Journal of Economic Behavior & Organization
Vol. 58 (2005) 117–132

JOURNAL OF Economic Behavior & Organization

www.elsevier.com/locate/econbase

Exchange rate dynamics, central bank interventions and chaos control methods☆

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Received 28 January 2003; received in revised form 30 May 2003; accepted 15 December 2003
Available online 22 December 2004

Abstract

We use a simple chartist–fundamentalist model developed by Day and Huang to explore recent chaos control algorithms as potential candidates for central bank intervention rules. We find that methods such as delayed feedback control, OGY and constant feedback have, in principle, the potential to reduce exchange rate variability and deviations from fundamentals even in the presence of large dynamic noise.

JEL classification: F31; E58; G18

Keywords: Exchange rate dynamics; Central bank interventions; Chaos control; Technical and fundamental trading rules

1. Introduction

The chartist–fundamentalist approach (e.g., Day and Huang, 1990; Huang and Day, 1993; Brock and Hommes, 1998; Lux and Marchesi, 2000; Chiarella et al., 2002; Farmer and Joshi, 2002; Rosser et al., 2003) has proven to be quite successful in replicating the

☆ Presented at the Complexity 2003 Workshop, Aix-en-Provence, May 2003, and at the 8th Viennese Workshop on Optimal Control, Dynamic Games and Nonlinear Dynamics, Vienna, May 2003. We thank the participants for their helpful discussions, especially Richard Day, Cars Hommes, Erik Mosekilde and Barkley Rosser.

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doi:10.1016/j.jebo.2003.12.002

stylized facts of financial markets. For instance, some of the more recent contributions generate artificial data that is hard to discriminate from actual data. Although buffeted with dynamic noise, price dynamics are at least partially due to an endogenous nonlinear law of motion. Such nonlinearity may originate from the fact that traders use nonlinear trading rules to determine their investment position.

If price fluctuations are stimulated endogenously, central authorities may have some chance to control the dynamics. Indeed, recently some methods to stabilize chaotic behavior have been introduced (Schuster, 1999). First, the OGY method, named after Ott et al. (1990), or the delayed feedback control (DFC) method of Pyragas (1992) may be used to stabilize unstable periodic orbits embedded within a chaotic attractor. While the OGY method slightly perturbs an accessible system parameter, the DFC method adds a linear feedback to the system. Second, the constant feedback (CF) method (e.g., Parthasarathy and Sinha, 1995; Wieland, 2002) is used to suppress chaos.

The aim of this paper is to investigate chaos control methods within the chartist–fundamentalist approach. Specifically, we study whether the chaos control literature offers ways to improve the effectiveness of central bank interventions. Our analysis is based on the seminal work of Day and Huang, which we adjust to foreign exchange markets. The contribution of Day and Huang not only established the study of models with chartists and fundamentalists on a sophisticated scientific level but also produced many descendants (Lux and Marchesi, 2002).

Although the empirical literature is ambivalent about the usefulness of intervention operations, central banks intervene quite frequently in foreign exchange markets (e.g., LeBaron, 1999; Neely, 2001; Sarno and Taylor, 2001). As it turns out, the two most common heuristic intervention strategies “leaning against the wind” and “targeting long-run fundamentals” are somehow related to concepts discussed in the chaos control literature. Our paper provides an analytical and numerical underpinning of central bank intervention strategies. Given the policy importance of central bank interventions, it is surprising that this aspect has until now received only scant attention in the literature.

Our main results are as follows. While “leaning against the wind” fails to stabilize the market, “targeting long-run fundamentals” may reduce both volatility and distortion. In order for the latter method to work, however, central banks have to intervene quite considerably. If they fail to do so, the exchange rate may not be driven towards fundamentals. The OGY method in our model is a more sophisticated version of “targeting long-run fundamentals.” For instance, this rule is only activated if the exchange rate lies within a promising intervention zone. With the CF method, central banks have the opportunity to direct the exchange rate towards a desired level while simultaneously reducing volatility.

The paper is organized as follows. In Section 2, we briefly present the model of Day and Huang. In Section 3, we extend the model by noise traders and a central bank. In addition, we explore the workings of DFC (Section 3.2), OGY (Section 3.3) and CF (Section 3.4). In Section 4, we discuss the methods. The final section concludes the paper.

2. The model

According to questionnaire studies such as Taylor and Allen (1992), Menkhoff (1997) and Lui and Mole (1998), professional foreign exchange traders surprisingly rely on rather simple technical and fundamental trading rules to determine their orders. While technical analysis suggests going with the current exchange rate trend, fundamental analysis is built on the premise that prices converge towards their fundamentals. Both concepts appear to be equally important. Interestingly, agents display a similar behavior in asset pricing experiments, producing complex price fluctuations (Smith, 1991; Sonnemans et al., 2003). Note that survey studies on expectation formation (Ito, 1990; Takagi, 1991) also report that many people adhere to destabilizing bandwagon expectations in the short run, but display stabilizing regressive expectations in the long run.

Due to such robust evidence, a number of theoretical exchange rate models now explore the interactions between chartists and fundamentalists. In Frankel and Froot (1986), a portfolio manager aggregates the predictions of technical and fundamental forecast rules, the weighting scheme being updated on the rules' past prediction success. Kirman (1991) studies changes in market opinion caused by stochastic interactions between the traders. For instance, if agents talk to each other, they may adopt the trading strategy of their opponent. Kirman's model has the potential to generate bubbles via opinion swings. De Grauwe et al. (1993) assume that fundamentalists are heterogeneous concerning their perception of the fundamental value. For instance, if the exchange rate is equal to its fundamental value, half of the fundamentalists believe that the exchange rate is overvalued whereas the other half believe that the exchange rate is undervalued. Hence, the positions of the fundamentalists cancel out so that the market is dominated by chartists' trading only, but as the exchange rate drifts away from its fundamental value, the impact of fundamentalists increases and eventually outweighs that of chartists.

In this paper, we apply the framework of Day and Huang to foreign exchange markets. Let us briefly recall its basic elements. The model incorporates two types of market participants: fundamentalists and chartists (trend chasers). Exchange rates adjust with respect to excess demand via a linear price impact function. The exchange rate p for period $t+1$ is given as

$$p_{t+1} = p_t + cE[p_t], \tag{1}$$

where c is a positive price adjustment coefficient. The excess demand is defined as the sum of the orders of the fundamentalists and the chartists

$$E[p_t] = \alpha(p_t) + \beta(p_t). \tag{2}$$

From (1) and (2) it follows that excess buying drives the exchange rate up and excess selling drives it down.

The net orders of fundamentalists are formalized as

$$\alpha(p_t) = \begin{cases} \dfrac{a(u-p_t)}{(p_t - m + \epsilon)^{d_1}(M + \epsilon - p_t)^{d_2}} & m < p_t < M \\ 0 & \text{otherwise} \end{cases}. \tag{3}$$

According to (3), fundamentalists submit buy (sell) orders if the exchange rate is below (above) its perceived fundamental value u. However, the excess demand function of the fundamentalists is nonlinear. The more the exchange rate converges towards its estimated bottoming (m) or topping (M) price, the more aggressively the fundamentalists trade. The parameters a, d_1, d_2, and ϵ are positive and characterize the slope of (3). Precisely, (3) falls rapidly near M, flattens out near u (for $p = u$, the excess demand is zero), and then falls rapidly again near m.

The excess demand of chartists is linear and expressed as

$$\beta(p_t) = b(p_t - v), \tag{4}$$

where b denotes a positive reaction coefficient of the chartists. As long as the exchange rate is above the perceived fundamental value v, chartists believe in bullish fundamentals and submit buying orders. If the exchange rate drops below v, chartists judge fundamentals as bearish and engage in selling. By "chasing" exchange rates up and down, chartists contribute to bull and bear markets.

Combining (1)–(4) delivers the one-dimensional nonlinear map

$$p_{t+1} = \begin{cases} M & p_t > M \\ p_t + c\left(\dfrac{a(u - p_t)}{(p_t - m + \epsilon)^{d_1}(M + \epsilon - p_t)^{d_2}} + b(p_t - v)\right) & m < p_t < M \\ m & p_t < m \end{cases} \tag{5}$$

that describes the law of motion of the exchange rate. For our analysis we adopt the parameter setting of Day and Huang: $c = 1$, $a = 0.2$, $u = 0.5$, $m = 0$, $M = 1$, $\epsilon = 0.01$, $d_1 = 0.5$, $d_2 = 0.5$, $b = 0.88$, $v = 0.5$.

Solving (5) for $p_{t+1} = p_t = p_*$, three fixed points can be identified at $p_{*_1} \approx 0.0434$, $p_{*_2} \approx 0.5$ and $p_{*_3} \approx 0.957$. Graphically, fixed points are located at the intersection of the map with the 45°-line. However, only p_{*_2} corresponds to the true fundamental value of the exchange rate. Since the absolute values of the slopes of the map at the fixed points are greater than 1, all three fixed points are unstable. Note that the above coefficients yield chaotic dynamics: the exchange rate fluctuates in a complex fashion either above or below $p_{*_2} = 0.5$, whereby the duration of a bull or a bear market is unpredictable.

3. Central bank interventions

3.1. Preliminaries

Central bank interventions are motivated by the desire to check short-run trends or to correct longer-term misalignments (Neely). To evaluate the effectiveness of central bank operations, we define the following performance measures. A stabilization of the exchange rate may be captured by the volatility. We calculate volatility V as average absolute price change

$$V = \frac{1}{T}\sum_{t=1}^{T}|p_t - p_{t-1}|. \tag{6}$$

The distortion D in the market is measured as the average absolute distance between the exchange rate and the target exchange rate of the central bank p_*

$$D = \frac{1}{T}\sum_{t=1}^{T} |p_t - p_*|. \tag{7}$$

Central bank interventions do not necessarily average out over time. To keep track of the accumulated position of the central bank, we compute a buffer B as

$$B = \sum_{t=1}^{T} \gamma(p_t), \tag{8}$$

where $\gamma(p_t)$ indicates the intervention volume of the central bank in period t. In agreement with the empirical evidence, we consider only sterilized and secret interventions. Sterilized interventions do not affect the domestic monetary base so that we may exclude any feedback from the real economy. Since interventions are carried out secretly, traders have no opportunity to exploit them strategically. Note further that central banks intervene quite frequently in foreign exchange markets. For example, together the Federal Reserve Bank and the Deutsche Bundesbank intervened in the period between 1979 and 1996 on 1 day in four (LeBaron, 1999; Saacke, 2002).

The evolution of exchange rates is, of course, not entirely deterministic, as indicated in (5). For instance, the excess demand functions of fundamentalists and chartists represent only a simplification in the actual behavior of the traders. The same holds for the linear price impact function. Therefore, we include so-called noise traders in the model. The excess demand of noise traders may be written as

$$\delta(p_t) = d_t, \tag{9}$$

where the random variable d_t is normally distributed with mean zero and (constant) variance. (See Farmer and Joshi for a similar modeling approach.)

Due to the central bank and the noise traders, (2) modifies to

$$E[p_t] = \alpha(p_t) + \beta(p_t) + \gamma(p_t) + \delta(p_t). \tag{10}$$

In the following we thus study the system

$$p_{t+1} = \begin{cases} M & p_t > M \\ p_t + c(\alpha(p_t) + \beta(p_t) + \gamma(p_t) + \delta(p_t)) & m < p_t < M \\ m & p_t < m \end{cases}, \tag{11}$$

with the right hand side denoted as $f(p)$ for short. Next, we specify the intervention strategies of the central bank.

3.2. The delayed feedback control method

The delayed feedback control method (DFC) was originally proposed by Pyragas as a heuristic method to control chaos in continuous time systems by adding a feedback signal at each computational step to a system variable. The aim of DFC is to stabilize unstable

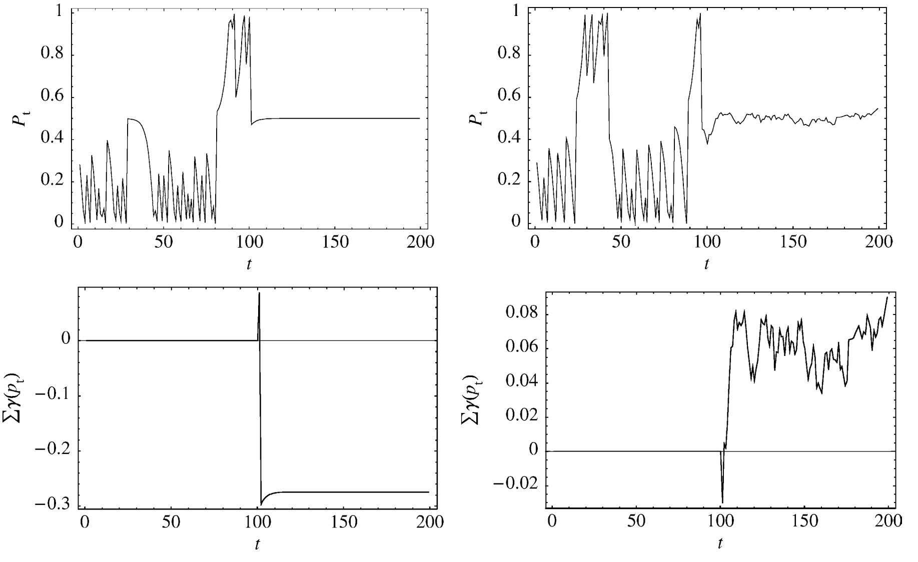

Fig. 1. Price evolution with "targeting long-run fundamentals" activated at $t = 100$ (top panels). Development of the intervention buffer (bottom panels). Without (left-hand side) and with (right-hand side) participation of noise traders ($\sigma = 0.01$). Parameter setting as in Section 2 and $x_0 = 0.3$.

periodic orbits embedded in a chaotic attractor. In our work, we distinguish between two feedback signals that resemble the well-known "targeting long-run fundamentals" and the "leaning against the wind" intervention strategies of central banks (Neely).

3.2.1. Targeting long run fundamentals

The "targeting long-run fundamentals" strategy may be formalized as

$$\gamma(p_t) = k(p_* - p_t), \tag{12}$$

where k is a positive reaction coefficient. The intervention is positive if the exchange rate is below its fundamental value and vice versa. Appropriate parameter values of k may be obtained by solving $|\partial f(p)/\partial p| < 1$ with respect to k (which is the condition for local asymptotic stability of one-dimensional maps). Inserting the numerical parameter values yields that a stabilization of the exchange rates at $p_* = p_{*_2} = 0.5$ is theoretically possible for $0.487843 < k \leq 2.48784$.

Fig. 1 displays an example with $k = 0.75$ without ($\sigma = 0$, top left-hand panel) and with ($\sigma = 0.01$, top right-hand panel) participation of noise traders. Central bank interventions start after 100 periods, and the system rapidly converges to the long-run equilibrium. In the stochastic scenario, the exchange rate fluctuates in a narrow band around its fundamental value. While accumulated interventions of the central bank converge towards a constant value in the deterministic setting (bottom left-hand panel), they follow a stochastic process in the presence of noise traders (bottom right-hand panel).

Since the aim of the central bank is to reduce volatility and distortion, we compute these measures from time series of 5000 iterations for a 50×50 grid for $0 \leq k \leq 1$ and

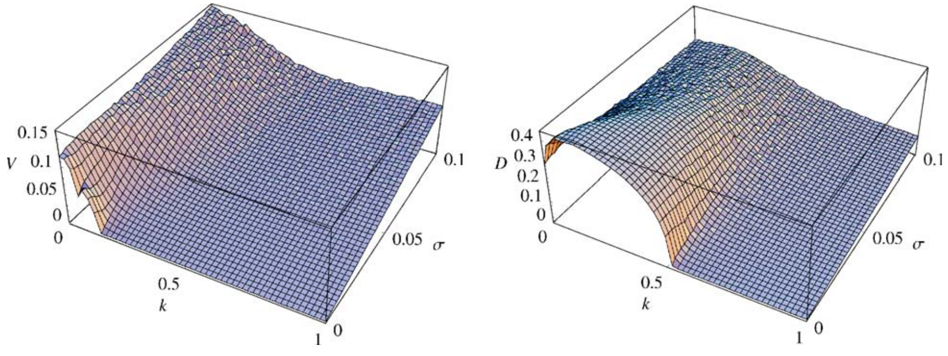

Fig. 2. Volatility (left) and distortion (right) for the “targeting long-run fundamentals” strategy. Parameter setting as in Section 2 and as indicated on the axes (50 × 50 grid and 5000 iterations).

$0 \leq \sigma \leq 0.1$. The results of this exercise are displayed in Fig. 2. Volatility already decreases for small values of k. However, this is accompanied by an increase in the distortion. Hence, if the central bank does not intervene courageously enough, then a stabilization far away from the true fundamental value occurs. In fact, further simulations reveal that the exchange rate gets stuck either in the bull or the bear market. For $0.487843 < k \leq 1$, volatility and distortion decrease, even in the case of a high noise level. To sum up, if k is large enough, “targeting long-run fundamentals” is able to filter most of the endogenous fluctuations.

3.2.2. *Leaning against the wind*

The second feedback signal can be interpreted as the “leaning against the wind” strategy. In this case, the demand function of the central bank is

$$\gamma(p_t) = k(p_{t-1} - p_t). \tag{13}$$

Since the reaction coefficient k is positive, the central bank always trades against past trends. Note that (13) increases the dimension of the system to 2. A stabilization policy could be successful if the modulus of both eigenvalues of the system at the fixed point for a given k is smaller than 1. However, both eigenvalues at $p_* = 0.5$ are greater than one for all $k > 0$. Thus, the “leaning against the wind” strategy fails to calm down foreign exchange market dynamics.

3.3. *The OGY method*

The OGY method by Ott et al. controls unstable periodic orbits embedded in a chaotic attractor by slightly changing an accessible system parameter. In our model, the central bank is not able to manipulate any of the (given) system parameters that describe the other market participants, yet the central bank may alter a parameter characterizing its own behavior. The central bank intervention is

$$\gamma(p_t) = q_t. \tag{14}$$

Linearizing $f(p, q)$ from (11) gives

$$p_{t+1} - p_* = \frac{\partial f(p,q)}{\partial p}(p_t - p_*) + \frac{\partial f(p,q)}{\partial q}(q_t - q_0) \tag{15}$$

where the partial derivatives are evaluated at $p = p_*$ and $q = q_0$ (Kopel, 1996). q_0 is the nominal value of the parameter q (i.e. the base level of the central bank interventions) that is here taken to be $q_0 = 0$. Since the central bank only intervenes in a small neighborhood of the fixed point, it seeks q_t so that in this neighborhood $p_{t+1} - p_* = 0$. Thus, solving (15) with respect to q_t gives the intervention rule

$$q_t = \begin{cases} \dfrac{\partial f(p,q)/\partial p}{\partial f(p,q)/\partial q}(p_* - p_t) & p_t \in Q \\ 0 & \text{otherwise} \end{cases}. \tag{16}$$

Here Q is the bank's intervention zone, whose determination is discussed below.

For simplicity we set $\bar{k} = (\partial f(p,q)/\partial p)/(\partial f(p,q)/\partial q)$.[1] Then (16) is quite simple to interpret. If the exchange rate falls within a relatively small neighborhood Q of the equilibrium, then the central bank trades in the direction of the fundamental value. The OGY method may thus be interpreted as a more sophisticated version of the simple "targeting long-run fundamentals" method. The intervention zone can be specified for a given constant maximum intervention level of the central bank $|q|$. Inserting $|q_t| = |q|$ in (16) and solving for p_t gives

$$Q = [p_* - \bar{k}^{-1}q, p_* + \bar{k}^{-1}q]. \tag{17}$$

Thus, Q obviously grows with increasing $|q|$, which is, however, accompanied by a growing error of the linear approximation.

Inserting our parameter setting, we obtain the intervention strategy

$$q_t = \begin{cases} 1.488(0.5 - p_t) & 0.5 - 1.488^{-1}q \le p_t \le 0.5 + 1.488^{-1}q \\ 0 & \text{otherwise} \end{cases}. \tag{18}$$

For $q = 0.744$, OGY and "targeting long-run fundamentals" are congruent. Note that $\bar{k} = 1.488$ lies within the interval $0.487843 < k \le 2.48784$, where "targeting long-run fundamentals" is successful.

Let us consider the numerical example presented in Fig. 3. The central bank intervention strategy is activated in $t = 100$ with $|q| = 0.25$. The left-hand panel shows a deterministic regime ($\sigma = 0$) and the right-hand panel demonstrates a simulation run with noise traders ($\sigma = 0.01$). The corresponding accumulated intervention positions are plotted below. After the exchange rate has entered the intervention zone $0.331972 \le p_t \le 0.668028$, the control algorithm starts and rapidly stabilizes the market at the fundamental value. As in the case of "targeting long-run fundamentals," the accumulated interventions of the central bank converge towards a constant value in the deterministic setting, but it follows a stochastic process in the presence of noise traders.

[1] Note that at the fixed point $p_{*_2} = 0.5$, $\partial f(p,q)/\partial p = 1 + c(-a(\epsilon + M - u)^{d2}(\epsilon - m + u)^{d1} + b)$ and $\partial f(p,q)/\partial q = c$.

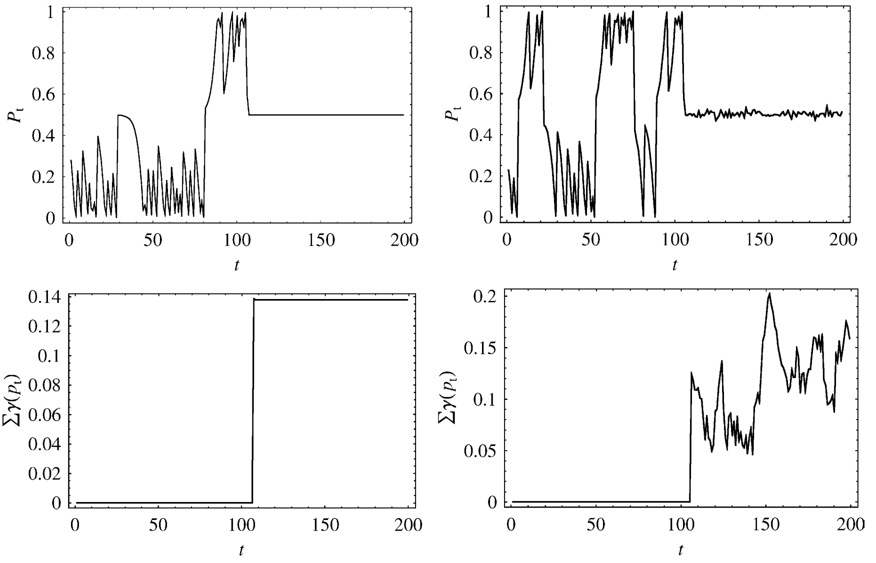

Fig. 3. Price evolution with OGY activated at $t = 100$ (top panels). Development of the intervention buffer (bottom panels). Without (left-hand side) and with (right-hand side) participation of noise traders ($\sigma = 0.01$). Parameter setting as in Section 2 and $x_0 = 0.3$.

Fig. 4 shows volatility and distortion computed from a time series of 5000 iterations for a 50×50 grid for $0 \leq |q| \leq 0.5$ and $0 \leq \sigma \leq 0.1$. Exchange rate fluctuations and deviations from fundamentals are clearly reduced by the OGY method. Note that for increasing noise levels, $|q|$ has to be increased to achieve stabilization. The reason for this is that the exchange rate is at a high noise level that is often bounced out of the intervention zone so that interventions are paused ($q_t = 0$).

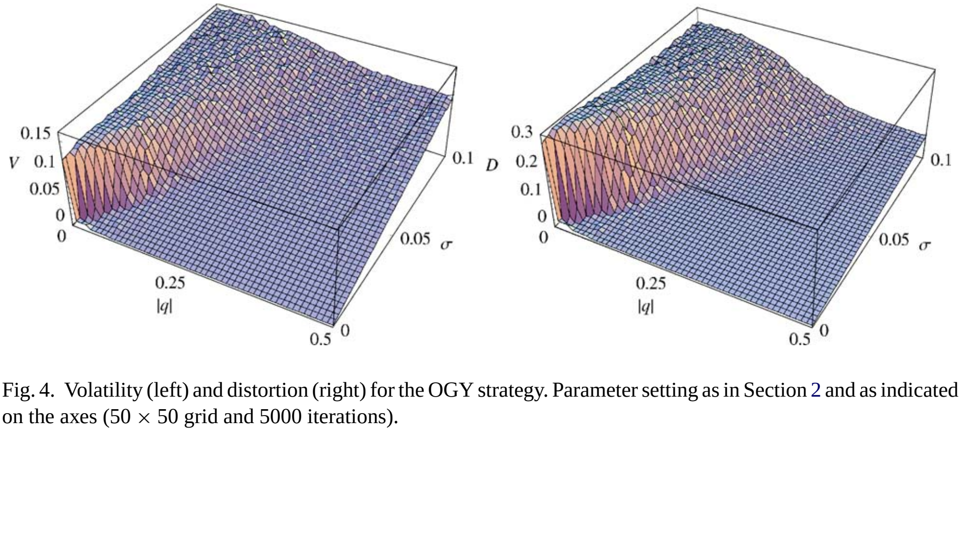

Fig. 4. Volatility (left) and distortion (right) for the OGY strategy. Parameter setting as in Section 2 and as indicated on the axes (50×50 grid and 5000 iterations).

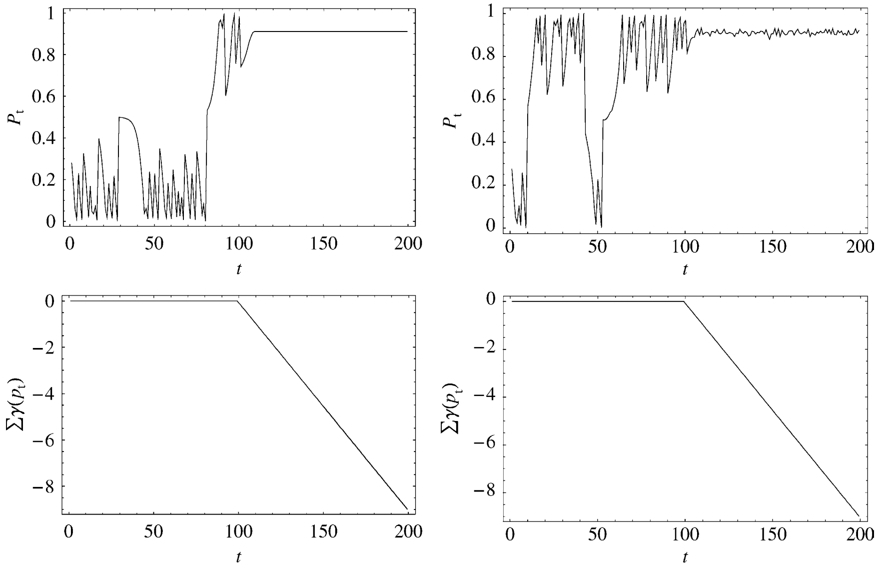

Fig. 5. Price evolution with CF activated at $t = 100$ (top panels). Development of the intervention buffer (bottom panels). Without (left-hand side) and with (right-hand side) participation of noise traders ($\sigma = 0.01$). Parameter setting as in Section 2 and $x_0 = 0.3$.

3.4. The constant feedback method

So far we have considered methods that enable a central bank to stabilize the fundamental value p_{*2}, but sometimes central banks attempt to manipulate exchange rates in order to promote their domestic economy. Such a "beggar your neighbor" policy may be achieved via the constant feedback method. This method adds a constant amount to the system equation at each discrete time step (Parthasarathy and Sinha; Wieland). Applying CF to our model, the demand of the central bank becomes

$$\gamma(p_t) = k. \tag{19}$$

Hence, central banks either constantly buy ($k > 0$) or sell ($k < 0$) a certain amount of currency.

Fig. 5 shows an example where the central bank aims at stabilizing the exchange rate within the bull market. The intervention starts in period 100 with a volume of $k = -0.09$ (left-hand panel: $\sigma = 0$, right-hand panel: $\sigma = 0.01$). By selling small currency positions the central bank prevents the market from overheating. Put differently, central bank operations weaken the buying pressure of the chartists at the cost of a growing negative buffer (bottom panels).

Both over- and undervaluation of the currency may occur. Fig. 6 presents all p_0/k combinations that yield a stabilization within the bear market (white dots) and bull market (gray dots) for the deterministic setting. Black dots indicate p_0/k combinations in which stabilization fails. Clearly, to fix the exchange rate within the bull (bear) area, the central bank should first wait until the exchange rate is near this re-

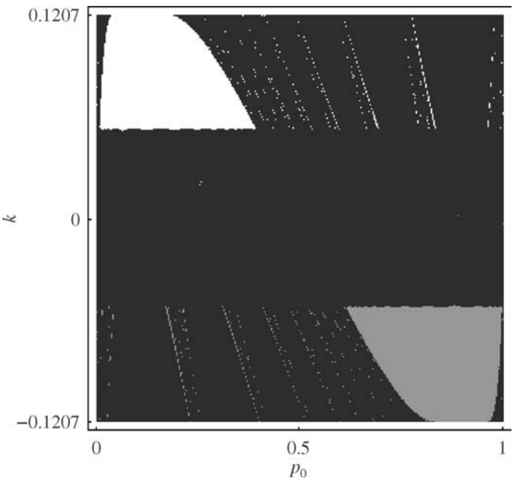

Fig. 6. Basin of attraction for the CF strategy. White (gray) dots: stabilization in the bear (bull) market. Black dots: no stabilization. Parameter setting as in Section 2 and as indicated on the axes (100 × 100 grid).

gion and then start selling (buying). We are able to characterize successful intervention levels somewhat more strongly. Solving $|\partial f(p)/\partial p| < 1$ with respect to p yields $p \in [0.0626692, 0.170832] \cup [0.829168, 0.937331]$, which may be denoted as areas of potential new fixed points. Computing $k = (p - f(p))/c$ gives the corresponding constant feedback parameters $-0.120668 \leq k \leq -0.0514906$ and $0.120668 \leq k \leq 0.0514906$, respectively (Wieland). The central bank is thus able to stabilize a point in a desired market region.

In Fig. 7 volatility and distortion are computed from a time series of 5000 iterations for a 50×50 grid for $0 \leq k \leq 0.1207$ and $0 \leq \sigma \leq 0.1$. We assume that the central bank aims to stabilize the exchange rate at $p_* = 0.1$. The measure of distortion is corrected accordingly. For a low level of noise traders, CF strongly reduces volatility and distortion. The distortion minimum is achieved for $k \approx 0.1$, but the effectiveness of CF declines with increasing noise level. Volatility is somewhat higher than in the case of DFC and OGY.

4. Discussion

Most empirical studies are skeptical about the usefulness of central bank interventions; see Schwartz (2000) for a pessimistic and Dominguez (2003) for a more optimistic perspective. One explanation for this finding could be that not all intervention strategies have the power to calm down the market. For instance, within our model the “leaning against the wind” rule does not stabilize the market. An alternative explanation could be that central banks do not use the correct level of intervention. Nevertheless, the high degree of intervention in the past reveals central banks’ conviction that buying and selling currency is an effective policy tool.

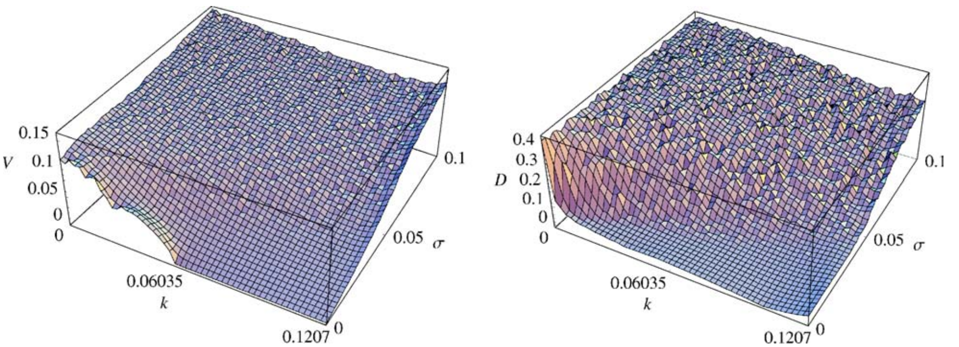

Fig. 7. Volatility (left) and distortion (right) for the CF strategy. Parameter setting as in Section 2 and as indicated on the axes (50×50 grid and 5000 iterations).

This paper thus explores whether chaos control methods may explain or even improve the working of central bank operations. In general, chaos control has been demonstrated in a wide variety of areas including mechanics, electronics, lasers, biology and chemistry (for a comprehensive introduction into the theory, definition, empirical estimation, and the control of chaos, see Rosser, 2000). There also exists a number of applications of chaos control methods in economic contexts. Holyst et al. (1996) apply the OGY method to a model of two competing firms. Kopel (1997) shows in a competitive market model how firms can improve their performance measures by the use of the target method. Bala et al. (1998) apply the same method to a model with exchange economies and tatonnement adjustment. Within a macroeconomic disequilibrium model, Kaas (1998) shows that a government can stabilize the dynamics by varying income tax rates or government expenditure.

Compared to a macroeconomic case, where frequent changes of parameters such as the income tax rate are presumably difficult to accomplish, we would like to point out that central bank operations appear not only as an important but also as a natural candidate for the application of chaos control methods. Nevertheless, most economic applications face two severe problems: first, how much information about the dynamical system is needed to carry out successful stabilization? Related questions include how to obtain such information (e.g., either from economic considerations or from time series analysis) and what are the consequences of approximation errors? Do we still observe a stabilization, or do small errors already produce large fluctuations? Second, what happens if the methods are executed in a situation where the underlying dynamics are not chaotic but, say, periodic? In the remainder of this section, we critically study these questions with respect to our case.

Although "targeting long-run fundamentals" has the potential to reduce volatility and distortion, the permanent activity of the central bank even far away from the desired fundamental exchange rate is required. Unfortunately, in order to compute an appropriate reaction coefficient one would need a priori information on the system equation. Since such knowledge is not available, the central bank has to learn k iteratively by experimentation.

The OGY method is only activated in a small neighborhood of the fundamental value. Note that due to the linearization around the fixed point, the error of approximation grows with both increasing $|q|$ and increasing noise level. However, one advantage of this method is that the necessary parameter perturbations (Ott et al.) as well as the fixed points (So et al., 1997) may, in principle, be retrieved from time series information.

The constant feedback method allows the central bank to steer the system towards a distorted equilibrium.[2] Wieland shows that time series information may be used both to identify possible new fixed points and to calculate appropriate parameter settings for k. Moreover, the method is able to stabilize underlying periodic and chaotic systems. The constant feedback method has already successfully been applied in open ecosystems that may suffer from changing system equations and exogenous shocks (e.g., Hudson et al., 1998).

Does the model of Day and Huang have empirical relevance? Surveying numerous empirical studies, Rosser concludes that evidence for bubbles and crashes on foreign exchange markets is great. Moreover, Gu (1993) reports empirical support for the model's application to stock market data. Gu (1995) argues that to make a market viable, market makers have to churn the market (i.e. produce endogenous fluctuations). Is it therefore likely that financial markets are chaotic? Besides the theoretical arguments put forward by Day and Huang or de Grauwe et al., the behavior of real agents seems to generate chaos in experimental settings (Sonnemans et al.). However, the question of chaotic dynamics in financial markets is highly controversial. Although early empirical contributions (Scheinkman and LeBaron, 1989) claim to have found positive Lyapunov exponents and low dimensional attractors, the estimates typically lack statistical reliability tests (e.g., Barnett and Serletis, 2000; Rosser, 2000). At least evidence for nonlinearities, a necessary condition for chaos, is solid.

Let us finally address both issues together. Fig. 8 investigates whether the intervention rules are still able to stabilize the markets when central banks have only incomplete knowledge about the law of motion and when the underlying dynamics is not chaotic. We use the same parameter setting as in Section 2, but now the price adjustment coefficient is $c = 0.836$. As a result, we observe two coexisting period three cycles, one in the bull market and one in the bear market (0.184792, 0.0843517, 0.0137281 and 0.815208, 0.915648, 0.986272). The top panel shows the dynamics for 400 observations in the time domain. Due to the dynamic noise ($\sigma = 0.02$), the system again switches erratically between bull and bear markets. The second, third and fourth panels present the impact of DFC, OGY and CF on the dynamics, respectively. The central bank applies the same parameter setting for interventions as in Section 3, thus using falsely $c = 1$ instead of $c = 0.836$. The results are striking. Although the dynamics is not chaotic and although the intervention strength is not optimal, the market becomes much more stable.[3] Hence, the strategies discussed in this paper seem to be robust.

[2] DFC and CF are related for certain parameter conditions. Remember that for small values of k, "targeting long-run fundamentals" leads to a higher distortion. For instance, if the system is stabilized in the bull market then p_t does not fluctuate around u but around a higher value. The distance between that value and u acts like a constant feedback term.

[3] At first sight, this result may appear surprising for the OGY method since this method normally needs to exploit the sensitive dependence on initial conditions. However, this requirement is substituted by the added noise.

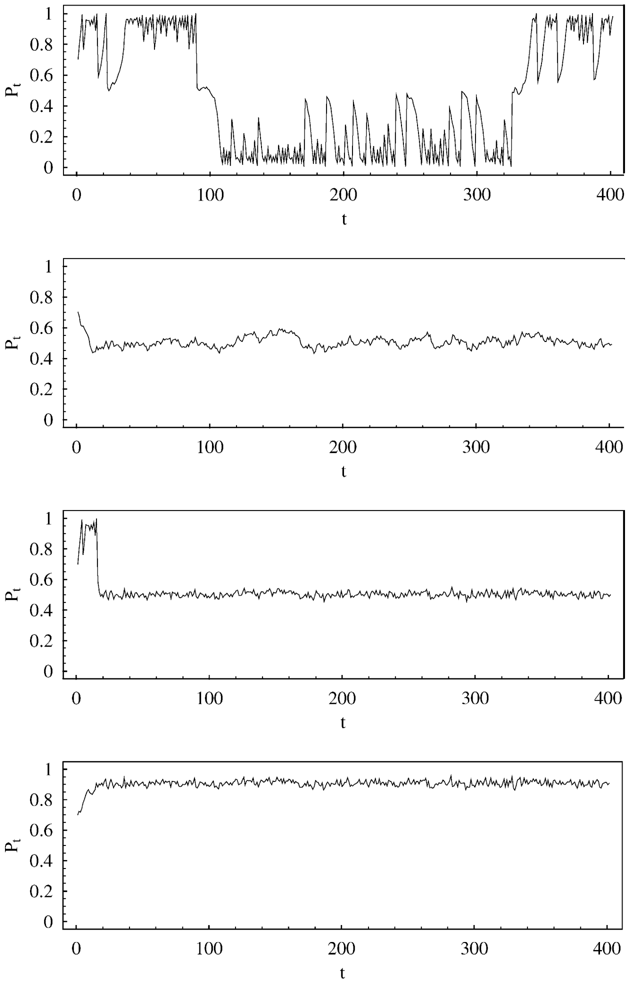

Fig. 8. Price evolution without interventions (first panel), with DFC interventions (second panel), with OGY interventions (third panel) and with CF interventions (fourth panel). Parameter setting as in Sections 2 and 3, but $c = 0.836$.

5. Conclusion

According to the chartist–fundamentalist approach, exchange rate fluctuations are at least partially due to an endogenous nonlinear law of motion. Clearly, if the dynamics is not fully exogenous, central authorities may have the opportunity to stabilize the markets. The aim of this paper is to explore and design central bank intervention strategies, taking into account recent findings from the chaos control literature.

Using the model of Day and Huang as a foreign exchange market laboratory, we compare the working of three recently developed chaos control mechanisms. Delayed feedback control in the form of "targeting long-run fundamentals" proved to be effective if applied courageously enough. "Leaning against the wind" has no potential to stabilize the market. The OGY method allows the central bank to reduce both distortion and volatility. With the constant feedback method, the central bank has an instrument to shift the exchange rate away from fundamentals. At least in the short run, central banks may thus conduct a "beggar your neighbor policy."

Empirical studies do not agree whether central bank interventions may stabilize the market or not. We would like to stress that chaos control methods deliver a strong theoretical background for the working of these methods. We therefore hope that our paper may stimulate further research (e.g. within a standard foreign exchange rate model, such as the model of Dornbusch, 1976), enriched by chartists and fundamentalists so that the effectiveness of central bank interventions can be better understood.

Acknowledgement

We thank two anonymous referees for their constructive comments.

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