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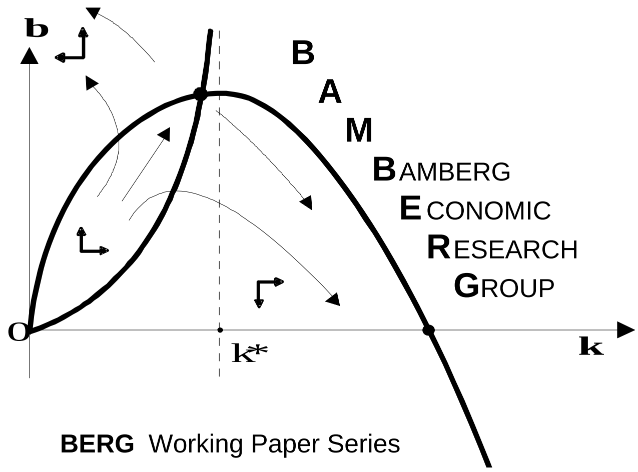

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Evolutionary competition and profit taxes: market stability versus tax burden☆

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Abstract

The seminal cobweb model by Brock and Hommes reveals that fixed-point dynamics may turn into increasingly complex dynamics as firms switch more quickly between competing expectation rules. While policy-makers may be able to manage such rational routes to randomness by imposing a proportional profit tax, the stability-ensuring tax rate may cause a very high tax burden for firms. Using a mix of analytical and numerical tools, we show that a rather small profit-dependent lump-sum tax may even be sufficient to take away the competitive edge of cheap destabilizing expectation rules, thereby contributing to market stability.

Keywords: Cobweb models, discrete choice approach, intensity of choice, profit taxes, tax burden, stability analysis.

JEL classification: D84; E30; Q11

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1. Introduction

Brock and Hommes (1997) show that an evolutionary competition between heterogeneous expectation rules may create complex endogenous dynamics. They consider a cobweb model in which firms have the choice between naive and rational expectations. Due to positive information costs, rational expectations pro-

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duce lower steady-state profits than naive expectations. If firms react strongly to this profit differential, the majority of them opt for naive expectations, making the market unstable. An important question is whether policy-makers can stabilize such dynamics. Extending the model by Brock and Hommes (1997), Schmitt and Westerhoff (2015) demonstrate that policy-makers may be able to manage rational routes to randomness by introducing a proportional tax on firms' profits. The basic idea of their paper is that an increase in the tax rate reduces profit differentials between free naive expectations and costly rational expectations. As a result, more firms rely on rational expectations, bringing stability to the market. However, the stability-ensuring tax rate may become quite high and policy-makers may need to impose a high tax burden on firms in order to calm markets.

The aim of our paper is to show that policy-makers may be able to stabilize markets by imposing even a rather small profit-dependent lump-sum tax. To reduce the profit differential between competing expectation rules, it is not necessary to tax away a large part of firms' profits, but simply to eliminate the competitive edge of cheap destabilizing expectation rules. Our approach is also based on the model by Brock and Hommes (1997) and we seek to clarify our main argument by using the following examples. Suppose that the steady-state profit of a firm relying on naive expectations is 21, while a firm with rational expectations has, due to constant per period information costs of $F = 1$, a steady-state profit of 20. Under these assumptions, many firms may decide in favor of naive expectations and the market may, consequently, become unstable. If policy-makers impose a lump-sum tax of $\lambda = 1$ on firms' profits exceeding a threshold value of $\hat{\pi}^L = 20$, the steady-state profits of firms using naive and rational expectations both become equal to 20. As a result, firms split evenly between rational and naive expectations, and the market becomes more stable. In contrast, a complete leveling of the firms' steady-state profits via a proportional profit tax necessitates a tax rate of 100 percent. Alternatively, if we assume that the market remains stable for profit differentails up to 0.5, a small lump-sum tax of $\lambda = 0.5$ or a rather substantial tax rate of 50 percent is needed to stabilize markets. These examples suggest that a profit-dependent lump-sum tax may cause a significantly lower tax burden for firms than a proportional profit tax, yet still manages to reduce the profit differentials of the expectation rules and to foster market stability.

Using a combination of analytical and numerical methods, we show that this elementary insight functions quite well, also out of equilibrium and in the presence of exogenous noise. In a deterministic environment, there exists a robust set of λ and $\hat{\pi}^L$ combinations for which the model's steady state is stable and for which the firms' tax burden may be regarded as modest. For our leading parameter setting, taken from Brock and Hommes (1997), we find that a stability-ensuring profit-dependent lump-sum tax - in relation to a stability-ensuring proportional profit tax - reduces the firms' tax burden by more than 95 percent. Of course, the challenge for policy-makers is to find suitable levels for the lump-sum tax and the profit threshold. Our results suggest that, in a noisy environment, policy-makers should increase λ and/or decrease $\hat{\pi}^L$ to establish stable markets. Nevertheless, the firms' tax burden remains considerably lower than in the case of a proportional profit tax.

Our insights are not restricted to the specific cobweb model by Brock and Hommes (1997); they should also work in other models in which cheap destabilizing rules compete with costly stabilizing rules. For instance, Goeree and Hommes (2000) generalize this model to the case of non-linear demand and supply, while Lasselle et al. (2005) focus on a switching between free adaptive expectations and costly rational expectations. Moreover, Droste et al. (2002) consider a duopoly model in which firms can switch between free simple rules and more sophisticated rules that require extra information costs. In the asset-pricing model by Brock and Hommes (1998), market participants adapt their price expectations by a profit-based switching behavior between free technical and costly fundamental expectation rules. Hommes and Zeppini (2014) propose a model of technological change with evolutionary switching between costly innovators and free imitators to study technological progress. Many more models in this line of research, together with corroborating empirical evidence, are summarized in Hommes (2006), Chiarella et al. (2009) and Hommes and Wagner (2009). Westerhoff and Franke (2015) provide an overview of how such types of models may be used to analyze economic policy questions.

The remainder of this paper is organized as follows. In Section 2, we introduce the model by Brock and Hommes (1997) with profit-dependent lump-sum taxes and derive its dynamical system. Our analytical results are presented in Section 3. In particular, we compute the model's steady state and discuss how the intensity of choice and profit-dependent lump-sum taxes may affect its local asymptotic stability. In Section 4, we illustrate how the model's global dynamics depends on profit-dependent lump-sum taxes, and how policy-makers may control the market's volatility and the firms' tax burden. In Section 5, we summarize our main results and suggest a few avenues for future research.

2. A cobweb model with profit-dependent lump-sum taxes

We now extend the seminal cobweb model by Brock and Hommes (1997) by assuming that policy-makers may impose a profit-dependent lump-sum tax. After presenting the setup of the model in Section 2.1, we will derive its dynamical system in Section 2.2.

2.1. The setup of the model

Cobweb models describe the dynamics of a competitive market for a non-storable consumption good. Since the good takes one period to produce, firms must form their price expectations one period ahead. Brock and Hommes (1997) assume that firms can choose between two different expectation rules. They can either buy a rational expectations forecast or freely obtain a naive forecast. The firms' rule selection is repeated at the beginning of each period, and depends on the past performance of the rules: the higher the profits of an expectation rule, the more firms will rely on it. Brock and Hommes (1997) show that the firms' rule selection behavior may cause complex dynamics. For increasing values of the intensity choice, price dynamics become more and more complicated.

Schmitt and Westerhoff (2015) demonstrate that policy-makers may be able to manage such rational routes to randomness by imposing a proportional tax on positive profits. Unfortunately, the stability-ensuring profit tax rate increases with the firms' intensity of choice, and may become quite high. Since a high tax burden may be harmful for firms, in this paper we consider an alternative tax function. In particular, policy-makers may impose a profit-dependent lump-sum tax according to which firms have to pay a limited profit tax if their profits exceed a certain threshold value.

Let us turn to the details of the model. Market clearing takes place in every period, implying that

$$D_t = S_t, \tag{1}$$

where D_t and S_t denote demand and supply at time step t, respectively. Consumer demand depends negatively on the current market price p_t, and is expressed as

$$D_t = a - bp_t, \tag{2}$$

where a and b are positive parameters. In determining their production decisions, firms need to form price expectations one period ahead and choose between two different expectation rules. By normalizing the number of firms to $N = 1$, we can formalize their total supply as

$$S_t = n^N_{t-1} q^N_t + n^R_{t-1} q^R_t. \tag{3}$$

While q^N_t and q^R_t represent the quantities supplied by firms with naive and rational expectations, n^N_{t-1} and n^R_{t-1} stand for their respective market shares.

As in Brock and Hommes (1997), firms face a quadratic cost function, i.e. $C_t = \frac{1}{2c} q_t^2$ with $c > 0$. Additionally, firms may have to pay a profit-dependent lump-sum tax, where λ_t denotes the amount to be paid. Since firms expect with a (small) probability *prob* that policy-makers will abandon the tax, their profit maximization problem takes the form

$$\underset{q_t}{argmax}\ \pi^e_t = \underset{q_t}{argmax}\ (1 - prob)(p^e_t q_t - C_t - \lambda_t) + prob(p^e_t q_t - C_t), \tag{4}$$

where π_t^e and p_t^e stand for expected profits and expected prices, respectively. As it turns out, a firm's optimal supply is given by $q_t = cp_t^e$.[1]

To form their price expectations, firms can either rely on a naive expectation rule by simply taking the last observed price as a forecast, i.e. $p_t^e = p_{t-1}$, or they can use a rational expectation (perfect foresight) rule, i.e. $p_t^e = p_t$. Quantities supplied by firms with naive or rational expectations can thus be expressed as

$$q_t^N = cp_{t-1} \tag{5}$$

and

$$q_t^R = cp_t, \tag{6}$$

respectively. While naive expectations are freely available, using rational expectations incurs constant per period information costs $F > 0$ (which do not influence firms' supply decisions and, for notational simplicity, have been neglected in (4)).

The fractions of firms with naive and rational expectations are updated over time according to an evolutionary fitness measure. Firms are boundedly rational in the sense that they tend to select the expectation rule with the highest fitness. As in Brock and Hommes (1997), firms use realized profits as the performance criterion. Since firms may have to pay a profit-dependent lump-sum tax, it is convenient to introduce first pre-tax profits. For the two expectation rules, firms' pre-tax profits in period t can be formalized as

$$\hat{\pi}_t^N = 0.5cp_{t-1}(2p_t - p_{t-1}) \tag{7}$$

and

$$\hat{\pi}_t^R = 0.5cp_t^2 - F, \tag{8}$$

respectively.

[1] As we will see, the lump-sum tax is either given by $\lambda_t = \lambda$, $\lambda_t = 0$ or $\lambda_t = p_t^e q_t - C_t - \hat{\pi}^L$. Since firms expect with a certain probability that policy-makers will abolish the tax, $q_t = cp_t^e$ is indeed the solution of (4).

Firms only have to pay a profit tax if their pre-tax profits exceed the threshold value $\hat{\pi}^L$. The profit-dependent lump-sum tax is limited to λ. If firms' pre-tax profits are higher than $\hat{\pi}^H = \hat{\pi}^L + \lambda$, firms need to pay the full amount λ. If their profits exceed $\hat{\pi}^L$, but fall short of $\hat{\pi}^H$, they only have to pay the difference between their pre-tax profits and the threshold value. Accordingly, the tax functions of firms using naive and rational expectation are given by

$$\lambda_t^N = \begin{cases} \lambda & for \;\; \hat{\pi}_t^N > \hat{\pi}^H \\ \hat{\pi}_t^N - \hat{\pi}^L & for \;\; \hat{\pi}^L \leq \hat{\pi}_t^N \leq \hat{\pi}^H \\ 0 & for \;\; \hat{\pi}_t^N < \hat{\pi}^L \end{cases} \tag{9}$$

and

$$\lambda_t^R = \begin{cases} \lambda & for \;\; \hat{\pi}_t^R > \hat{\pi}^H \\ \hat{\pi}_t^R - \hat{\pi}^L & for \;\; \hat{\pi}^L \leq \hat{\pi}_t^R \leq \hat{\pi}^H \\ 0 & for \;\; \hat{\pi}_t^R < \hat{\pi}^L \end{cases} . \tag{10}$$

Note that policy-makers have two control parameters: they can adjust the profit-dependent lump-sum tax by shifting the threshold value $\hat{\pi}^L$ and/or by changing the maximal tax payment λ.

Realized profits of naive and rational firms in period t can now easily be defined by

$$\pi_t^N = 0.5cp_{t-1}(2p_t - p_{t-1}) - \lambda_t^N \tag{11}$$

and

$$\pi_t^R = 0.5cp_t^2 - F - \lambda_t^R, \tag{12}$$

respectively.

Brock and Hommes (1997) determine the fractions of firms choosing naive or rational expectations by the discrete choice model by Manski and McFadden (1981), resulting in

$$n_t^N = \frac{exp[\beta\pi_t^N]}{exp[\beta\pi_t^N] + exp[\beta\pi_t^R]} \tag{13}$$

and

$$n_t^R = \frac{exp[\beta\pi_t^R]}{exp[\beta\pi_t^N] + exp[\beta\pi_t^R]}. \tag{14}$$

The key feature of this evolutionary approach is that more firms will choose the expectation rule that has the higher fitness. Parameter β denotes the firms' intensity of choice and measures how sensitively they select the most profitable rule. For $\beta = 0$, firms do not observe any profit differentials between the two rules, and (13) and (14) imply that $n_t^N = n_t^R = 0.5$. When the intensity of choice increases, more and more firms opt for the rule that yields a higher profit. For $\beta = \infty$, firms observe fitness differentials perfectly, and all of them select the rule with the higher profit.

2.2. The model's dynamical system

Let us next derive the model's dynamical system. Substituting (5) and (6) into (3) and combining this expression with (1) and (2) leads to

$$a - bp_t = n_{t-1}^N c p_{t-1} + n_{t-1}^R c p_t. \tag{15}$$

Due to $n_{t-1}^N + n_{t-1}^R = 1$, the model's steady-state price turns out to be

$$p^* = \frac{a}{b+c}. \tag{16}$$

Furthermore, solving (15) explicitly for p_t yields

$$p_t = \frac{a - n_{t-1}^N c p_{t-1}}{b + c n_{t-1}^R}. \tag{17}$$

Accordingly, the current price p_t depends on the expectation rules' market shares of the previous period, i.e. n_{t-1}^N and n_{t-1}^R. Once p_t is known, the fitness of the two expectation rules can be identified, and the new market shares n_t^N and n_t^R follow via (13) and (14). Given n_t^N and n_t^R, the next equilibrium price p_{t+1} can be determined, and so on.

Since it is convenient to rewrite the model's dynamical system in deviations from the steady-state price, we follow Brock and Hommes (1997) and introduce $\tilde{p}_t = p_t - p^*$. If we, furthermore, define the difference between the fractions of firms using rational and naive expectations as $m_t = n_t^R - n_t^N$, where $m_t = 1$ ($m_t = -1$) corresponds to all firms holding rational (naive) expectations, we can represent our model by the two-dimensional nonlinear map:

$$\tilde{p}_t = \frac{-(1-m_{t-1})c\tilde{p}_{t-1}}{(m_{t-1}+1)c+2b} \tag{18}$$

and

$$m_t = tanh(0.5\beta((\hat{\pi}_t^R - \lambda_t^R) - (\hat{\pi}_t^N - \lambda_t^N))), \tag{19}$$

where

$$\lambda_t^N = \begin{cases} \lambda & for \ \ \hat{\pi}_t^N > \hat{\pi}^H \\ \hat{\pi}_t^N - \hat{\pi}^L & for \ \ \hat{\pi}^L \leq \hat{\pi}_t^N \leq \hat{\pi}^H \\ 0 & for \ \ \hat{\pi}_t^N < \hat{\pi}^L \end{cases}, \tag{20}$$

$$\lambda_t^R = \begin{cases} \lambda & for \ \ \hat{\pi}_t^R > \hat{\pi}^H \\ \hat{\pi}_t^R - \hat{\pi}^L & for \ \ \hat{\pi}^L \leq \hat{\pi}_t^R \leq \hat{\pi}^H \\ 0 & for \ \ \hat{\pi}_t^R < \hat{\pi}^L \end{cases}, \tag{21}$$

$$\hat{\pi}_t^N = 0.5c(\tilde{p}_{t-1} + p^*)(2\tilde{p}_t - \tilde{p}_{t-1} + p^*) \tag{22}$$

and

$$\hat{\pi}_t^R = 0.5c(\tilde{p}_t + p^*)^2 - F. \tag{23}$$

Note that (19) depends on the relative fitness of rational expectations over naive expectations, which we define as $\pi_t^\Delta = (\hat{\pi}_t^R - \lambda_t^R) - (\hat{\pi}_t^N - \lambda_t^N)$. If all firms are always exempt from profit taxes ($\lambda_t^N = \lambda_t^R = 0$ for all t), the relative fitness function simplifies to $\pi_t^\Delta = \hat{\pi}_t^R - \hat{\pi}_t^N$ and our model corresponds exactly to the original model by Brock and Hommes (1997). Of course, the same dynamics emerge if all firms always pay the same amount of profit taxes ($\lambda_t^N = \lambda_t^R = \lambda$ for all t). Since (20) and (21) consist of three branches, the relative fitness function π_t^Δ gives rise to $3 \times 3 = 9$ possible out-of-equilibrium constellations.

3. Analytical insights

The two-dimensional nonlinear map (18)-(23) admits the unique steady state:

$$(\tilde{p}^*, m^*) = (0, tanh(-0.5\beta(F - (\lambda^{N^*} - \lambda^{R^*})))) \tag{24}$$

with

$$\lambda^{N^*} = \begin{cases} \lambda & for \;\; \hat{\pi}^{N^*} > \hat{\pi}^H \\ \hat{\pi}^{N^*} - \hat{\pi}^L & for \;\; \hat{\pi}^L \leq \hat{\pi}^{N^*} \leq \hat{\pi}^H \\ 0 & for \;\; \hat{\pi}^{N^*} < \hat{\pi}^L \end{cases}, \tag{25}$$

$$\lambda^{R^*} = \begin{cases} \lambda & for \;\; \hat{\pi}^{R^*} > \hat{\pi}^H \\ \hat{\pi}^{R^*} - \hat{\pi}^L & for \;\; \hat{\pi}^L \leq \hat{\pi}^{R^*} \leq \hat{\pi}^H \\ 0 & for \;\; \hat{\pi}^{R^*} < \hat{\pi}^L \end{cases}, \tag{26}$$

$$\hat{\pi}^{N^*} = 0.5cp^{*^2} \tag{27}$$

and

$$\hat{\pi}^{R^*} = 0.5cp^{*^2} - F. \tag{28}$$

While the steady-state price $\tilde{p}^*$ is independent of the profit-dependent lump-sum tax, the steady-state fraction m^* depends on λ and $\hat{\pi}^L$.

To determine the stability properties of the steady state, we have to find the two eigenvalues of the Jacobian matrix of the model's dynamical system at the steady state. Straightforward computations yield

$$z_1 = 0 \tag{29}$$

and

$$z_2 = -\frac{(1-m^*)c}{(1+m^*)c + 2b}. \tag{30}$$

A necessary and sufficient condition for the model's steady state to be locally asymptotically stable, henceforth denoted by LAS, is given by $|z_{1,2}| < 1$ (see, e.g. Gandolfo 2009 or Medio and Lines 2001). From $-1 \leq m^* \leq 1$, we infer for z_2 that $-\frac{c}{b} \leq z_2 \leq 0$. However, we follow Brock and Hommes (1997) and assume that the market is unstable when all firms rely on naive expectations, i.e. $-\frac{c}{b} < -1$. Since $z_2 = -1$ if $m^* = -\frac{b}{c}$, LAS of the model's steady state requires that $m^* > -\frac{b}{c}$, which is equivalent to

$$\beta(F - (\lambda^{N^*} - \lambda^{R^*})) < ln\frac{c+b}{c-b}. \tag{31}$$

Referring to (31) immediately reveals a possible stability trade-off between the intensity of choice and profit-dependent lump-sum taxes. Suppose, for instance, that $\lambda^{R^*} = 0$. Policy-makers may then be able to compensate a destabilizing change in β by increasing λ^{N^*}. Since there are six possible steady-state combinations of λ^{N^*} and λ^{R^*}, stability condition (31) can take different forms.[2] In the following, we discuss these forms in detail.

If $\hat{\pi}^{N^*} > \hat{\pi}^{R^*} > \hat{\pi}^{H}$, referred to as Case 1, all firms need to pay the maximum amount of the lump-sum tax. Accordingly, $\lambda^{N^*} = \lambda^{R^*} = \lambda$ and stability condition (31) simplifies to

$$\beta F < ln\frac{c+b}{c-b}. \tag{32}$$

The same stability condition applies to Case 2 in which $\hat{\pi}^{R^*} < \hat{\pi}^{N^*} < \hat{\pi}^{L}$ and neither of the two types of firms must pay profit taxes, i.e. $\lambda^{N^*} = \lambda^{R^*} = 0$. In these two cases, profit-dependent lump-sum taxes obviously have no impact on the LAS of the model's steady state. Since $m^* = -\frac{b}{c}$ if $\beta = \beta_c = \frac{ln\frac{c+b}{c-b}}{F}$, the model's steady state is LAS for $\beta < \beta_c$. Note that Case 2, and thus stability condition (32), correspond to the original model by Brock and Hommes (1997).

Case 3, given by $\hat{\pi}^{L} \leq \hat{\pi}^{R^*} < \hat{\pi}^{N^*} \leq \hat{\pi}^{H}$, implies that both types of firms only need to pay part of the maximum tax amount, i.e. $\lambda^{N^*} = \hat{\pi}^{N^*} - \hat{\pi}^{L}$ and $\lambda^{R^*} = \hat{\pi}^{R^*} - \hat{\pi}^{L}$. Consequently, stability condition (31) turns into

$$0 < ln\frac{c+b}{c-b}. \tag{33}$$

Note that the model's steady state is now always LAS, irrespective of the firms' intensity of choice. The reason for this outcome is quite simple. Steady-state profits of naive and rational firms are both equal to $\hat{\pi}^{L}$, which is why half of these rely on stabilizing rational expectations.

[2] In contrast to the nine possible out-of-equilibrium combinations of λ_t^N and λ_t^R, the three steady-state combinations $\lambda^{N^*} = 0$ and $\lambda^{R^*} = \lambda$, $\lambda^{N^*} = 0$ and $\lambda^{R^*} = \hat{\pi}^{R^*} - \hat{\pi}^{L}$, and $\lambda^{N^*} = \hat{\pi}^{N^*} - \hat{\pi}^{L}$ and $\lambda^{R^*} = \lambda$ do not exist. This is because, while rational expectations may outperform naive expectations out of equilibrium, naive expectations are always more profitable than (costly) rational expectations at the steady state.

Case 4 applies if $\hat{\pi}^{R^*} < \hat{\pi}^L < \hat{\pi}^H < \hat{\pi}^{N^*}$. Naive firms must then pay the maximum tax amount, while rational firms are exempt from taxation. Since $\lambda^{N^*} = \lambda$ and $\lambda^{R^*} = 0$, stability condition (31) can be expressed as

$$\beta(F - \lambda) < ln\frac{c+b}{c-b}. \tag{34}$$

Stability condition (34) depends on the lump-sum tax. In particular, a destabilizing increase in β may be compensated by an increase in λ such that the steady state remains LAS. Since Case 4 only holds if $F > \lambda$, policy-makers are (in this parameter range) unable to accommodate for every increase in the firms' intensity of choice.

For $\hat{\pi}^{R^*} < \hat{\pi}^L \leq \hat{\pi}^{N^*} < \hat{\pi}^H$, referred to as Case 5, we have $\lambda^{N^*} = \hat{\pi}^{N^*} - \hat{\pi}^L$ and $\lambda^{R^*} = 0$. Accordingly, only naive firms must pay profit taxes and the tax payments by these firms are below the maximum lump-sum tax. Stability condition (31) can thus be rewritten as

$$\beta(F - x\lambda) < ln\frac{c+b}{c-b}, \tag{35}$$

where $x = \frac{\hat{\pi}^{N^*} - \hat{\pi}^L}{\lambda}$. Since $\hat{\pi}^{R^*} < \hat{\pi}^L$, we have $\hat{\pi}^{N^*} - \hat{\pi}^L < F$, and we therefore know that $x\lambda < F$. In Case 6, i.e. $\hat{\pi}^L \leq \hat{\pi}^{R^*} \leq \hat{\pi}^H < \hat{\pi}^{N^*}$, both groups need to pay profit taxes, but only naive firms have to pay the maximum tax amount. Since $\lambda^{N^*} = \lambda$ and $\lambda^{R^*} = \hat{\pi}^{R^*} - \hat{\pi}^L$, we obtain from stability condition (31)

$$\beta(F - y\lambda) < ln\frac{c+b}{c-b}, \tag{36}$$

where $y = \frac{\hat{\pi}^H - \hat{\pi}^{R^*}}{\lambda}$. It follows from $\hat{\pi}^H < \hat{\pi}^{N^*}$ that $\hat{\pi}^H - \hat{\pi}^{R^*} < F$ and thus $y\lambda < F$. It becomes evident from (35) and (36) that policy-makers can influence the LAS of the model's steady state by adjusting λ and $\hat{\pi}^L$. Due to $F > x\lambda$ and $F > y\lambda$, however, the policy-makers' ability to counter an increase in β is also limited under the parameter constellations of Cases 5 and 6.

To demonstrate the relation between Cases 1 to 6, we adapt the parameter setting of Brock and Hommes (1997), i.e. we set $a = 10$, $b = 0.5$, $c = 1.35$ and $F = 1$. Since the model's steady-state price is given by $p^* = \frac{a}{b+c} = 5.41$, steady-state pre-tax profits of naive and rational firms amount to $\hat{\pi}^{N^*} = 0.5cp^{*^2} =$

19.72 and $\hat{\pi}^{R^*} = 0.5cp^{*^2} - F = 18.72$, respectively. Moreover, we assume that the firms' intensity of choice is higher than its critical value, i.e. $\beta > \beta_c = 0.78$. Based on our analytical results, we depict in panel (a) of Figure 1 combinations of λ and $\hat{\pi}^L$ for which the model's steady state is always unstable (dark gray area), always LAS (light gray area) and for which its LAS depends on the intensity of choice (white area).

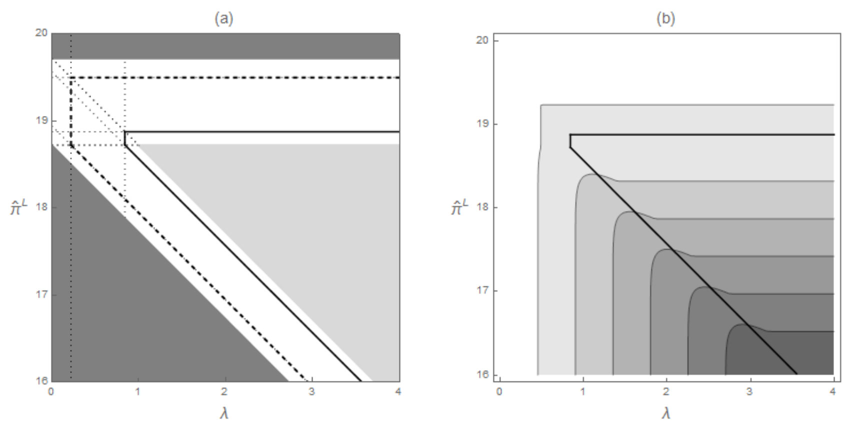

Figure 1: Panel (a) shows combinations of λ and $\hat{\pi}^L$ for which the steady state is unstable (dark gray), stable (light gray) and for which its stability depends on β (white). Panel (b) shows different categories for the tax burden at the steady state for $\beta = 5$. Category borders are given by 0.45, 0.9, 1.35, 1.8, 2.25 and 2.7. The solid (dashed) line represents the stability border for $\beta = 5$ ($\beta = 1$). Parameters are $a = 10$, $b = 0.5$, $c = 1.35$ and $F = 1$.

Note that the two regions in dark gray result from Cases 1 and 2, i.e. from $\hat{\pi}^L < 18.72 - \lambda$ and $\hat{\pi}^L > 19.72$. Recall that stability condition (32) is never fulfilled if we set $\beta > \beta_c = 0.78$. Combinations from Cases 1 and 2 are therefore always unstable. Case 3, i.e. $\hat{\pi}^L < 18.72$ and $\hat{\pi}^L > 19.72 - \lambda$, bounds the area colored in light gray. Since the corresponding stability condition (33) is always fulfilled, the model's steady state is LAS for parameter combinations satisfying Case 3. The white area represents Cases 4, 5 and 6 for which the steady state's LAS depends on the intensity of choice. To make this fact clear, we choose two different values for the intensity of choice. The solid (dashed) line is obtained when $\beta = 5$ ($\beta = 1$) is plugged into stability conditions (34), (35) and (36) of Cases 4, 5 and 6, respectively. All parameter combinations that are to the right of/below the solid (dashed) line are LAS, while those left/above are unstable. Obviously, the higher the intensity of choice, the smaller the area representing LAS parameter combinations of λ and $\hat{\pi}^L$. For $\beta = \beta_c = 0.78$, the stability borders of Cases 4, 5 and 6 are equal to the lines limiting the region that is always unstable. If the intensity of choice increases to $\beta = \infty$, the stability borders move to the right until they coincide with the borders of the area in light gray.

Policy-makers may furthermore wish to keep the firms' tax burden low, which is defined by

$$TB = \frac{m^* + 1}{2}\lambda^{R^*} + \frac{1 - m^*}{2}\lambda^{N^*}, \tag{37}$$

where $\frac{m^*+1}{2}$ and $\frac{1-m^*}{2}$ express the steady-state market shares of rational and naive firms, respectively. Of course, the tax burden at the steady state depends on λ and $\hat{\pi}^L$. To illustrate the influence of these two parameters on the firms' tax burden, we set $\beta = 5$ and plot in panel (b) of Figure 1 the firms' steady-state tax burden in (λ, $\hat{\pi}^L$)-parameter space. To be precise, this panel shows seven categories for the tax burden in different shades of gray. Category borders are given by 0.45, 0.9, 1.35, 1.8, 2.25 and 2.7. The higher the tax burden, the darker the gray tone of the corresponding category. Once again, the solid black line displays the stability border for $\beta = 5$.

Overall, Figure 1 offers the following policy-relevant insights. For a given value of the intensity of choice, policy-makers can set λ and $\hat{\pi}^L$ such that the model's steady state is LAS. If the intensity of choice changes, for whatever reason, the model's steady state may become unstable. However, policy-makers can always find new combinations of λ and $\hat{\pi}^L$ for which the LAS of the model's steady state is re-established. In particular, high lump-sum taxes λ, combined with low threshold values $\hat{\pi}^L$, promote market stability. In order to obtain stable dynamics with a minimum tax burden, policy-makers should choose combinations of λ and $\hat{\pi}^L$ that are located within the stable area and that are colored in the second lightest shade of gray (e.g. $\lambda = 2$ and $\hat{\pi}^L = 18.8$).

4. Numerical insights

In this section, we first present a number of simulations to illustrate how the model's global dynamics depends on the intensity of choice and on profit-dependent lump-sum taxes. We then explore the impact of profit-dependent lump-sum taxes on the market's volatility and the firms' tax burden, and attempt to derive a number of policy recommendations.

4.1. Global dynamics

Our analytical results suggest that policy-makers may use profit-dependent lump-sum taxes to support the LAS of the model's steady-state. As we will see, our local stability results are quite robust, and help us to understand the global behavior of the model. Figure 2 contains examples of how the intensity of choice and profit-dependent lump-sum taxes may influence the model's dynamics. Panel (a) of Figure 2 presents a bifurcation diagram for the intensity of choice.[3] Since $\lambda = 0$ and $\hat{\pi}^L = 0$, we observe the same rational route to randomness discussed in Brock and Hommes (1997), i.e. a stable steady state evolves into chaotic fluctuations as β increases. The primary bifurcation towards instability is a period-doubling bifurcation at which the steady state becomes unstable and a stable 2-cycle emerges. If β becomes larger, further bifurcations occur and the model dynamics becomes more and more complicated. The destabilizing impact of the intensity of choice can be explained as follows. Recall that naive expectations generate higher steady-state profits than rational expectations. For increasing values of β, firms react more sensitively to profit differences between their expectations rules and a larger number of them will opt for the expectation rule with the higher profit. Consequently,

[3] In all bifurcation diagrams, the bifurcation parameter is increased in 500 discrete steps. For each parameter combination, 30 observations are plotted after a transient phase of 1,000 periods has been omitted.

the steady-state difference in market shares m^*, defined as $m^* = n^{R^*} - n^{N^*}$, decreases. For $\beta = \beta_c = 0.78$, we have $m^* = -\frac{b}{c} = -0.37$, and the steady state becomes unstable. To show the model's dynamics for high values of the intensity of choice, we set $\beta = 5$ and plot an exemplary time series in panel (d) of Figure 2. Apparently, prices evolve quite erratically.

We explore the effect of profit-dependent lump-sum taxes by setting $\beta = 5$ and using λ and $\hat{\pi}^L$ as bifurcation parameters. To keep the analysis consistent with Figure 1, we consider the parameter space $0 \leq \lambda \leq 4$ and $16 \leq \hat{\pi}^L \leq 20$. In panel (g) of Figure 2, we present a bifurcation diagram for λ with $\hat{\pi}^L = 18$. By increasing λ from 0 to 4, we observe that chaotic fluctuations abruptly turn into a 2-cycle with a very low amplitude that converges almost immediately to $\tilde{p}^* = 0$. We know from our analytical results that the model's steady state becomes LAS when the lump-sum tax exceeds $\lambda_c = 1.57$. To be more precise, if $\hat{\pi}^L = 18$ and λ increases beyond 0.72, the dynamical system moves from Case 1 to Case 6, i.e. $\hat{\pi}^L \leq \hat{\pi}^{R^*} = 18.72 \leq \hat{\pi}^H < \hat{\pi}^{N^*} = 19.72$. Thus, the stability condition is given by (36), yielding $\lambda_c = 0.5cp^{*^2} - \hat{\pi}^L - \frac{ln\frac{c+b}{c-b}}{\beta} = 1.57$. The stabilizing impact of the lump-sum tax can be explained as follows. In Case 6, the relative fitness function is given by $\pi_t^\Delta = (\hat{\pi}_t^R - (\hat{\pi}_t^R - \hat{\pi}^L)) - (\hat{\pi}_t^N - \lambda)$, implying that $m_t = tanh(-0.5\beta(\hat{\pi}_t^N - \hat{\pi}^L - \lambda))$. If λ increases, the profit differential $\pi_t^\Delta = \hat{\pi}_t^N - \hat{\pi}^L - \lambda$ decreases. Consequently, more firms opt for rational expectations, and the dynamics stabilizes. For $\lambda \geq 1.72$, the system eventually enters Case 3 in which the model's steady state is stable for all values of β.

Policy-makers can also influence the dynamics by varying the profit threshold. Panel (j) of Figure 2 contains a bifurcation diagram in which $\hat{\pi}^L$ is increased from 16 to 20 while $\lambda = 2$. As can be seen, chaotic dynamics abruptly turns into a 2-cycle, which transitions into $\tilde{p}^* = 0$ before another 2-cycle emerges that passes into chaotic fluctuations again. These observations correspond to our analytical results. If $\lambda = 2$ and $\hat{\pi}^L$ exceeds 16.72, the dynamical system moves again from Case 1 to Case 6. We know from (36) that $\hat{\pi}_c^L = 0.5cp^{*^2} - \lambda - \frac{ln\frac{c+b}{c-b}}{\beta} = 17.57$, i.e. the model dynamics are LAS as soon

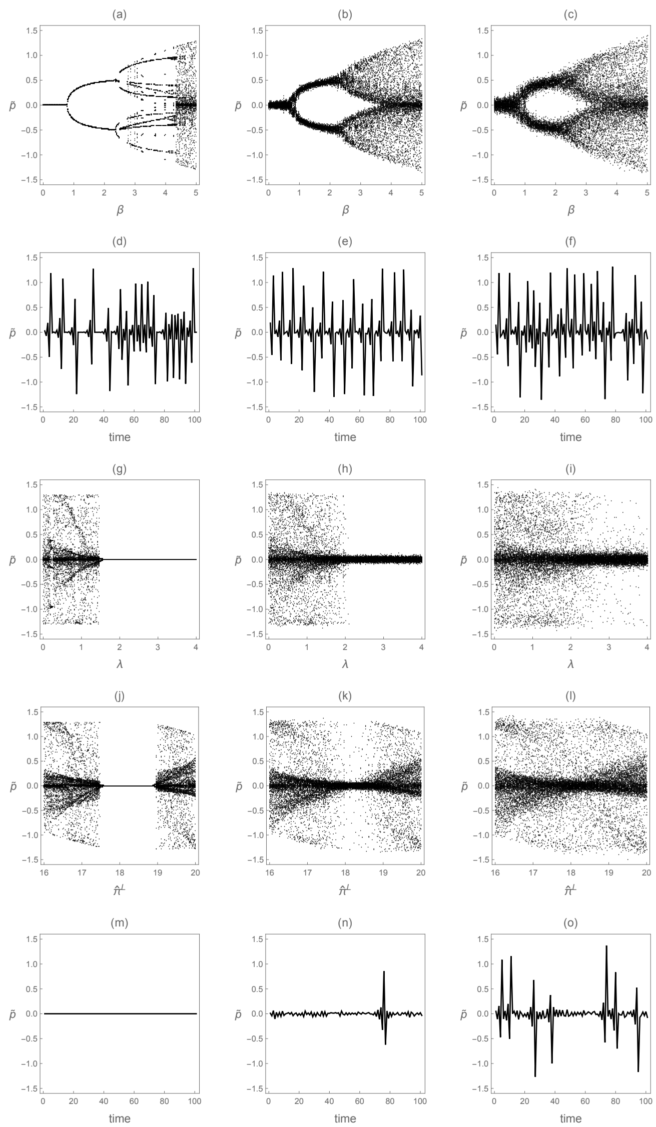

Figure 2: Panels (a), (d), (g), (j) and (m) show a bifurcation diagram for β with $\lambda = 0$ and $\hat{\pi}^L = 0$, a time series for $\beta = 5$, $\lambda = 0$ and $\hat{\pi}^L = 0$, a bifurcation diagram for λ with $\beta = 5$ and $\hat{\pi}^L = 18$, a bifurcation diagram for $\hat{\pi}^L$ with $\beta = 5$ and $\lambda = 2$, and a time series for $\beta = 5$, $\lambda = 2$ and $\hat{\pi}^L = 18$, respectively. The second and third column repeat the simulations from the first column with 5 and 10 percent exogenous noise, respectively. Parameters are $a = 10$, $b = 0.5$, $c = 1.35$ and $F = 1$.

as the profit threshold exceeds this value. The stabilizing impact of the profit threshold also becomes clear from $m_t = tanh(-0.5\beta(-\hat{\pi}^L + \hat{\pi}_t^N - \lambda))$. Obviously, policy-makers can reduce the profit differential by increasing $\hat{\pi}^L$. For $17.72 \leq \hat{\pi}^L \leq 18.72$, the dynamics is stable since the dynamical system is in Case 3. However, if the profit threshold exceeds 18.72, the dynamical system enters Case 5. The requirement for the model's steady state to be LAS is now obtained from (35), yielding $\hat{\pi}^L < \frac{ln\frac{c+b}{c-b}}{\beta} + 0.5cp^{*^2} - F = 18.88$. Accordingly, the dynamics are unstable for $\hat{\pi}^L \geq 18.88$. In this case, the relative fitness function is given by $\pi_t^\Delta = \hat{\pi}_t^R - (\hat{\pi}_t^N - (\hat{\pi}_t^N - \hat{\pi}^L))$. It becomes evident from $m_t = tanh(-0.5\beta(\hat{\pi}^L - \hat{\pi}_t^R))$ that an increasing profit threshold leads to higher profit differentials. Hence, the profit threshold needs to be decreased in order to stabilize the dynamics. For $\hat{\pi}^L > 19.72$, the dynamical system arrives at Case 2 and endogenous dynamics set in. Panel (m) of Figure 2 depicts the model dynamics for profit-dependent lump-sum taxes with $\lambda = 2$ and $\hat{\pi}^L = 18$. Since the dynamical system is in Case 3, the dynamics converges to the model's steady state, irrespective of the firms' intensity of choice.

The numerical insights obtained from the bifurcation diagrams depicted in panels (g) and (j) in Figure 2 can also be related to the analytical stability results summarized in panel (a) of Figure 1. Suppose first that policy-makers set $\hat{\pi}^L = 18$ and increase the lump-sum tax from 0 to 4. For $\lambda < 0.72$, $0.72 \leq \lambda < 1.72$ and $\lambda \geq 1.72$, the model is in Case 1 (dark gray area, always unstable), Case 6 (white area, stability depends on β) and Case 3 (light gray area, always stable), respectively. Since $\beta = 5$, the model's steady state is LAS for values for the lump-sum tax to the right of the solid line, i.e. for $\lambda \geq 1.57$. Suppose next that policy-makers set $\lambda = 2$ and increase the profit threshold from 16 to 20. The dynamical system then attains the following scenarios: Case 1 for $\hat{\pi}^L < 16.72$, Case 6 for $16.72 \leq \hat{\pi}^L < 17.72$, Case 3 for $17.72 \leq \hat{\pi}^L \leq 18.72$, Case 5 for $18.72 < \hat{\pi}^L \leq 19.72$ and Case 2 for $\hat{\pi}^L > 19.72$. As can be seen, the model's steady state is LAS for $17.57 \leq \hat{\pi}^L < 18.88$.

To investigate whether these results are robust, we add exogenous noise to the dynamics. In the second and third column of Figure 2, we repeat our simulations from the first column, but add a normally distributed random variable with mean zero and a standard deviation of 0.025 and 0.05 to the evolution of $\tilde{p}_t$ in (18), respectively. Panels (b) and (c) reveal that the rational route to randomness survives in a noisy environment. Panels (h), (i), (k) and (l) show that the stabilizing effect of profit-dependent lump-sum taxes is robust with respect to exogenous shocks. However, the stabilizing impact of λ and $\hat{\pi}^L$ decreases with the noise level. For instance, we only observe one volatility outbreak in panel (n), while there are a few more volatility outbreaks in panel (o). Compared to the unregulated market, depicted in panel (f), the market displays less wild fluctuations.

4.2. Volatility, tax burden and policy implications

Our analytical and numerical analysis reveals the existence of a large/robust set of (λ, $\hat{\pi}^L$)-parameter combinations that calm the model's dynamics. Next, we extend our analysis and study how profit-dependent lump-sum taxes jointly affect the market's volatility and the firms' tax burden. As a measure for the market's volatility, we use the standard deviation of the price (in deviations from its steady state)

$$V = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\tilde{p}_t - \bar{\tilde{p}})^2}, \tag{38}$$

where T and $\bar{\tilde{p}}$ represent the length and the mean of the underlying sample. We compute the firms' mean tax burden as

$$B = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{m_t + 1}{2}\lambda_t^R + \frac{1 - m_t}{2}\lambda_t^N\right). \tag{39}$$

Our simulations are based on a sample length of $T = 10,000$ periods, where a transient phase of 1,000 periods has been omitted.

Figure 3 provides examples of how policy-makers may influence the market's volatility and the firms' mean tax burden by adjusting λ and $\hat{\pi}^L$. In all simulations, we set $\beta = 5$. For illustrative reasons, the mean tax burden is divided by three. The simulations depicted in the first column are based on our deterministic model. As a robustness check, we repeat these simulations in the second and third column with 5 and 10 percent exogenous noise, respectively.[4] Panels (a) to (c) of Figure 3 correspond to panels (g) to (i) of Figure 2. Setting $\hat{\pi}^L = 18$ and increasing the lump-sum tax from 0 to 4 decreases the volatility (black line) and increases the mean tax burden (gray line). The trade-off between the volatility of the market and the firms' mean tax burden is also evident when exogenous noise is added to the dynamics. However, the potential reduction in volatility decreases with the noise level. Panels (d) to (f) of Figure 3 refer to panels (j) to (l) of Figure 2, i.e. we now set $\lambda = 2$ and increase the profit threshold from 16 to 20. As can be seen, the firms' mean tax burden (gray line) decreases while the volatility of the market (black line) first decreases and then increases. While higher noise levels diminish the possible reduction in volatility, the mean tax burden basically remains constant.

Policy-makers wishing to minimize both the volatility of the market and the firms' mean tax burden should be aware of inefficient λ and $\hat{\pi}^L$ combinations. For instance, panel (f) of Figure 3 reveals that a volatility of $V = 0.42$ emerges for $\hat{\pi}^L = 17.28$ and $\hat{\pi}^L = 19.00$, producing, in turn, mean tax burdens of $B = 0.48$ and $B = 0.25$, respectively. In the following, we assume that policy-makers prefer the alternative with the lower mean tax burden. To identify all efficient combinations of λ and $\hat{\pi}^L$, we use a grid resolution of 100×100 for the parameter space $0 \leq \lambda \leq 4$ and $16 \leq \hat{\pi}^L \leq 20$ and represent all 10,000 realizations of V and B in panels (g) to (i) of Figure 3 by a gray dot. For instance, panel (g) indicates that for a volatility level of $V = 0$, the lowest possible value for the mean tax burden is $B = 0.50$. Linking all efficient combinations yields an efficient frontier of λ and $\hat{\pi}^L$ combinations, represented by the black line.

The efficient frontier indicates the price (the firms' mean tax burden) for a reduction in the volatility of the market. In this sense, the price for a complete stabilization in the deterministic case is given by $B = 0.50$. This number can be

[4] Since the standard deviation of price fluctuations is about 0.5 when there is no profit-dependent lump-sum tax, $\sigma = 0.025$ and $\sigma = 0.05$ correspond to noise levels of roughly 5 and 10 percent.

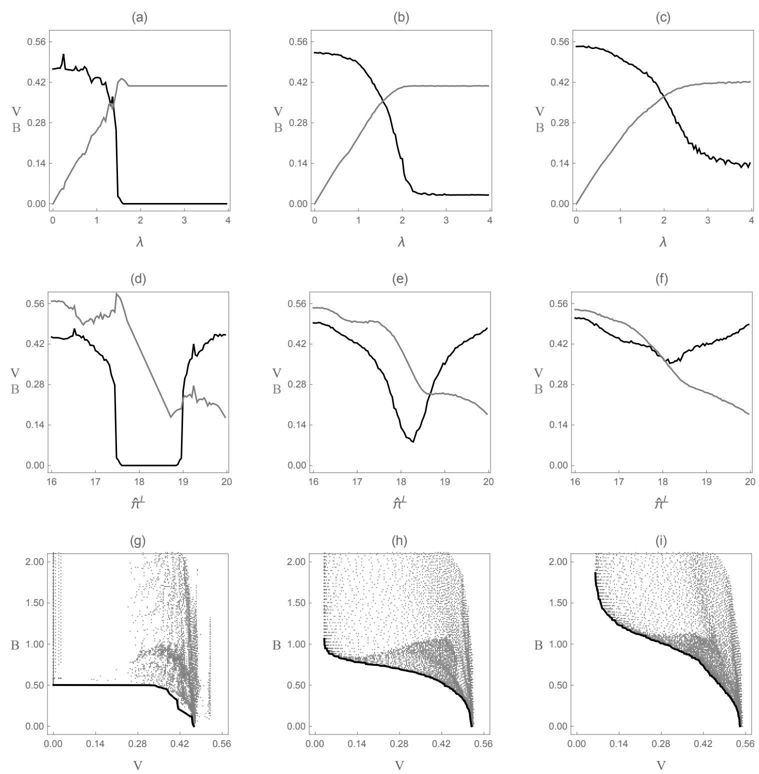

Figure 3: Panel (a) shows the volatility (black line) and the mean tax burden (gray line), divided by three, as a function of λ with $\hat{\pi}^L = 18$. Panel (d) shows the same as a function of $\hat{\pi}^L$ with $\lambda = 2$. Panel (g) shows all possible combinations of V and B for $0 \leq \lambda \leq 4$ and $16 \leq \hat{\pi}^L \leq 20$. The black line represents the efficient frontier. The second and third column repeats the simulations from the first column with 5 and 10 percent exogenous noise, respectively. Parameters are $a = 10$, $b = 0.5$, $c = 1.35$, $F = 1$ and $\beta = 5$.

compared directly to the approach by Schmitt and Westerhoff (2015). In their setup, in which firms face a proportional tax on positive profits, a tax rate of 84.4 percent is required to completely stabilize the dynamics, implying a mean tax burden of $B = 16.39$. Putting these numbers into perspective reveals that a profit-dependent lump-sum tax can reduce the firms' mean tax burden by more than 95 percent. Panels (h) and (i) reveal that the higher the exogenous noise, the higher the mean tax burden for a volatility reduction. However, if policy-makers still regard a mean tax burden of 1.5 as acceptable for firms, they are able to significantly reduce the volatility of the market even in the presence of substantial exogenous noise.

Figure 4 continues the analysis of Figure 3. In panels (a) to (f) of Figure 4, we define seven different categories for the volatility and the mean tax burden and plot them in (λ, $\hat{\pi}^L$)- parameter space. While volatility category borders are given by 0.075, 0.15, 0.225, 0.3, 0.375 and 0.45, we define 0.45, 0.9, 1.35, 1.8, 2.25 and 2.7 as category borders for the mean tax burden. Accordingly, the lower the volatility (mean tax burden), the lighter the gray tone of the corresponding category. Efficient combinations of λ and $\hat{\pi}^L$ are represented by the black dots. Obviously, efficient combinations of λ and $\hat{\pi}^L$ associated with a low mean tax burden (light area) tend to correspond to a high volatility (dark area), while combinations of λ and $\hat{\pi}^L$ associated with a high mean tax burden (dark area) tend to correspond to a low volatility (light area).

Panels (a) and (d), depicting the outcome for the deterministic case, can be compared with the two panels of Figure 1. As can be seen, our numerical analysis also reveals a triangle-shaped area in which combinations are located for which the model's steady state is LAS. In order to ensure stable dynamics, policy-makers should choose combinations within the white area. If policy-makers additionally aim at minimizing the firms' tax burden, they should choose one of the efficient combinations located within the stable area. Panel (d) reveals that those combinations are linked with the second category for the mean tax burden. Panel (b) of Figure 4 shows that the white stability area shrinks when simulations are repeated with 5 percent exogenous noise. To reduce fluctuations, policy-makers now need to set the profit threshold lower and the lump-sum tax higher than in panel (a). As a result, the firms' mean tax burden increases, as revealed by panel (e). Efficient combinations for which the dynamics is relatively stable are now located in the third category of the firms' mean tax burden. Panels (c) and (f) indicate that the stabilizing effects of profit-dependent lump-sum taxes weaken further when the noise level is set to 10 percent. Accepting a higher mean tax burden for firms, policy-makers are nevertheless able to reduce the volatility of the market. For instance, selecting a profit-dependent lump-

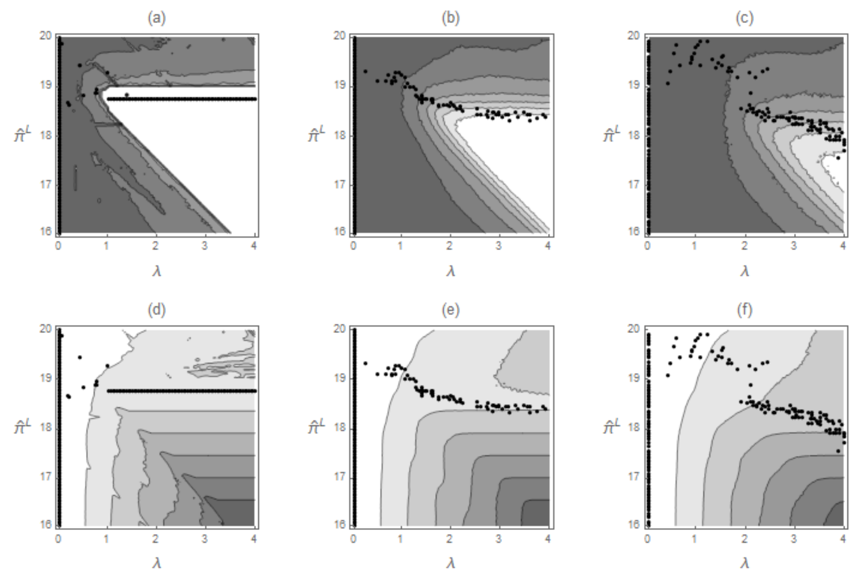

Figure 4: Panels (a) and (d) show different categories for the volatility and the mean tax burden in (λ, $\hat{\pi}^L$)-space, respectively. Category borders for volatility are given by 0.075, 0.15, 0.225, 0.3, 0.375 and 0.45, while 0.45, 0.9, 1.35, 1.8, 2.25 and 2.7 represent category borders for the mean tax burden. Black dots represent combinations of the efficient frontier. The second and third column repeats the simulations from the first column with 5 and 10 percent exogenous noise, respectively. Parameters are $a = 10$, $b = 0.5$, $c = 1.35$, $F = 1$ and $\beta = 5$.

sum tax in the range of $17.8 < \hat{\pi}^L < 18$ and $3.6 < \lambda < 4$ yields a volatility level lower than 0.15 and a mean tax burden of around 1.35.

5. Conclusions

The seminal cobweb model by Brock and Hommes (1997) reveals that an evolutionary competition between cheap destabilizing and costly stabilizing expectations rules may create complex endogenous dynamics. In recent years, this approach has gained tremendous empirical support, as documented in Hommes (2013), for instance. Schmitt and Westerhoff (2015) show that policy-makers may be able to calm such dynamics by imposing a proportional profit tax. Since high tax rates reduce the profit advantage of cheap destabilizing expectation rules, costly stabilizing expectation rules gain in popularity and, as a result, markets become more stable. In this paper, we show that policy-makers may also stabilize markets by imposing profit-dependent lump-sum taxes. An important insight of our analysis is that it is not necessary to impose high profit taxes to reduce profit differentials between competing expectation rules. If destabilizing expectation rules outperform stabilizing expectation rules because stabilizing expectation rules incur positive per period information costs, as assumed in Brock and Hommes (1997) and many other related papers, policy-makers may reduce the expectation rules' profit differentials by imposing a rather modest profit-dependent lump-sum tax. Our analysis suggests that such a tax does not only work locally in a deterministic framework, but also in a noisy out-of-equilibrium environment. While the effectiveness of profit-dependent lump-sum taxes depends on the noise level, policy-makers may even be able to stabilize markets and limit firms' tax burden for higher noise levels.

Our analysis is based on the cobweb model by Brock and Hommes (1997), and may be extended in various directions. First of all, one could consider the case that firms have a choice between a different pair of competing expectation rules, say naive and adaptive expectations. Similarly, one could study a model version in which firms can apply more than two expectations rules. It would be interesting to assume that these rules have different constant or even flexible cost advantages. In our setup, firms only take the last observed profit of the expectations rules into account. However, firms may use a smoothed measure of past realized profits as a fitness criterion. Relatedly, the firms' rule selection behavior may depend on additional socio-economic principles such as current market circumstances or herding effects. One could also investigate the effects of profit-dependent lump-sum taxes for other markets. For instance, can a profit-dependent lump-sum tax be used to influence the behavior of financial market participants who switch between technical and fundamental trading rules? In this paper we ask whether policy-makers can stabilize markets by imposing a small profit-dependent lump-sum tax. We believe that market stability is an important policy goal in reality but, of course, it would also be interesting to explore this issue in more detail from a welfare perspective. To conclude, the basic message of our paper - policy-makers' opportunity to reduce cost disadvantages of stabilizing expectation rules via profit-dependent lump-sum taxes and thereby to support market stability without causing high tax burdens - seems to be quite useful and worthwhile for further investigations.

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