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The effectiveness of Keynes–Tobin transaction taxes when heterogeneous agents can trade in different markets: A behavioral finance approach ☆

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Abstract

We develop a model in which boundedly rational agents apply technical and fundamental analysis to identify trading signals in two different speculative markets. Whether an agent trades and, if so, in which market with which strategy depends on profit considerations. As it turns out, an ongoing evolutionary competition between the trading strategies causes complex price dynamics which closely resembles the behavior of actual speculative prices. Moreover, we find that if the agents have to pay a transaction tax in one market, price variability decreases in this market but increases in the other market. However, the imposition of a uniform tax on all transactions stabilizes both markets. Our results suggest that if regulators of a market introduce a transaction tax, other markets are likely to follow.

JEL classification: D84; G14; G18

Keywords: Speculative markets; Transaction tax; Heterogeneous agents; Technical and fundamental analysis

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1. Introduction

According to classical theory, asset prices properly reflect their fundamentals since arbitrageurs quickly counter mispricings. Keynes (1936) doubted this hypothesis and provided a contrasting view of financial markets. He argued that many persons lack the capability to compute fundamentals correctly. Instead, they are subject to waves of optimistic and pessimistic sentiment. Prices may thus change violently as the result of a sudden shift of opinion. In addition, day-to-day changes in fundamentals may have an excessive impact on prices. Incentives to correct the vagaries of 'ignorant' investors are limited. On the contrary, most 'expert' investors are concerned with outwitting the crowd. What matters is not what an investment is really worth but what the crowd thinks how the crowd will evaluate it.

Keynes concluded that pure laissez-faire capitalism does not fulfill its social purpose. The introduction of a transaction tax might thus prove a serviceable reform, mitigating the predominance of destabilizing short-term speculation over stabilizing long-term investment. Keynes' suggestion obtained new momentum when Tobin (1978) proposed the imposition of a uniform tax of around 1 percent on all currency transactions in order to placate foreign exchange dynamics. Nowadays, a levy of between 0.05 and 0.5 percent is discussed (Eichengreen et al., 1995; Haq et al., 1996; Frankel, 1996; Spahn, 2002). Even such small tax rates still have a strong impact on high-frequency trading. For example, if the tax rate is 0.05 percent, then a person who goes in and out of the market once a day accumulates an annual tax burden of about 43 percent. Long-term investors are, of course, less strongly penalized.

Although the Keynes–Tobin transaction tax mechanism is vividly debated in the popular media, academic scrutiny has remained scant. One reason was the lack of theoretical models that take into account the action of heterogeneous speculators. But this obstacle has dissolved with the advent of the chartist–fundamentalist approach. Contributions by, e.g. Day and Huang (1990), Kirman (1991), Lux (1995) or Brock and Hommes (1997) show that the behavior of heterogeneous boundedly rational speculators may endogenously create complex price dynamics. These models, which are in harmony with Keynes' (1936) view of financial markets, have clearly improved our understanding of what is going on in the markets.

The goal of this paper is twofold: To develop a simple model in which agents can trade in two speculative markets and to investigate how transaction taxes alter the dynamics. Within our model, agents have five options. They may apply technical or fundamental analysis in market 1 or 2, or they may abstain from trading. The agents tend to pick those rules which did well in the past. Though the (deterministic) evolution of the prices in the two markets is governed by a 10-dimensional (10-D) non-linear map, we are able to derive analytical conditions for the local asymptotic stability of the steady state. In addition, we show that the steady state may lose stability through a Flip or a Neimark–Hopf bifurcation. If we add dynamic noise to the system, then the model's dynamics resembles those of actual markets closely. We observe intricate price motion, bubbles and crashes, high volatility, excess kurtosis, and clustered volatility. The driving force of the complex dynamics is an enduring evolutionary competition between the trading strategies.

Transaction taxes affect the competition between trading strategies in a non-trivial way. Suppose a small tax is imposed in market 1. Then market 1 is stabilized but market 2 is destabilized. Although some agents retreat from trading, some destabilizing speculators also migrate from market 1 to market 2. If the agents have to pay a uniform levy in both markets, chartism declines in favor of fundamentalism in both markets and thus both markets display lower price fluctuations and deviations from fundamentals. Hence, there is no reason for regulators of a market not to impose such a tax – at least the own market will benefit.

The remainder of this paper is organized as follows. Section 2 briefly reviews the chartist–fundamentalist approach. In Section 3, we present a model in which investors can switch between two markets. Section 4 contains our stability and bifurcation analysis. In Section 5, the model is calibrated to speculative markets. Section 6 studies the impact of transaction taxes and the last section concludes the paper.

2. Survey of the literature

Experimental evidence has long suggested that people generally lack the computational power to derive fully optimal actions (Simon, 1955). But this does not imply that they are irrational. In fact, people strive to do the right thing. Their behavior may better be described as a rule-governed behavior. As argued by psychologists, people rely on a limited number of simple heuristics which have proven to be useful in the recent past (Kahneman et al., 1986). This observation may be crucial since if one is able to identify the agents' main heuristics, then it should be possible to model their behavior. In our case, two related strands of research are relevant: survey studies and laboratory experiments.

Questionnaire evidence informs us that financial market participants rely on technical and fundamental analysis to determine their orders. As reported by Taylor and Allen (1992), most professional traders are familiar with both concepts. For short-term predictions, technical and fundamental analysis are judged as equally important. Which rule is applied in a given situation depends on market circumstances. Technical analysis aims at deriving trading signals out of past price movements (Murphy, 1999). Such positive feedback rules are likely to destabilize the markets. The intention of fundamental analysis is to exploit deviations between prices and fundamentals. Betting on mean reversion tends to stabilize the markets.

Smith (1991) pioneered the use of laboratory asset markets to observe investor behavior in a well-defined and controlled environment. In the experiments, each participant receives an initial portfolio of cash and stocks and is free to trade. The trading is conducted by computer through local networks. When all participants have entered their action, the next period's price is revealed. The experiments indicate that people use simple forms of forecast rules such as extrapolative or regressive predictors. Furthermore, they frequently drove asset prices far above fundamentals, after which the markets crashed. Bubble-and-crash sequences appear to be quite robust, even when all agents know the asset's true value (see, e.g. Caginalp et al., 2001; Sonnemans et al., 2004; Hommes et al., 2005).

The chartist–fundamentalist approach is based on these observations and aims at explaining the complicated price dynamics as observed in actual speculative markets. Some of these models highlight the fact that the agents' trading rules may be non-linear (Day and Huang, 1990; Chiarella, 1992; Farmer and Joshi, 2002). For instance, Day and Huang (1990) assume that chartists generate their orders according to linear trading rules but that the demand of fundamentalists increases exponentially with respect to mispricings.[1] If the price is close to its fundamental, chart orders outweigh fundamental orders and the price is driven away from fundamentals. But the higher the distortion is, the stronger the demand of the fundamentalists becomes and prices are – temporarily – pushed back to fundamentals.

But agents also switch between trading rules. Kirman (1991, 1993) explores sudden swings in market opinion due to social interactions. Speculators frequently talk to each other and observe what others do. Coordinated behavior may arise, triggering severe bubbles and crashes. In Brock and Hommes (1997, 1998), agents choose between cheap naive predictors and costly sophisticated predictors. The agents are rational in the sense that they prefer profitable predictors. Suppose that the majority of agents rely on precise predictors. Then prices are driven towards fundamentals. But with prices close to fundamentals, prediction errors of naive rules become small. Since they are relatively cheap, higher profits are generated than with expensive predictors. As the impact of naive rules increases, prices disconnect from fundamentals. Lux (1995, 1997, 1998) combines both economic and social factors. On the one hand, agents switch between technical and fundamental analysis because of profit differentials. On the other hand, the mood of chartists depends on social interactions within their group. They may turn from an optimistic into a pessimistic mood and vice versa. Significant bubbles may occur if agents increasingly turn into optimistic chartists. The bubble is stopped when expected arbitrage opportunities make fundamental analysis appear superior.

These contributions do not only produce intricate motion, some of them even generate time series which are not distinguishable from actual time series: Artificial as well as actual prices show bubbles and crashes, excess volatility, fat tails for the distribution of the returns, uncorrelated price changes and volatility clustering. Recent interesting refinements – incorporating wealth, learning or other aspects – include Caginalp et al. (2000), Cont and Bouchaud (2000), Gaunersdorfer (2000), Lux and Marchesi (2000), De Grauwe and Grimaldi (2002), Chiarella and He (2003), or Rosser et al. (2003).

To sum up, the chartist–fundamentalist approach gives a realistic picture of speculative behavior. Therefore, it seems to be reasonable to employ such a setup as an artificial laboratory to study the effectiveness of certain regulatory policies.

[1] In De Grauwe et al. (1993), the market impact of fundamentalists is non-linear since their estimates of the long-run equilibrium price are normally distributed around its true value.

Besides theoretical reasoning, empirical studies or laboratory experiments, computer simulations may be regarded as an additional instrument to design successful new trading institutions. Simulation studies have the advantage that one can control for all kinds of shocks, measure the variables precisely and produce as many observations as required. A couple of studies have already followed this avenue. Westerhoff (2003a) reports that trading breaks may successfully limit the destabilizing activity of chartists while Ehrenstein (2002) finds that transaction taxes may reduce exchange rate volatility and generate significant tax revenues. Westerhoff (2003b) obtains similar findings but warns that if the tax rate is set too high then too many stabilizing fundamental traders leave the market so that prices stop tracking their fundamentals. Ehrenstein et al. (2004) take into account the feedback that reduced speculation via reduced market depth may increase exchange rate variability. They show that a transaction tax may still reduce volatility and distortion.

The goal of this paper is to analyze the effect of transaction taxes when speculators can trade in more than one market. Multi-asset market models with interacting chartists and fundamentalists are, unfortunately, rare. Westerhoff (2004) develops a model in which fundamentalists are restricted to a certain market and chartists can wander between markets. Chiarella et al. (2004) present an interesting framework in which the agents' orders depend on the assets' cross-correlation. Both approaches reveal novel generators of complex endogenous dynamics. For our purpose, however, we have to develop a new model which will be presented in the next section. Our model is inspired by the aforementioned contributions, especially by the type of models surveyed in Hommes (2001).

3. The model

For simplicity, we consider only trading in two speculative markets. The agents can rely on technical or fundamental trading strategies to determine their orders. While technical analysis goes with past price trends, fundamental analysis predicts a convergence towards fundamentals. But the agents are free to trade and so they have five options in total. The agents decide on a certain option, depending on its relative fitness, where the fitness is given as a weighted average of current and past profits. For instance, the strategy 'no trading' produces no profits and consequently has a fitness of zero. The decision of the agents is rational in the sense that the higher the fitness of a rule, the more agents will select it. After the agents have decided on a strategy, they submit their orders. Prices adjust with respect to excess demand via a price impact function: Excess buying drives prices up and excess selling drives them down. After the new prices have been revealed, the next trading round begins.

Let us now develop the model. We approximate the price adjustment process by a log-linear price impact function (Farmer and Joshi, 2002). Such a function describes the relation between the number of assets bought or sold in a given time interval and the price change caused by these orders. The log prices of markets 1 and 2 in period $t+1$ are given as

$$S^1_{t+1} = S^1_t + a^M(W^{C,1}_t D^{C,1}_t + W^{F,1}_t D^{F,1}_t) + r^{M,1}_t \tag{1}$$

and

$$S^2_{t+1} = S^2_t + a^M(W^{C,2}_t D^{C,2}_t + W^{F,2}_t D^{F,2}_t) + r^{M,2}_t, \tag{2}$$

where a^M is a positive price adjustment coefficient, $D^{C,1}$, $D^{C,2}$, $D^{F,1}$ and $D^{F,2}$ stand for the orders due to technical or fundamental analysis rules in markets 1 or 2, respectively, and $W^{C,1}$, $W^{C,2}$, $W^{F,1}$ and $W^{F,2}$ denote the fractions of agents who follow these rules. Accordingly, excess buying (selling) increases (decreases) prices. Since our model represents only a simplification of actual order matching mechanism (e.g. market maker or limit order book) we add random variables $r^{M,1} \sim \mathrm{N}(0, \sigma^{M,1})$ and $r^{M,2} \sim \mathrm{N}(0, \sigma^{M,2})$ to the systems' equations.

Orders generated by technical analysis in markets 1 and 2 are specified as

$$D^{C,1}_t = a^C(S^1_t - S^1_{t-1}) + r^{C,1}_t \tag{3}$$

and

$$D^{C,2}_t = a^C(S^2_t - S^2_{t-1}) + r^{C,2}_t. \tag{4}$$

The reaction coefficient a^C is positive and captures the strength of positive feedback trading. Although the 'philosophy' of technical analysis it to ride on a trend, there exist a myriad of different technical trading rules (Murphy, 1999). To capture part of the variety in technical analysis we include random variables $r^{C,1} \sim \mathrm{N}(0, \sigma^{C,1})$ and $r^{C,2} \sim \mathrm{N}(0, \sigma^{C,2})$.

Orders produced by fundamental analysis in markets 1 and 2 are written as

$$D^{F,1}_t = a^F(F^1_t - S^1_t) + r^{F,1}_t \tag{5}$$

and

$$D^{F,2}_t = a^F(F^2_t - S^2_t) + r^{F,2}_t, \tag{6}$$

where a^F is a positive reaction coefficient and F^1 and F^2 are the log fundamental values of markets 1 and 2, respectively. For instance, if an asset is undervalued ($S < F$), buying is suggested. Fundamental values may change over time due to real shocks. Since modeling the evolution of the fundamentals as random walks has no impact on the results, we set – for the sake of convenience – the fundamental values at constant. As already remarked by Keynes (1936), it is quite difficult for investors to determine the fundamentals. To incorporate errors in the perception of fundamentals, we buffet (5) and (6) with random variables $r^{F,1} \sim \mathrm{N}(0, \sigma^{F,1})$ and $r^{F,2} \sim \mathrm{N}(0, \sigma^{F,2})$.

The agents do not stick to a certain rule/market combination but compare their past performance. To be precise, the agents assign the 'no trading' alternative a fitness of zero and compute the fitness of the remaining options as follows:

$$\begin{aligned} A^{C,1}_t = {} & (\mathrm{Exp}[S^1_t] - \mathrm{Exp}[S^1_{t-1}])D^{C,1}_{t-2} - tax^1(\mathrm{Exp}[S^1_t] \\ & + \mathrm{Exp}[S^1_{t-1}])|D^{C,1}_{t-2}| + bA^{C,1}_{t-1}, \end{aligned} \tag{7}$$

$$A_t^{F,1} = (\text{Exp}[S_t^1] - \text{Exp}[S_{t-1}^1])D_{t-2}^{F,1} - tax^1(\text{Exp}[S_t^1] + \text{Exp}[S_{t-1}^1])|D_{t-2}^{F,1}| + bA_{t-1}^{F,1}, \tag{8}$$

$$A_t^{C,2} = (\text{Exp}[S_t^2] - \text{Exp}[S_{t-1}^2])D_{t-2}^{C,2} - tax^2(\text{Exp}[S_t^2] + \text{Exp}[S_{t-1}^2])|D_{t-2}^{C,2}| + bA_{t-1}^{C,2} \tag{9}$$

and

$$A_t^{F,2} = (\text{Exp}[S_t^2] - \text{Exp}[S_{t-1}^2])D_{t-2}^{F,2} - tax^2(\text{Exp}[S_t^2] + \text{Exp}[S_{t-1}^2])|D_{t-2}^{F,2}| + bA_{t-1}^{F,2}. \tag{10}$$

The first terms of the above equations indicate the most recent performance. Note the timing of the model: Orders submitted in period $t-2$ are filled at prices in period $t-1$.[2] Profits then depend on prices in period t.[3] The second term reflects the costs of trading with respect to transaction taxes, where tax^1 is the tax rate of market 1 and tax^2 is the tax rate of market 2. The fitness of the strategies furthermore depends on their past performance (Hommes, 2001). The memory parameter b measures how fast current fitness is discounted for strategy selection. For $b=0$, the fitness equals current profits. But the larger the memory of the agents, the more strongly the fitness depends on its past performance. If $b=1$, then the fitness is calculated as the sum over all past profits.

We are interested in how the importance of the strategies evolves over time. In order to simplify the model as far as possible, we do not keep track of the positions of individual agents. As will become clear later, it is the composition of strategies in a market that matters for stability. The percentage of agents choosing a certain option is expressed by a discrete choice model (Manski and McFadden, 1981)

$$W_t^{C,1} = \frac{\text{Exp}[cA_t^{C,1}]}{\text{Exp}[cA_t^{C,1}] + \text{Exp}[cA_t^{F,1}] + \text{Exp}[cA_t^{C,2}] + \text{Exp}[cA_t^{F,2}] + \text{Exp}[0]}, \tag{11}$$

$$W_t^{F,1} = \frac{\text{Exp}[cA_t^{F,1}]}{\text{Exp}[cA_t^{C,1}] + \text{Exp}[cA_t^{F,1}] + \text{Exp}[cA_t^{C,2}] + \text{Exp}[cA_t^{F,2}] + \text{Exp}[0]}, \tag{12}$$

$$W_t^{C,2} = \frac{\text{Exp}[cA_t^{C,2}]}{\text{Exp}[cA_t^{C,1}] + \text{Exp}[cA_t^{F,1}] + \text{Exp}[cA_t^{C,2}] + \text{Exp}[cA_t^{F,2}] + \text{Exp}[0]}, \tag{13}$$

$$W_t^{F,2} = \frac{\text{Exp}[cA_t^{F,2}]}{\text{Exp}[cA_t^{C,1}] + \text{Exp}[cA_t^{F,1}] + \text{Exp}[cA_t^{C,2}] + \text{Exp}[cA_t^{F,2}] + \text{Exp}[0]} \tag{14}$$

and

$$W_t^O = 1 - W_t^{C,1} - W_t^{F,1} - W_t^{C,2} - W_t^{F,2}. \tag{15}$$

[2]A so-called market order is a request to transact immediately at the best available price. In fact, the fill price is typically unknown to the speculator.

[3]Alternatively, one may use risk adjusted profits as a fitness measure. In real markets, however, pure profits seem to be what investors care most about (see again the discussion in Hommes, 2001).

The higher the fitness of a strategy, the more agents will rely on it. Parameter c captures how sensitive the mass of traders is to selecting the most attractive strategy. The higher c is, the more agents will select the option with the highest fitness. If c goes to plus infinity, all agents follow the option with the highest fitness. For $c = 0$, each option is used by 20 percent of the agents, regardless of its profitability. In this sense, c reflects the rationality of the agents.

4. Stability and bifurcation analysis

In this section, we analyze the underlying deterministic system without transaction taxes and we characterize the unique steady state of the model; we also derive analytical conditions for the local asymptotic stability of the steady state and highlight their dependence on the key parameters of the model (i.e. the reaction coefficients of chartists and fundamentalists and the price adjustment coefficient). Though the evolution of the prices is due to high-dimensional non-linear laws of motion, we shall see that closed analysis is not precluded due to the particular structure of the Jacobian of the deterministic system, evaluated at the steady state.

4.1. The underlying deterministic system

In order to get some insight into the underlying deterministic system, we drop all random terms from (1) to (6), as well as the terms which capture transaction taxes from (7) to (10). Taking into account also (11)–(14), we obtain a non-linear dynamic model with a high number of equations, some of which are second-order difference equations. However, the model can be reduced to a 10-D discrete-time dynamical system through suitable changes of variables. For $i = 1, 2$ we set

$$X^i_{t+1} = S^i_t,$$

$$Y^i_{t+1} = X^i_t (= S^i_{t-1})$$

and we rewrite one period ahead (7)–(10), i.e. we obtain

$$A^{C,i}_{t+1} = (\mathrm{Exp}[S^i_{t+1}] - \mathrm{Exp}[S^i_t]) D^{C,i}_{t-1} + b A^{C,i}_t,$$

$$A^{F,i}_{t+1} = (\mathrm{Exp}[S^i_{t+1}] - \mathrm{Exp}[S^i_t]) D^{F,i}_{t-1} + b A^{F,i}_t.$$

The orders by technical analysis at time t can thus be expressed as

$$D^{C,i}_t = a^C (S^i_t - X^i_t),$$

while the orders by technical analysis at time $(t-1)$, which appear in the dynamic equation for $A^{C,i}_{t+1}$, are given as $D^{C,i}_{t-1} = a^C(S^i_{t-1} - X^i_{t-1}) = a^C(X^i_t - Y^i_t)$, due to our changes of variables. On the other hand, the orders by fundamental traders at time $(t-1)$, which appear in the dynamic equation for $A^{F,i}_{t+1}$, are rewritten as $D^{F,i}_{t-1} = a^F(F^i - X^i_t)$. Therefore, we obtain the following 10-D system in the dynamic variables S^i, X^i, Y^i, $A^{C,i}$ and $A^{F,i}$:

$$S^1_{t+1} = S^1_t + a^M(W^{C,1}_t a^C(S^1_t - X^1_t) + W^{F,1}_t a^F(F^1 - S^1_t)), \tag{16}$$

$$X^1_{t+1} = S^1_t, \tag{17}$$

$$Y^1_{t+1} = X^1_t, \tag{18}$$

$$S^2_{t+1} = S^2_t + a^M(W^{C,2}_t a^C(S^2_t - X^2_t) + W^{F,2}_t a^F(F^2 - S^2_t)), \tag{19}$$

$$X^2_{t+1} = S^2_t, \tag{20}$$

$$Y^2_{t+1} = X^2_t, \tag{21}$$

$$A^{C,1}_{t+1} = (\mathrm{Exp}[S^1_{t+1}] - \mathrm{Exp}[S^1_t])a^C(X^1_t - Y^1_t) + bA^{C,1}_t, \tag{22}$$

$$A^{F,1}_{t+1} = (\mathrm{Exp}[S^1_{t+1}] - \mathrm{Exp}[S^1_t])a^F(F^1 - X^1_t) + bA^{F,1}_t, \tag{23}$$

$$A^{C,2}_{t+1} = (\mathrm{Exp}[S^2_{t+1}] - \mathrm{Exp}[S^2_t])a^C(X^2_t - Y^2_t) + bA^{C,2}_t, \tag{24}$$

$$A^{F,2}_{t+1} = (\mathrm{Exp}[S^2_{t+1}] - \mathrm{Exp}[S^2_t])a^F(F^2 - X^2_t) + bA^{F,2}_t. \tag{25}$$

Notice that the dynamical model (16)–(25) is driven by the iteration of a 10-D map, which gives the state of the system at time $(t+1)$, described by S^i_{t+1}, X^i_{t+1}, Y^i_{t+1} and $A^{K,i}_{t+1}$ for $i = 1, 2$ and $K \in \{C, F\}$ as a function of the state of the system at time t, i.e. S^i_t, X^i_t, Y^i_t and $A^{K,i}_t$. In fact, the other variables and quantities $W^{C,1}_t$, $W^{F,1}_t$, $W^{C,2}_t$, $W^{F,2}_t$, $(\mathrm{Exp}[S^1_{t+1}] - \mathrm{Exp}[S^1_t])$ and $(\mathrm{Exp}[S^2_{t+1}] - \mathrm{Exp}[S^2_t])$, which appear in the right-hand sides of (16), (19) and (22)–(25), are functions of the state at time t according to

$$W^{C,1}_t = \frac{\mathrm{Exp}[cA^{C,1}_t]}{\mathrm{Exp}[cA^{C,1}_t] + \mathrm{Exp}[cA^{F,1}_t] + \mathrm{Exp}[cA^{C,2}_t] + \mathrm{Exp}[cA^{F,2}_t] + \mathrm{Exp}[0]},$$

$$W^{F,1}_t = \frac{\mathrm{Exp}[cA^{F,1}_t]}{\mathrm{Exp}[cA^{C,1}_t] + \mathrm{Exp}[cA^{F,1}_t] + \mathrm{Exp}[cA^{C,2}_t] + \mathrm{Exp}[cA^{F,2}_t] + \mathrm{Exp}[0]},$$

$$W^{C,2}_t = \frac{\mathrm{Exp}[cA^{C,2}_t]}{\mathrm{Exp}[cA^{C,1}_t] + \mathrm{Exp}[cA^{F,1}_t] + \mathrm{Exp}[cA^{C,2}_t] + \mathrm{Exp}[cA^{F,2}_t] + \mathrm{Exp}[0]},$$

$$W^{F,2}_t = \frac{\mathrm{Exp}[cA^{F,2}_t]}{\mathrm{Exp}[cA^{C,1}_t] + \mathrm{Exp}[cA^{F,1}_t] + \mathrm{Exp}[cA^{C,2}_t] + \mathrm{Exp}[cA^{F,2}_t] + \mathrm{Exp}[0]}$$

and

$$\begin{aligned}\mathrm{Exp}[S^1_{t+1}] - \mathrm{Exp}[S^1_t] &= \mathrm{Exp}[S^1_t]\{\mathrm{Exp}[a^M(W^{C,1}_t a^C(S^1_t - X^1_t)\\ &\quad + W^{F,1}_t a^F(F^1 - S^1_t))] - 1\},\\ \mathrm{Exp}[S^2_{t+1}] - \mathrm{Exp}[S^2_t] &= \mathrm{Exp}[S^2_t]\{\mathrm{Exp}[a^M(W^{C,2}_t a^C(S^2_t - X^2_t)\\ &\quad + W^{F,2}_t a^F(F^2 - S^2_t))] - 1\}.\end{aligned}$$

4.2. *Steady state and local stability analysis*

Throughout this section we assume that the reaction parameters a^M, a^C and a^F are strictly positive, and that the memory parameter b is strictly lower than unity, $0 \leqslant b < 1$. The unique steady state of the dynamical model can easily be determined by looking for constant solutions to system (16)–(25). The stationary levels of the dynamic variables turn out to be

$$\bar{S}^1 = \bar{X}^1 = \bar{Y}^1 = F^1, \quad \bar{S}^2 = \bar{X}^2 = \bar{Y}^2 = F^2$$

and

$$\bar{A}^{C,1} = \bar{A}^{F,1} = \bar{A}^{C,2} = \bar{A}^{F,2} = 0,$$

i.e. prices are at their fundamental levels and agents make no profits, so that the average realized profits (which measure the fitness of the rules) in each market for each agent-type are zero in the long run. As a consequence we obtain at the steady state

$$\bar{W}^{C,1} = \bar{W}^{F,1} = \bar{W}^{C,2} = \bar{W}^{F,2} = \bar{W}^O = 0.2,$$

implying that the agents are uniformly distributed among all available strategies.

The local stability analysis of the steady state is performed via the localization, in the complex plane, of the eigenvalues of the Jacobian (evaluated at the steady state) of the map associated with the dynamical system. As it is known, a sufficient condition for the local asymptotic stability is that all of the (real or complex) eigenvalues of the Jacobian lie inside the 'unit circle' in the complex plane, i.e. they are all smaller than one in modulus.

In the Appendix A it is shown that the Jacobian matrix evaluated at the steady state is *block diagonal*, which makes it possible to characterize analytically its eigenvalue structure, and that all of the eigenvalues are smaller than one in modulus if and only if the following set of inequalities is satisfied

$$0.2a^M a^F > 0, \tag{26a}$$

$$a^F < 10/a^M + 2a^C, \tag{26b}$$

$$a^C < 5/a^M. \tag{26c}$$

Condition (26a) is always true (for strictly positive reaction coefficients a^M and a^F). Based on conditions (26b) and (26c), for fixed values of a^M, the region of local

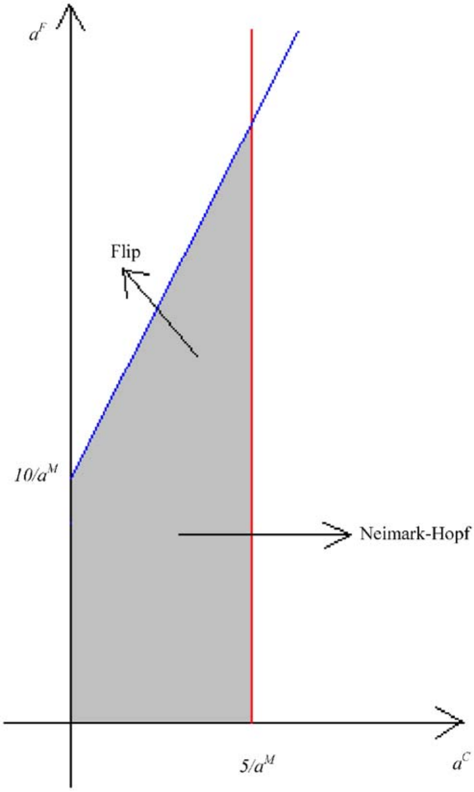

Fig. 1. Stability region in the (a^C, a^F) parameter plane.

asymptotic stability of the steady state and the bifurcation curves can be represented in the space of the parameters a^C and a^F (see Fig. 1).

In particular, as stressed in the Appendix A, a Neimark–Hopf bifurcation occurs, followed by the birth of a stable limit cycle, when a^C becomes larger than the bifurcation value $a^C_{\mathrm{NH}} = 5/a^M$. Notice that the larger is the price adjustment coefficient a^M, the lower is the bifurcation value a^C_{NH} of the reaction coefficient of the chartists, and that the local stability properties are not affected by the parameters c and b, which instead may play a role in the global dynamics of the system.

We also remark that when the Neimark–Hopf curve is crossed in Fig. 1, then two pairs of (complex) eigenvalues become simultaneously of modulus greater than one, because the reaction parameters are assumed to be the same for the two markets (see again the Appendix A for details). Things would be different if the parameters were assumed different in the two markets.[4]

[4]When a^C is increased beyond the Neimark–Hopf curve, assuming a high value of a^F, then the attracting curve becomes more and more irregular and may evolve into a chaotic attractor. For instance, for $a^M = 1$, $a^C = 5.1$, $a^F = 16.925$, $b = 0.975$ and $c = 300$ we detect a positive Lyapunov exponent and complex structure in phase space.

Finally, we want to stress the fact that the stability property of the steady state, which has been proved in this section, is only local. With such a high-dimensional non-linear system, nothing can be said in general about the global behavior, in particular about the long-run behavior associated with initial conditions taken sufficiently far away from the steady state (for instance with initial prices much higher than fundamental values, or with a high initial relative importance of the technical trading rules).

5. The dynamics without transaction taxes

5.1. Calibration

We continue our study with simulation analysis on the basis of the parameter setting presented in Table 1. Unfortunately, empirical guidance on how to select the parameters is limited.[5] We have attempted to calibrate the model such that it produces reasonable dynamics.[6] Note that the better the model matches the behavior of real prices, the more reliable the policy experiments are. Although numerical investigations are sometimes criticized one should note that it is often quite simple to replicate the results and to test their robustness.

5.2. How the model works

Let us first try to understand the working of the model. Fig. 2 depicts a typical simulation run with 5000 observations. Since the model is calibrated to daily data, 5000 observations corresponds to a time span of about 20 years. The first two panels show the development of log prices in markets 1 and 2, respectively. Prices apparently circle in an intricate way around their fundamentals (i.e. $F^1 = F^2 = 0$). In some periods, prices are close to fundamentals, yet also pronounced bubbles and crashes occasionally occur. Market 2, for example, is overvalued by more than 70 percent around period 1400. The last two panels display log price changes, i.e. returns, in markets 1 and 2, respectively. Returns are often used to measure volatility. As can be seen, extreme price changes may be as large as 10 percent. Moreover, volatility tends to cluster, that is, periods of low volatility alternate with periods of high volatility.

What drives the dynamics is the evolution of the importance of the strategies. The central panel presents the impact of the five strategies in the time domain. From

[5]Empirical studies based on daily (monthly) data indicate that the price impact of technical and fundamental trading rules ranges between 0 and 0.1 (0 and 0.5) (see, e.g., Westerhoff and Reitz, 2003, 2005).

[6]In the absence of noise, the underlying steady state of the model, with $S^1 = F^1$ and $S^2 = F^2$, would thus be locally asymptotically stable. Of course, nothing can be said in general about transient motion (before convergence to the steady state) or about global stability of the deterministic system, and therefore about the long-run behavior of the model in the presence of exogenous noise. If, e.g., the system is not in the 'neighborhood' of its steady state (due to shocks), irregular transient dynamics may appear.

Table 1
Parameter setting

$a^M = 1$	$a^C = 0.05$	$a^F = 0.05$
$b = 0.975$	$c = 300$	$\sigma^{M,1} = \sigma^{M,2} = 0.01$
$\sigma^{C,1} = \sigma^{C,2} = 0.05$	$\sigma^{F,1} = \sigma^{F,2} = 0.01$	$tax^1 = 0$
$tax^2 = 0$	$F^1 = 0$	$F^2 = 0$

bottom to top we see the weight of chartism in market 1 (black), fundamentalism in market 1 (white), 'no trading' (gray), fundamentalism in market 2 (gray) and chartism in market 2 (black). On average, each strategy is used in about 20 percent of the time. Destabilizing speculation in form of technical analysis obviously does survive natural selection pressure. Moreover, it is at least as profitable as fundamental analysis. The importance of the strategies, however, varies gradually over time. Note that whenever the composition of chartism, fundamentalism and no trading in a market changes, the stability of the market is affected. For instance, many agents rely on technical analysis in market 1 around period 500. Since technical analysis is destabilizing, the price of market 1 is pushed away from its fundamental value and volatility is relatively high. Between periods 4000 and 4500, chartism is neither popular in market 1 nor in market 2. Instead agents prefer fundamentalism or simply abstain from trading. Now prices are close to fundamentals and volatility is low.

5.3. *Stylized facts*

Next we conduct a more comprehensive Monte Carlo analysis to check to which extent our model is able to match the statistical properties of real speculative prices. According to Cont (2001), Lux and Ausloos (2002) and Sornette (2003), speculative markets are characterized by five universal features: (1) prices are distorted in the form of bubbles and crashes; (2) price volatility is higher than warranted by fundamentals; (3) the distribution of log price changes is leptokurtic; (4) daily log price changes are close to serially independent; and (5) daily absolute log price changes display strong autocorrelation.

With the help of some simple statistics we are able to quantify these phenomena. We define returns r as log price changes, volatility V as average absolute return, and distortion D as average absolute distance between log prices and log fundamentals. Moreover, leptokurtic behavior is given if the kurtosis K exceeds 3. Temporal dependence for lag size k in the return process may be captured by autocorrelation coefficients ac^k. Memory in absolute returns, as a proxy for volatility, may be identified analogously.

Table 2 presents some estimates of these statistics. Since markets 1 and 2 are symmetrical we can restrict the analysis to one market. We have generated 1000 simulations runs with 5000 observations each. Reported are the 5, 25, 50, 75 and 95 percent quantiles. Overall, there is little variation in the main statistics. Extremely

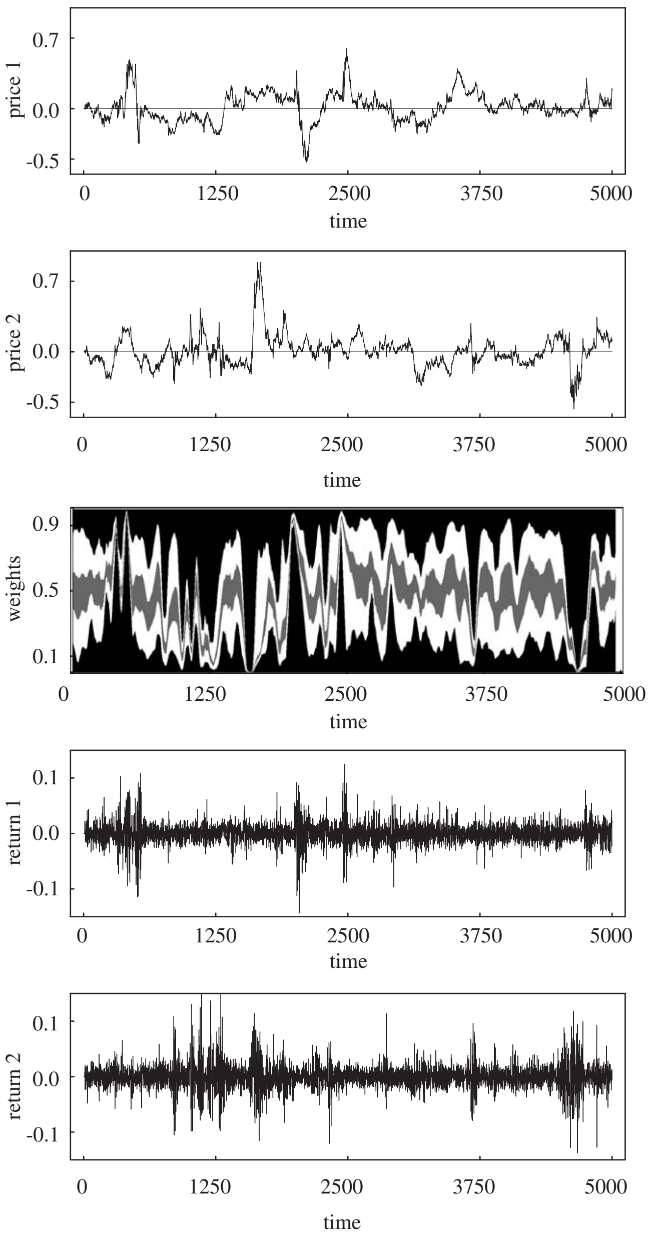

Fig. 2. The first, second, third, fourth and fifth panels show log prices of market 1, log prices of market 2, weights of the traders' strategies (from bottom to top: chartism in market 1 (black), fundamentalism in market 1 (white), no trading (gray), fundamentalism in market 2 (white), and chartism in market 2), returns of market 1 and returns of market 2, respectively. Parameter setting as in Table 1, 5000 observations.

Table 2
Stylized facts of artificial financial markets. Since both markets are symmetrical, we only report estimates for one market

Quantile	r_{min}	r_{max}	V	D	K
0.05	−16.9	09.8	1.15	07.9	06.5
0.25	−14.7	11.7	1.23	09.5	07.8
0.50	−13.1	13.1	1.30	10.7	08.6
0.75	−11.7	14.5	1.36	12.3	09.5
0.95	−09.7	17.3	1.50	15.7	10.9
Quantile	ac_r^1	ac_r^2	ac_r^3	ac_r^4	ac_r^5
0.05	−0.02	−0.04	−0.03	−0.03	−0.04
0.25	0.01	−0.01	−0.01	−0.01	−0.03
0.50	0.03	0.01	0.01	0.01	0.00
0.75	0.06	0.03	0.02	0.03	0.02
0.95	0.10	0.07	0.05	0.06	0.05
Quantile	$ac_{\|r\|}^1$	$ac_{\|r\|}^2$	$ac_{\|r\|}^3$	$ac_{\|r\|}^4$	$ac_{\|r\|}^5$
0.05	0.17	0.16	0.15	0.15	0.15
0.25	0.23	0.23	0.22	0.21	0.21
0.50	0.27	0.27	0.26	0.25	0.25
0.75	0.31	0.31	0.30	0.29	0.28
0.95	0.35	0.35	0.35	0.34	0.33

The quantiles of the statistics are calculated on the basis of 1000 simulation runs with 5000 observations each. Parameter setting as in Table 1.

negative returns hover between −16.9 and −9.7 percent whereas extreme positively returns scatter between 9.8 and 17.3 percent. The median volatility is 1.3 percent. Ninety percent of the mispricing estimates range between 7.9 and 15.7 percent. At least more than 95 percent of the simulation runs show excess kurtosis where the mean kurtosis is 8.6.[7] Almost all autocorrelation coefficients for raw returns are not significant, implying that the evolution of the prices is close to a random walk. The autocorrelation coefficients for absolute returns are significant. Further simulations taking into account higher lag sizes demonstrate that the autocorrelation function is slowly decaying, revealing quite long memory in volatility, up to 100 lags.

Table 3 contains similar estimates for time series of the Dow Jones industrial average, the German share price index, mark–dollar exchange rates, mark–yen exchange rates and crude oil, gold and silver prices. Depending on the time series, daily returns may be as large as 40 percent (crude oil). Extreme returns for

[7]As noted by Lux (2001), the fourth moment of the distribution of the returns may not exist. Thus, the Hill tail index estimator should be used to describe the fattailedness of the distribution of the returns. In our case, 90 percent of these estimates lie between 2.5 and 3.3, which is in good agreement with results obtained for actual speculative markets.

Table 3
Stylized facts of actual financial markets

Series	$r_{\min}$	$r_{\max}$	V	D	K
DJI	−25.6	09.7	0.70	—	71.1
DAX	−13.7	07.3	0.81	—	12.3
DEM/USD	−05.8	04.9	0.50	—	07.0
DEM/JPY	−03.8	08.9	0.44	—	14.0
Crude oil	−40.6	19.2	1.73	—	24.1
Gold	−14.2	12.5	0.82	—	15.9
Silver	−25.7	33.2	1.32	—	30.7
Series	ac_r^1	ac_r^2	ac_r^3	ac_r^4	ac_r^5
DJI	0.04	−0.04	−0.02	−0.02	0.02
DAX	0.03	−0.05	−0.01	0.01	0.02
DEM/USD	−0.03	0.00	0.03	−0.01	0.01
DEM/JPY	0.04	0.01	−0.02	0.00	0.00
Crude oil	0.00	−0.06	−0.07	0.03	−0.04
Gold	−0.07	0.01	0.04	−0.01	−0.01
Silver	0.00	−0.01	0.01	−0.04	0.01
Series	$ac_{\|r\|}^1$	$ac_{\|r\|}^2$	$ac_{\|r\|}^3$	$ac_{\|r\|}^4$	$ac_{\|r\|}^5$
DJI	0.17	0.16	0.17	0.12	0.17
DAX	0.24	0.27	0.26	0.22	0.22
DEM/USD	0.15	0.15	0.16	0.20	0.14
DEM/JPY	0.24	0.20	0.18	0.19	0.16
Crude oil	0.20	0.22	0.23	0.20	0.22
Gold	0.38	0.37	0.33	0.35	0.34
Silver	0.37	0.31	0.30	0.30	0.32

Daily data for the Dow Jones industrial average (1975–2000), the German share price index (1975–2000), mark–dollar exchange rates (1974–1998), mark–yen exchange rates (1974–1998), crude oil prices for 1 barrel in USD (1986–2000), gold prices for 1 troyounce in USD (1975–2000), and silver prices for 1 troyounce in USD (1975–2000).

commodities and stock markets scatter between 7.3 and 25.7 percent while exchange rates vary less dramatically. Overall, estimates of volatility hover between 0.44 and 1.73 percent. Since the fundamental values of these markets are unknown, we are unable to compute the distortion. However, both extreme returns and high volatility are indicators of distorted prices.

All time series display high kurtosis with estimates ranging from 7.0 to 71.1. The autocorrelation coefficients of raw returns are, with very few exceptions, not significant but temporal dependence in volatility is clearly visible. Comparing Tables 2 and 3, one may conclude that our simple model has the power to mimic some important stylized facts of speculative markets quite well. Therefore, we use the model as a computer laboratory to conduct some artificial policy experiments in the next section.

6. The dynamics with transaction taxes

We explore the effectiveness of transaction taxes in two steps. In Section 6.1, we investigate the consequences of transaction taxes imposed in one market only. In Section 6.2, the agents are confronted with uniform transaction taxes in both markets.

6.1. Transaction tax in one market

Let us start with an example. Fig. 3 displays a simulation run in which a regulator has imposed a transaction tax of 0.25 percent in market 1. Fig. 3 can be compared directly with Fig. 2 since it is based on the same seed of random variables; differences in the dynamics are solely due to taxation. As can be seen, even a low tax rate of 0.25 percent may have a quite dramatic impact: While market 1 has become less distorted and less volatile, market 2 shows stronger bubbles and crashes and higher volatility than before.

The central panel reveals the reasons for this outcome. Transaction taxes interfere with the evolutionary competition between the agents' strategies. Agents stop following technical analysis rules in market 1 and, as a result, price variability declines. Also the impact of fundamentalism has weakened in this market, yet not as strongly. Due to the reduction of fundamental trading in market 1 distortions do not vanish completely. Some agents retreat from trading but both chartism and fundamentalism increase in market 2. Hence, price fluctuations are more pronounced here. To sum up, market 1 benefits from the imposition of the transaction tax in terms of higher stability at the cost of market's 2 stability.

To evaluate the effect of transaction taxes a more general kind of analysis is needed. Fig. 4 presents the impact of increasing transaction taxes in market 1 on volatility, distortion, chartism and fundamentalism in market 1 (left-hand panels) and market 2 (right-hand panels), respectively. The tax rate is increased in 20 steps from 0 to 0.5 percent. All estimates are computed as averages over 10 simulation runs, each time series containing 5000 observations. For tax rates up to 0.5 percent, volatility in market 1 decreases from about 1.3 percent to 0.8 percent. But volatility in market 2 increases from 1.3 to 1.6 percent at the same time. The distortion in market 2 remains almost constant, but first decreases in market 1 from 12 to 7.5 percent. If the tax rate is set higher than 0.25 percent, mispricings in market 2 start to grow again. At a tax rate of 0.5 percent, the distortion has climbed to 10 percent.

The results may be understood by inspecting the bottom 4 panels. Transaction taxes obviously change the relative importance of the five strategies. We see that both technical and fundamental trading is crowded out in market 1. This first decreases volatility and distortion but when the impact of fundamentalism is too low, prices stop tracking their fundamentals. Furthermore, the migration of traders from market 1 to market 2 increases the volatility in market 2. Since the increments in fundamentalism and chartism are roughly equal, distortion remains almost constant.

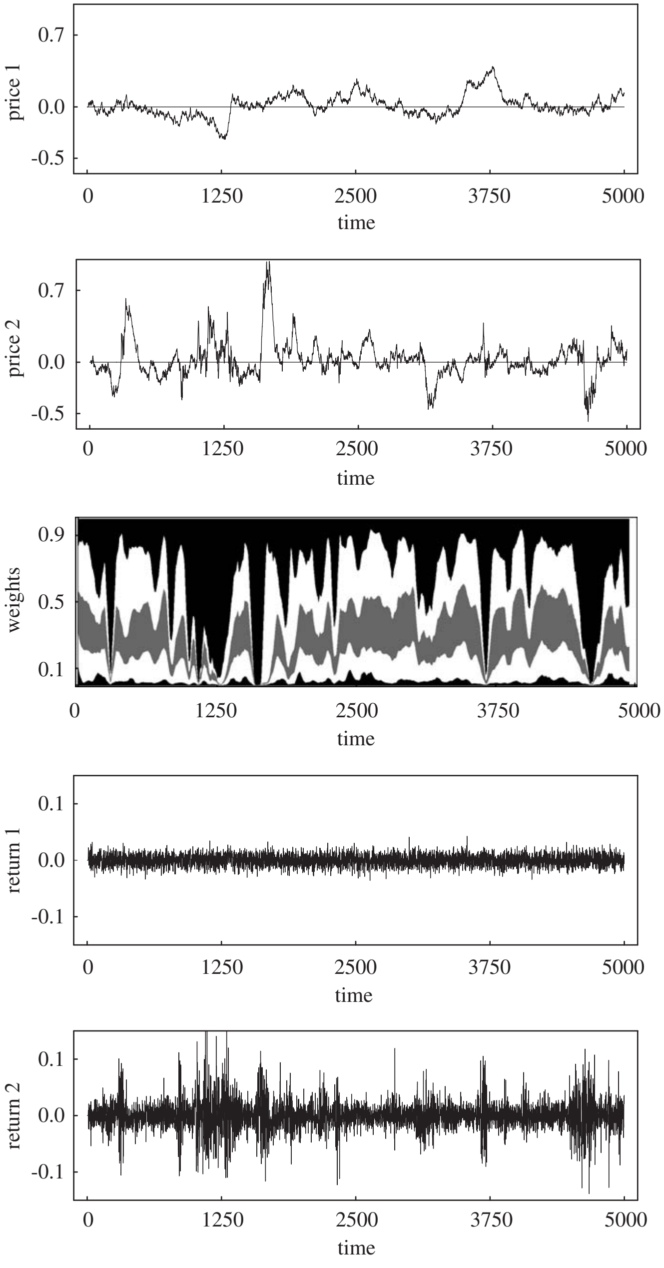

Fig. 3. The first, second, third, fourth and fifth panels show log prices of market 1, log prices of market 2, weights of the traders' strategies (from bottom to top: chartism in market 1 (black), fundamentalism in market 1 (white), no trading (gray), fundamentalism in market 2 (white), and chartism in market 2), returns of market 1 and returns of market 2, respectively. Parameter setting as in Table 1, but a transaction tax of 0.25 percent is imposed in market 1, 5000 observations.

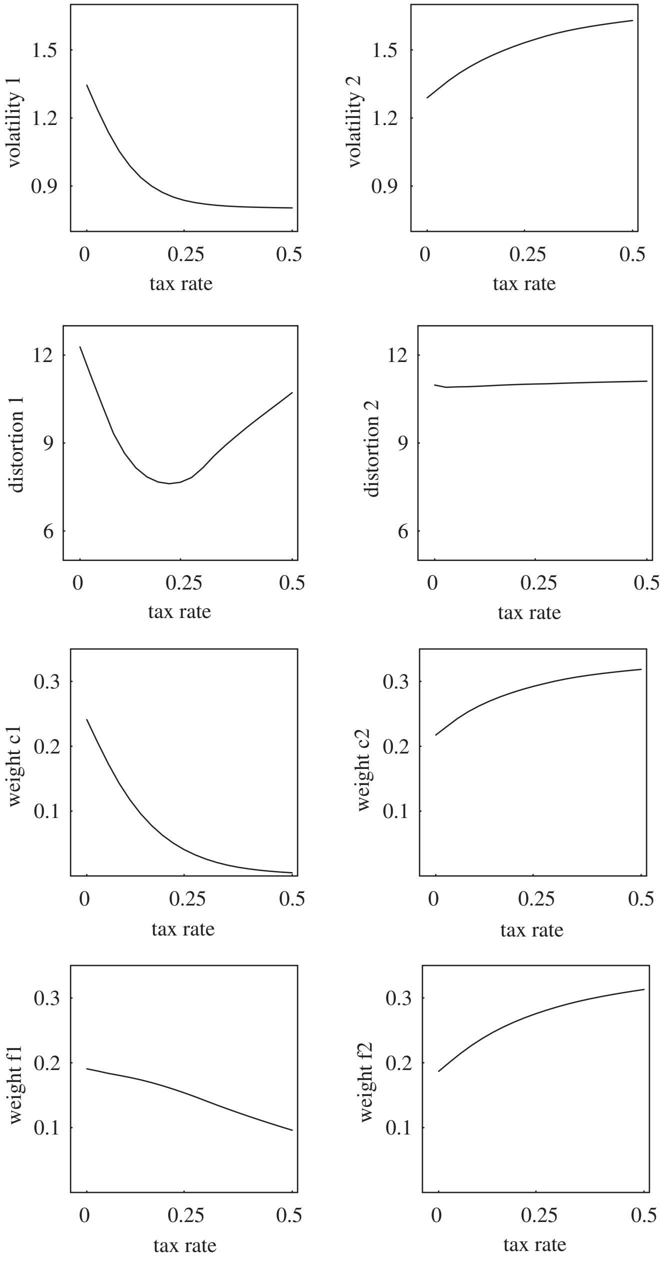

Fig. 4. The first, second, third and fourth lines of panels display the impact of a transaction tax imposed in market 1 on the volatility, distortion, weight of chartism and weight of fundamentalisms. The left-hand panels stand for market 1 while the right-hand panels stand for market 2. The tax rate is increased in 20 steps from 0 to 0.5 percent. The statistics are computed as averages over 10 simulation runs, each containing 5000 observations. The other parameters are as in Table 1.

6.2. Transaction tax in both markets

Note that when market 1 imposes a transaction tax, market 2 becomes destabilized. The regulators of market 2 may thus also want to introduce a transaction tax. Fig. 5 shows the dynamics when markets 1 and 2 impose a uniform transaction tax of 0.25 percent. Now both markets are stabilized, i.e. we observe less mispricing and price variability. The central panel depicts a strong increase in the weight of inactive traders. Many agents stop using technical analysis, but the impact of fundamental analysis is not diminished by the levy.

In Fig. 6 we explore the robustness of the findings in the same way as we did in Fig. 4. The results are striking. If regulators introduce a uniform tax on all transactions, then volatility and distortion decrease in both markets. Moreover, the bottom 4 panels indicate that agents stop using chart trading rules yet not fundamental trading rules. Remember that this important outcome has already been predicted by Keynes (1936). He argued that a transaction tax would mitigate the predominance of destabilizing short-term speculation over stabilizing long-term investment.

Our results allow to be even more optimistic about the usefulness of transaction taxes. Even if both technical and fundamental analysis are executed and evaluated on a daily basis, transaction taxes still stabilize speculative market. The reason is as follows. As reported in Table 4, the average weight of technical analysis is between 22 and 23 percent per market while the average weight of fundamental analysis is about 19 percent per market. Since the market impact of the 'no trading' alternative is only 17 percent, we can conclude that both technical and fundamental trading strategies produce positive (myopic) profits and, furthermore, that technical analysis is the most profitable strategy. But the picture changes if we inspect the profitability (fitness) of the trading rules per trading unit. Profits (fitness) per trading unit for technical analysis are much lower than for fundamental analysis. The profitability of technical analysis is obviously driven by a strong trading activity. By contrast, fundamental analysis produces lower but more consistent profits. As a result, the profitability (fitness) of fundamental analysis is less sensitive to transaction taxes than the profitability (fitness) of technical analysis.

Let us finally check the robustness of our results with respect to our parameter setting. Fig. 7 shows the impact of transaction taxes imposed in both market on volatility and distortion. The solid lines mark the estimates for the parameter setting of Fig. 6. In the first line of panels we see that if the reaction coefficient of the technical trading rules increases, then both volatility and distortion increase. However, changes in a^C do not affect the effectiveness of transaction taxes. The second line of panels reveals that an increase in the reaction parameter of the fundamental trading rule has no impact on volatility but decreases the distortion. Note that also a^F does not affect the power of transaction taxes. The third line of panels demonstrates that volatility and distortion increase with the memory of the agents. A similar result is found for an increase in the agents' rationality (fourth line of panels). For different values of b and c we again find that transaction taxes may stabilize the markets. Overall one may thus conclude that our findings are quite robust.

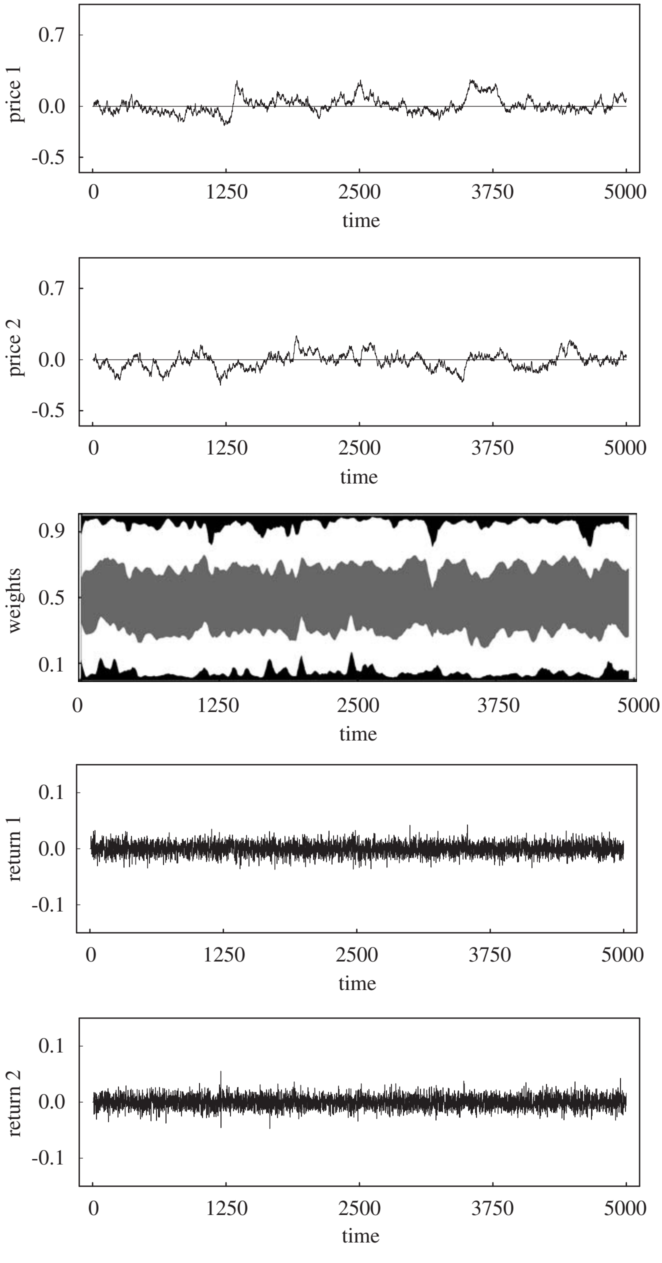

Fig. 5. The first, second, third, fourth and fifth panels show log prices of market 1, log prices of market 2, weights of the traders' strategies (from bottom to top: chartism in market 1 (black), fundamentalism in market 1 (white), no trading (gray), fundamentalism in market 2 (white), and chartism in market 2), returns of market 1 and returns of market 2, respectively. Parameter setting as in Table 1, but a transaction tax of 0.25 percent is imposed in both markets, 5000 observations.

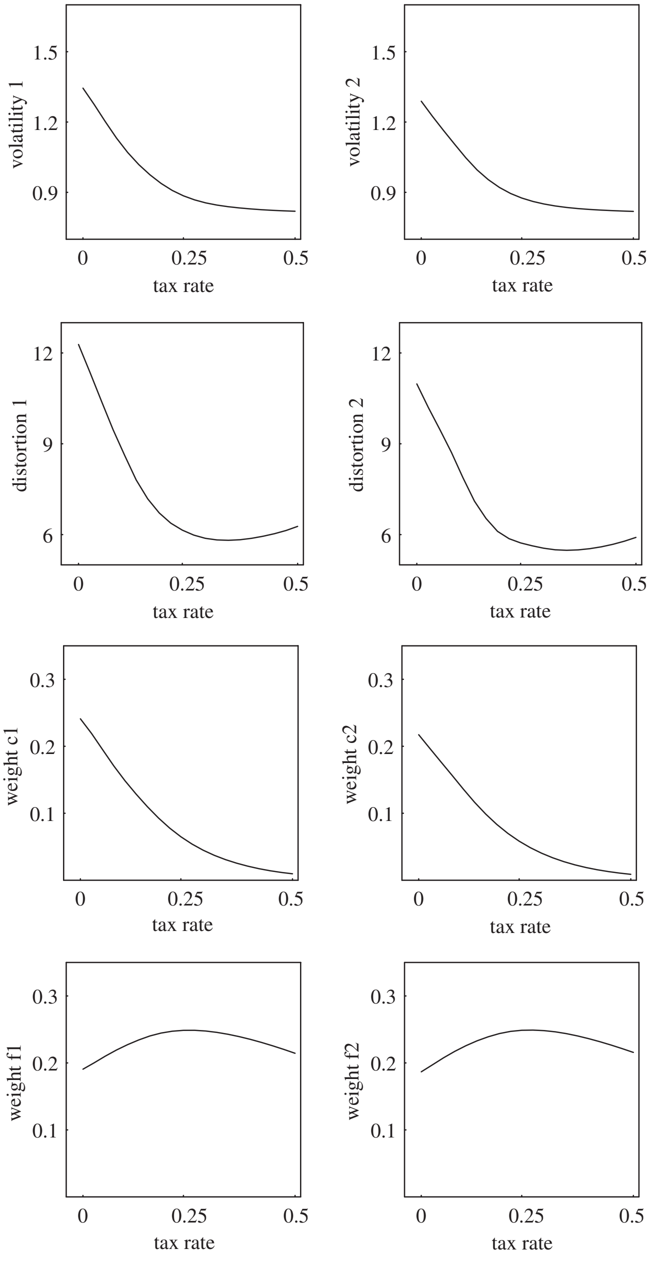

Fig. 6. The first, second, third and fourth lines of panels display the impact of a transaction tax imposed in both markets on the volatility, distortion, weight of chartism and weight of fundamentalisms. The left-hand panels stand for market 1 while the right-hand panels stand for market 2. The tax rate is increased in 20 steps from 0 to 0.5 percent. The statistics are computed as averages over 10 simulation runs, each containing 5000 observations. The other parameters are as in Table 1.

Table 4
Endogenous competition between strategies

Strategies	$C1$	$C2$	$F1$	$F2$	O
Weight	0.23	0.22	0.19	0.19	0.17
1000*profit trading unit	0.39	0.30	0.81	0.86	0
10*fitness trading unit	0.16	0.12	0.32	0.34	0

$C1$, $C2$, $F1$, $F2$ and O denote technical analysis in market 1, technical analysis in market 2, fundamental analysis in market 1, fundamental analysis in market 2 and no trading, respectively. Average values over 25 simulation runs with 5000 observations each. Parameter setting as in Table 1.

7. Conclusions

We believe that chartist–fundamentalist models can be instrumental in helping regulators of markets determine better policy. We present a model in which agents are free to trade in two different markets applying technical or fundamental analysis. The agents prefer rules which have performed well in the past. As it turns out, the agents judge on average technical and fundamental analysis as equally useful, which is consistent with survey and experimental data. An unending competition between the trading strategies creates, however, complex price dynamics. Simulations reveal that the model is able to mimic the behavior of real speculative markets quite well. In particular, artificial time series are portrayed by intricate price fluctuations, bubbles and crashes, excess volatility, leptokurtic returns and clustered volatility.

As ventured by Keynes (1936) and Tobin (1978), small transaction taxes reduce price variability by rendering high-frequency trend chasing unprofitable without affecting long-term fundamental investments. Our model allows this hypothesis to be tested. We find that if a transaction tax is imposed in one market, speculators leave this market. Hence, this market becomes less distorted and less volatile. However, the agents do not simply vanish but reappear on another market, which in turn becomes destabilized. Regulators who impose a levy in their market may force regulators of other markets to follow. If all markets introduce a uniform transaction tax, then economically unjustified speculation is dampened, agents focus more strongly on fundamental data, and all markets are stabilized.

Although increasing evidence suggests that transaction taxes may be an effective policy tool, more research is still needed. As stressed by Lyons (2001), transaction taxes are likely to reduce market liquidity. Especially in foreign exchange markets, turnover in the interdealer market may drop sharply. In fact, transaction taxes also decrease market depth within our model since speculators leave the market. As is well known, the less liquid is a market, the larger is the price impact of a given order size. Such a feedback may limit the stabilizing effect of transaction taxes and in the extreme, it may even overcompensate it. This aspects clearly needs more attention. In particular, empirical estimates of the relation between liquidity and price impact would be helpful. Furthermore, it would be interesting to see a model in which the behavior of individual agents is monitored. Wealth effects and demand functions

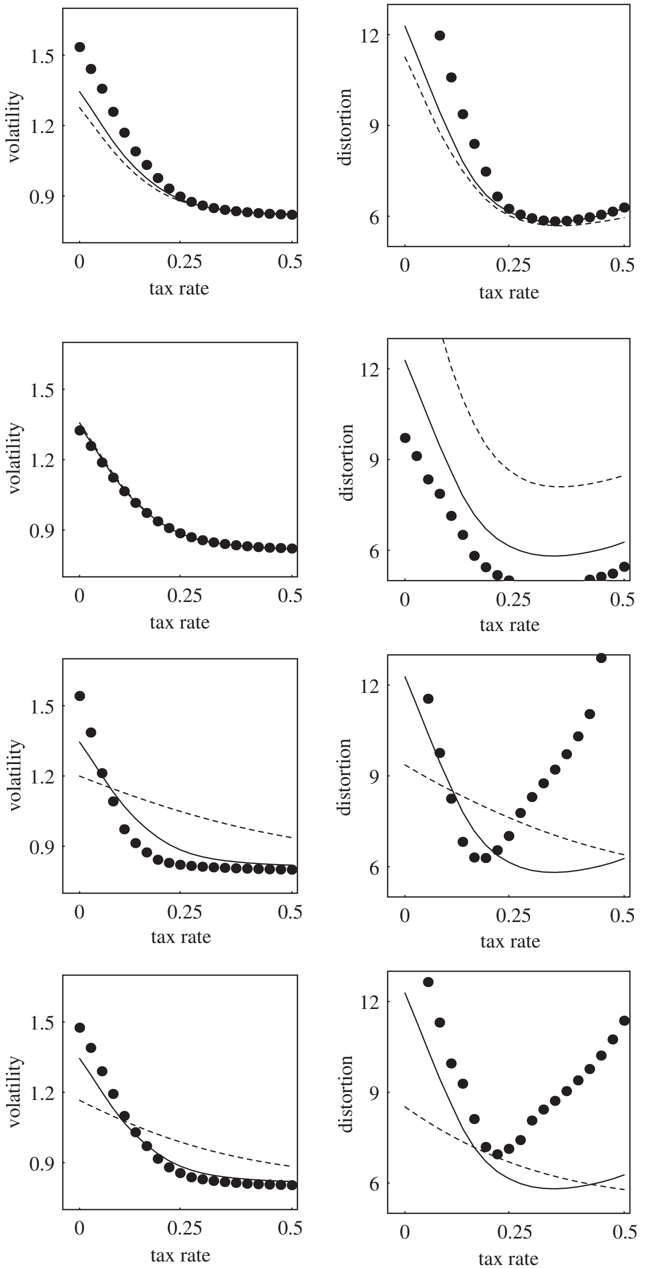

Fig. 7. The impact of a transaction tax imposed in both markets on volatility and distortion for different values of parameters. The tax rate is increased in 20 steps from 0 to 0.5 percent. First line of panels: $a^C = 0$ (dashed line) and $a^C = 0.15$ (dotted line). Second line of panels: $a^F = 0.025$ (dashed line) and $a^F = 0.075$ (dotted line). Third line of panels: $b = 0.9$ (dashed line) and $b = 0.99$ (dotted line). Fourth line of panels: $c = 100$ (dashed line) and $c = 600$ (dotted line). The solid lines are based on the parameter setting of Fig. 6.

depending on the assets' cross-correlation may be of relevance. In addition, one may try to test this policy within a laboratory setting with real agents. We hope that our study will attract more academic attention into this important and exciting research direction.

Acknowledgements

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Appendix A

This appendix contains the derivation of the Jacobian matrix of the map whose iteration determines the time evolution of the dynamical system (16)–(25), as well as the analysis of the eigenvalues of the Jacobian evaluated at the steady state.

Denoting by $'$ the unit time advancement operator,[8] the dynamics of the system is obtained by iteration of the following 10-D map

$$G : \begin{cases} S^{1\prime} = S^1 + a^M(W^{C,1}a^C(S^1 - X^1) + W^{F,1}a^F(F^1 - S^1)), \\ X^{1\prime} = S^1, \\ Y^{1\prime} = X^1, \\ S^{2\prime} = S^2 + a^M(W^{C,2}a^C(S^2 - X^2) + W^{F,2}a^F(F^2 - S^2)), \\ X^{2\prime} = S^2, \\ Y^{2\prime} = X^2, \\ A^{C,1\prime} = U^1a^C(X^1 - Y^1) + bA^{C,1}, \\ A^{F,1\prime} = U^1a^F(F^1 - X^1) + bA^{F,1}, \\ A^{C,2\prime} = U^2a^C(X^2 - Y^2) + bA^{C,2}, \\ A^{F,2\prime} = U^2a^F(F^2 - X^2) + bA^{F,2}, \end{cases} \tag{A.1}$$

where

$$W^{C,1} = \frac{\mathrm{Exp}[cA^{C,1}]}{Z}, \quad W^{F,1} = \frac{\mathrm{Exp}[cA^{F,1}]}{Z}, \quad W^{C,2} = \frac{\mathrm{Exp}[cA^{C,2}]}{Z},$$
$$W^{F,2} = \frac{\mathrm{Exp}[cA^{F,2}]}{Z},$$
$$Z = \mathrm{Exp}[cA^{C,1}] + \mathrm{Exp}[cA^{F,1}] + \mathrm{Exp}[cA^{C,2}] + \mathrm{Exp}[cA^{F,2}] + \mathrm{Exp}[0],$$
$$U^1 = \mathrm{Exp}[S^{1\prime}] - \mathrm{Exp}[S^1] = \mathrm{Exp}[S^1]\{\mathrm{Exp}[a^M(W^{C,1}a^C(S^1 - X^1) + W^{F,1}a^F(F^1 - S^1))] - 1\},$$

[8] I.e. if x is the value of a state variable at time t, then x' is the value of the same variable at time $(t+1)$.

$$U^2 = \mathrm{Exp}[S^{2\prime}] - \mathrm{Exp}[S^2] = \mathrm{Exp}[S^2]\{\mathrm{Exp}[a^M(W^{C,2}a^C(S^2 - X^2) + W^{F,2}a^F(F^2 - S^2))] - 1\}.$$

(i) First, let us consider the partial derivatives of $S^{i\prime}$, $i = 1, 2$, with respect to S^j, X^j, Y^j, $j = 1, 2$. One easily finds

$$\frac{\partial S^{1\prime}}{\partial S^1} = 1 + a^M(W^{C,1}a^C - W^{F,1}a^F) \quad \text{and} \quad \frac{\partial S^{1\prime}}{\partial X^1} = -a^M W^{C,1}a^C$$

which become at the steady state

$$\left.\frac{\partial S^{1\prime}}{\partial S^1}\right|_{\text{S.S.}} = 1 + 0.2a^M(a^C - a^F) \quad \text{and} \quad \left.\frac{\partial S^{1\prime}}{\partial X^1}\right|_{\text{S.S.}} = -0.2a^M a^C,$$

while $\partial S^{1\prime}/\partial Y^1 = \partial S^{1\prime}/\partial S^2 = \partial S^{1\prime}/\partial X^2 = \partial S^{1\prime}/\partial Y^2 = 0$. Similarly one gets

$$\left.\frac{\partial S^{2\prime}}{\partial S^2}\right|_{\text{S.S.}} = 1 + 0.2a^M(a^C - a^F), \quad \left.\frac{\partial S^{2\prime}}{\partial X^2}\right|_{\text{S.S.}} = -0.2a^M a^C$$

and $\partial S^{2\prime}/\partial Y^2 = \partial S^{2\prime}/\partial S^1 = \partial S^{2\prime}/\partial X^1 = \partial S^{2\prime}/\partial Y^1 = 0$.

(ii) Let us now compute the partial derivatives of $S^{i\prime}$, $i = 1, 2$, with respect to the variables $A^{K,j}$, $j = 1, 2$ and $K \in \{C, F\}$. They have the following general structure:

$$\frac{\partial S^{i\prime}}{\partial A^{K,j}} = a^M\left(a^C(S^i - X^i)\frac{\partial W^{C,i}}{\partial A^{K,j}} + a^F(F^i - S^i)\frac{\partial W^{F,i}}{\partial A^{K,j}}\right).$$

Since at the steady state $S^i = X^i = F^i$, it follows that all partial derivatives of this type vanish at the steady state.

(iii) Consider now the partial derivatives of $A^{K,i\prime}$, $i = 1, 2$ and $K \in \{C, F\}$ with respect to the variables $S^j = X^j = Y^j$, $j = 1, 2$. Notice first that the quantities $U^i \equiv (\mathrm{Exp}[S^{i\prime}] - \mathrm{Exp}[S^i])$ vanish at the steady state (where $S^{i\prime} = S^i$). For $i = 1, 2$ we obtain:

$$\frac{\partial A^{C,i\prime}}{\partial S^i} = a^C(X^i - Y^i)\frac{\partial U^i}{\partial S^i}, \quad \frac{\partial A^{C,i\prime}}{\partial X^i} = a^C\left((X^i - Y^i)\frac{\partial U^i}{\partial X^i} + U^i\right),$$
$$\frac{\partial A^{C,i\prime}}{\partial Y^i} = -a^C U^i,$$

$$\frac{\partial A^{F,i\prime}}{\partial S^i} = a^F(F^i - X^i)\frac{\partial U^i}{\partial S^i}, \quad \frac{\partial A^{F,i\prime}}{\partial X^i} = a^F\left((F^i - X^i)\frac{\partial U^i}{\partial X^i} - U^i\right), \quad \frac{\partial A^{F,i\prime}}{\partial Y^i} = 0.$$

Again, all the partial derivatives of this group vanish at the steady state, where $X^i = Y^i = F^i$ and $U^i = 0$. It can also easily be checked that for $K \in \{C, F\}$, $i, j = 1, 2$ and $i \neq j$

$$\frac{\partial A^{K,i\prime}}{\partial S^j} = \frac{\partial A^{K,i\prime}}{\partial X^j} = \frac{\partial A^{K,i\prime}}{\partial Y^j} = 0.$$

Very similar remarks hold for the partial derivatives of the type $\partial A^{K,i\prime}/\partial A^{L,j}$, $K, L \in \{C, F\}$ and $i,j = 1,2$. Due to the presence of coefficients of the type $(X^i - Y^i)$ or $(F^i - X^i)$, it turns out that the partial derivatives of this type are all equal to zero at the steady state, with the exception of

$$\left.\frac{\partial A^{F,1\prime}}{\partial A^{F,1}}\right|_{\text{S.S.}} = \left.\frac{\partial A^{C,1\prime}}{\partial A^{C,1}}\right|_{\text{S.S.}} = \left.\frac{\partial A^{F,2\prime}}{\partial A^{F,2}}\right|_{\text{S.S.}} = \left.\frac{\partial A^{C,2\prime}}{\partial A^{C,2}}\right|_{\text{S.S.}} = b.$$

Taking the dynamic variables in the same order in which they appear in (A.1), i.e. S^1, X^1, Y^1, S^2, X^2, Y^2, $A^{C,1}$, $A^{F,1}$, $A^{C,2}$ and $A^{F,2}$, one finds that the Jacobian matrix at the steady state (denoted by J) has the following *block diagonal* structure:

$$J = \begin{bmatrix} H_1 & 0_{(3,3)} & 0_{(3,4)} \\ 0_{(3,3)} & H_2 & 0_{(3,4)} \\ 0_{(4,3)} & 0_{(4,3)} & bI_4 \end{bmatrix},$$

where the matrices

$$H_1 = H_2 = \begin{bmatrix} 1 + 0.2a^M(a^C - a^F) & -0.2a^M a^C & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

collect the partial derivatives of the block of variables $(S^{i\prime}, X^{i\prime}, Y^{i\prime})$ with respect to (S^i, X^i, Y^i), $i = 1, 2, 0_{(m,n)}$ denotes the null (m,n) matrix, and I_4 is the 4-D identity matrix so that

$$bI_4 = \begin{bmatrix} b & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & b \end{bmatrix}.$$

Due to this particular structure, the eigenvalues can be obtained by computing separately the eigenvalues of each block H_1, H_2 and bI_4. One gets immediately that four of the ten eigenvalues are real and equal to b (and thus smaller than one in absolute value for $0 \leqslant b < 1$). Three of the eigenvalues are the ones of the block H_1 and the remaining three are the ones of the block $H_2 = H_1$. In turn, the 3-D matrix H_1 (or H_2) is *lower block triangular*, with one of the eigenvalues equal to 0 (and thus smaller than one in modulus). The two further eigenvalues are the ones of the following 2-D block

$$Q = \begin{bmatrix} 1 + 0.2a^M(a^C - a^F) & -0.2a^M a^C \\ 1 & 0 \end{bmatrix}.$$

Denote by $Tr(Q) = 1 + 0.2a^M(a^C - a^F)$, $Det(Q) = 0.2a^M a^C$ the trace and the determinant of Q, respectively. The characteristic polynomial Q is given by $P(z) =$

$z^2 - Tr(Q)z + Det(Q)$. A well known necessary and sufficient condition (see e.g. Gandolfo, 1996) to have both eigenvalues smaller than one in modulus, which implies a locally attracting steady state,[9] is the following:

$$\begin{aligned}
&P(1) = 1 - Tr(Q) + Det(Q) > 0,\\
&P(-1) = 1 + Tr(Q) + Det(Q) > 0,\\
&P(0) = Det(Q) < 1.
\end{aligned} \tag{A.2}$$

Conditions (A.2) may be rewritten in terms of the parameters, giving

$$\begin{aligned}
&0.2a^M a^F > 0,\\
&a^F < 10/a^M + 2a^C,\\
&a^C < 5/a^M.
\end{aligned}$$

For strictly positive reaction coefficients a^M, a^C and a^F, taking the price adjustment coefficient a^M as fixed, the local stability region and the bifurcation curves can be represented in the space of the parameters (a^C, a^F), as shown in Fig. 1. Starting from inside the stability region and varying the parameters a^C and a^F, the stability of the steady state is lost when the Flip-bifurcation curve of equation $a^F = 10/a^M + 2a^C$ is crossed (with one real eigenvalue which becomes lower than -1) or when the Neimark–Hopf bifurcation curve of equation $a^C = 5/a^M$ is crossed (with two complex conjugate eigenvalues which become of modulus higher than one). Notice in particular that the Neimark–Hopf bifurcation value $a^C_{\mathrm{NH}} \equiv 5/a^M$ is small for high values of the price adjustment coefficient a^M. We have numerical evidence that the Neimark–Hopf bifurcation is of *supercritical* type, i.e. the crossing of the bifurcation boundary is followed by the appearance of a *stable* limit cycle. Due to the assumption that the reaction parameters a^M, a^C and a^F are the same in the two markets, and thus $H_1 = H_2$, it follows that when the Neimark–Hopf curve is crossed in Fig. 1 two pairs of eigenvalues exit simultaneously the unit circle of the complex plane.[10] Things would, of course, be different if the reaction parameters were assumed to be different in the two markets.

Notice also that the stability properties of the steady state do not depend on the parameters c (sensitivity to the most attractive strategy) and b (memory parameter).

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[9]Since six of the ten eigenvalues have already been proved to be smaller than one in absolute value, and $H_1 = H_2$, it follows that in this particular model the set of inequalities (A.2) gives necessary and sufficient conditions for all of the eigenvalues to be inside the unit circle, which in turn implies a locally stable steady state.

[10]Similarly, when the Flip curve is crossed, two eigenvalues (one associated to H_1 and the other to H_2) become simultaneously lower than -1.

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