_x - 1}{a_x s_x}$ and therefore $J_{22} < 0$. (Q.E.D.)

Appendix B

In this appendix we present some additional simulations of the full model as well as bifurcation diagrams. Figure 9 illustrates the case where the relative population variable displays irregular yet persistent fluctuations. In this simulation, the adjustment speed of share price β_e is increased from 2 to 2.5, while the sensitivity of the sentiment switching index to the output gap, s_y, is reduced to 0.1. The fast adjustment of share price is a source of instability, which is counter-balanced by the nonlinearity in the opinion switching index ($s_{p_e} = 0.06$ and $s_{\pi^e_e} = 0.5$). The self-reflection parameter in the opinion switching index, s_x, is kept at 1.

The fluctuations in the population of traders are translated to capital gains expectations and the real economy. The relative size of the two groups (fundamentalists and chartists) fluctuates between -0.25 and 0 with oscillations differing in both amplitude and frequency. The stability in the fluctuation of the sentiment dynamics is related to the two volatility parameters in the switching equation – s_{p_e} and $s_{\pi^e_e}$ – which capture the idea that higher volatility leads agents to become fundamentalists.

We now turn to bifurcation diagrams based on the same calibration as in the lower panels of Figure 9 in order to further illustrate the properties of the full model. The top panel of Figure 10 show the bifurcation diagrams of population dynamics and output with respect to the sensitivity of the opinion switching index to the self-reference element, with s_x varying between 0.4 and 1.5. For values of s_x between 0 and 0.5 there are four local minima and maxima for x. This number doubles between 0.5 and 0.9. The number of local minima and maxima then goes back to four between 0.9 and 1 and

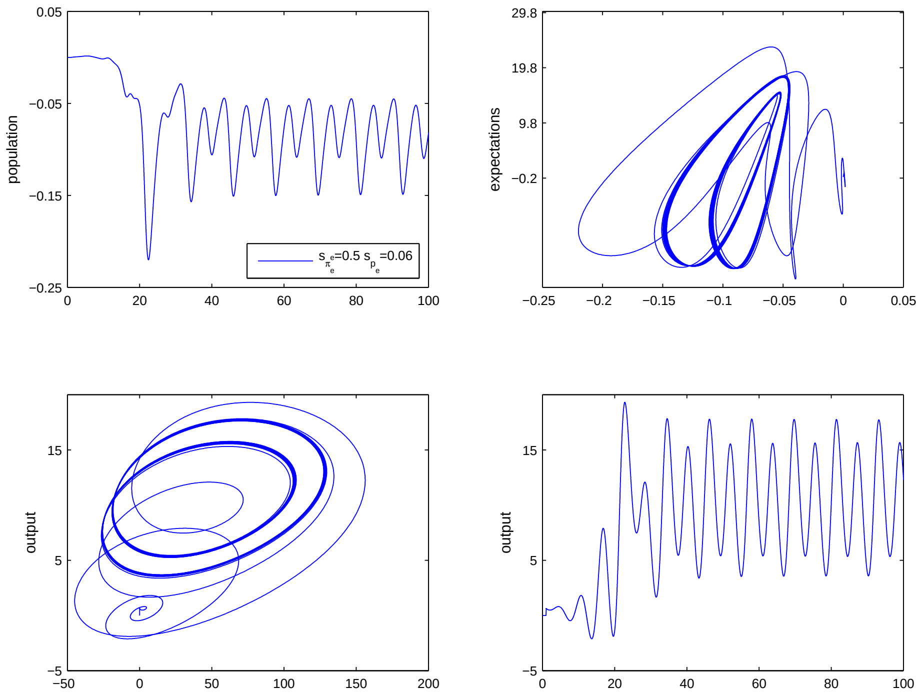

Figure 9: Complex dynamics in the 4D model (Y, p_e, π_e^e, x).

further reduces to two between 1 and 1.25. Beyond 1.25 there is a unique steady state. A similar pattern describes the oscillation of output.

As shown in the next two panels, the number of local minima and maxima decreases with a_x from four over the range 0.7-0.8 to two over the range 0.8-1 and one when $a_x > 1$. This result is also consistent with the analysis in section 3.3.

The third row of Figure 10 shows bifurcation diagrams of the population dynamics with respect to the sensitivity of the opinion switching index to the output gap, s_y, and to capital gains expectations $s_{\pi_e^e}$. Values of s_y in the range $[0.15; 0.2]$ and $[0.27; 0.32]$ produce large fluctuations in the opinion dynamic. The population variable x goes either to -1 or to positive values when $s_y > 0.34$. For values of $s_{\pi_e^e} < 0.3$, the opinion dynamics displays large fluctuations over the range [-0.6;0] in line with the result that excess volatility favors fundamentalist expectations.

The fourth and fifth rows of Figure 10 summarize additional sensitivity analysis. The population dynamics is stable for either low or high values of the speed of adjustment of expectations, $\beta_{\pi_e^e}$, and the speed of adjustment of the price of capital, β_e. Interestingly, only a high speed of adjustment of population dynamics ($\beta_x > 0.8$) produces stability. Finally, the system produces oscillations when the sensitivity of aggregate demand to Tobin's q, a_q, is either small or larger than 0.8.

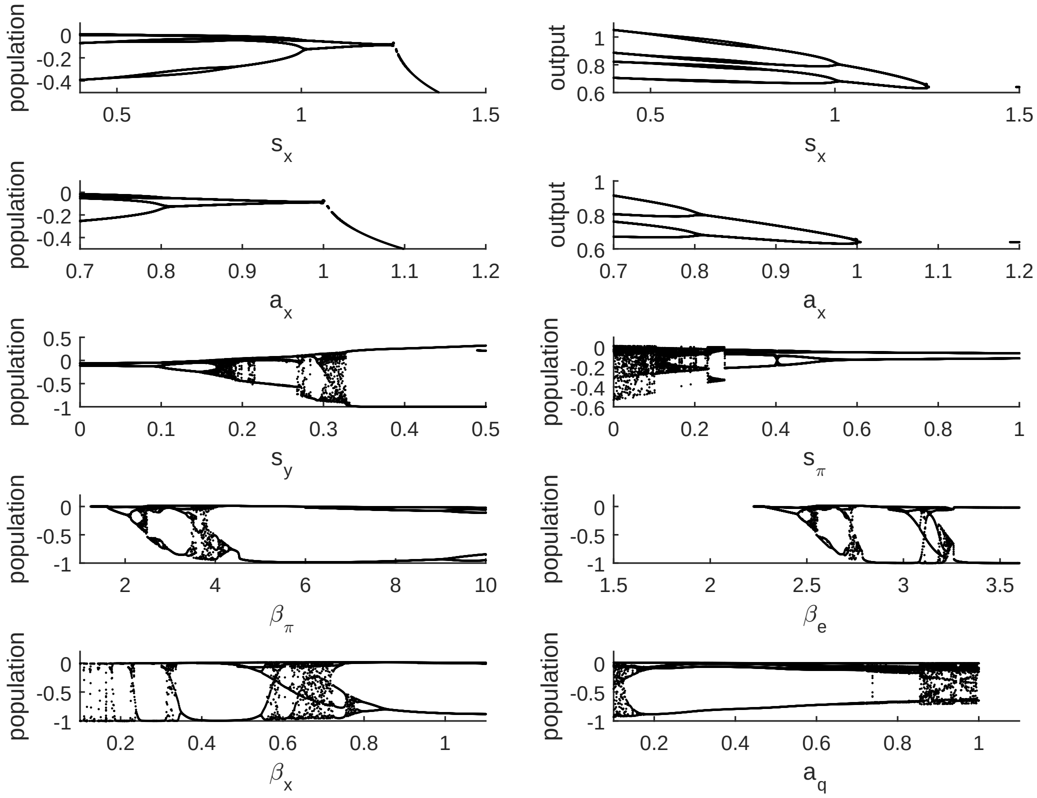

Figure 10: Bifurcation diagrams

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