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MARKET DEPTH AND PRICE DYNAMICS: A NOTE

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This note explores the consequences of nonlinear price impact functions on price dynamics within the chartist–fundamentalist framework. Price impact functions may be nonlinear with respect to trading volume. As indicated by recent empirical studies, a given transaction may cause a large (small) price change if market depth is low (high). Simulations reveal that such a relationship may create endogenous complex price fluctuations even if the trading behavior of chartists and fundamentalists is linear.

Keywords: Econophysics; market depth; price dynamics; nonlinearities; technical and fundamental analysis.

1. Introduction

Interactions between heterogeneous agents, so-called chartists and fundamentalists, may generate endogenous price dynamics either due to nonlinear trading rules or due to a switching between simple linear trading rules.[1,2] Overall, multi-agent models appear to be quite successful in replicating financial market dynamics.[3,4] In addition, this research direction has important applications. On the one hand, understanding the working of financial markets may help to design better investment strategies.[5] On the other hand, it may facilitate the regulation of disorderly markets. For instance, Ehrenstein shows that the imposition of a low transaction tax may stabilize asset price fluctuations.[6]

Within these models, the orders of the traders typically drive the price via a log–linear price impact function: buying orders shift the price proportionally up and selling orders shift the price proportionally down. Recent empirical evidence suggests, however, that the relationship between orders and price adjustment may be nonlinear. Moreover, as reported by Farmer *et al.*, large price fluctuations occur when market depth is low.[3,7] Following this observation, our goal is to illustrate a novel mechanism for endogenous price dynamics.

We investigate — within an otherwise linear chartist–fundamentalist setup — a price impact function which depends nonlinearly on market depth. To be precise, a given transaction yields a larger price change when market depth is low than when it is high. Simulations indicate that such a relationship may lead to complex price movements. The dynamics may be sketched as follows. The market switches back and forth between two regimes. When liquidity is high, the market is relatively stable. But low price fluctuations indicate only weak trading signals and thus the transactions of speculators decline. As liquidity decreases, the price responsiveness of a trade increases. The market becomes unstable and price fluctuations increase again.

The remainder of this note is organized as follows: Sec. 2 sketches the empirical evidence on price impact functions. In Sec. 3, we present our model, and in Sec. 4, we discuss the main results. The final section concludes the paper.

2. Empirical Evidence

Financial prices are obviously driven by the orders of heterogeneous agents. However, it is not clear what is the true functional form of price impact. For instance, Farmer proposes a log–linear price impact function for theoretical analysis while Zhang develops a model with nonlinear price impact.[8,9] His approach is backed up by empirical research that documents a concave price impact function. According to Hasbrouck, the larger is the order size, the smaller is the price impact per trade unit.[10] Kempf and Korn, using data on DAX futures, and Plerou *et al.*, using data on the 116 most frequently traded US stocks, find that the price impact function displays a concave curvature with increasing order size, and flattening out at larger values.[11,12] Weber and Rosenow fitted a concave function in the form of a power law and obtained an impressive correlation coefficient of 0.977.[13] For a further theoretical and empirical debate on the possible shape of the price impact function with respect to the order size, see Gabaix *et al.*, Farmer and Lillo, and Plerou *et al.*[14–16]

But these results are currently challenged by an empirical study which is crucial for this note. Farmer *et al.* present evidence that price fluctuations caused by individual market orders are essentially independent of the volume of the orders.[7] Instead, large price fluctuations are driven by fluctuations in liquidity, i.e., variations in the market's ability to absorb new orders. The reason is that even for the most liquid stocks there can be substantial gaps in the order book. When such a gap exists next to the best price — due to low liquidity — even a small new order can remove the best quote and trigger a large price change. These results are supported by Chordia, Roll and Subrahmanyam, who also document that there is considerable time variation in market wide liquidity and by Lillo, Farmer and Mantenga, who detect that higher capitalization stocks tend to have smaller price responses for the same normalized transaction size.[17,18]

Note that the relation between liquidity and price impact is of direct importance to investors developing trading strategies and to regulators attempting to stabilize financial markets. Farmer *et al.* argue, for instance, that agents who are trying to transact large amounts should split their orders and execute them a little at a time, watching the order book, and taking whatever liquidity which is available as it enters.[7] Hence, when there is a lot of volume in the market, they should submit large orders. Assuming a concave price impact function would obviously lead to quite different investment decisions. Ehrenstein, Westerhoff and Stauffer demonstrate, for instance, that the success of a Tobin tax depends on its impact on market depth.[19] Depending on the degree of the nonlinearity of the price impact function, a transaction tax may stabilize or destabilize the markets.

3. The Model

Following Simon, agents are boundedly rational and display a rule-governed behavior.[20] Moreover, survey studies reveal that financial market participants rely strongly on technical and fundamental analysis to predict prices.[21,22] Chartists typically extrapolate past price movements into the future. Let P be the log of the price. Then, their orders may be expressed as

$$D_t^C = a(P_t - P_{t-1})\,, \tag{1}$$

where a is a positive reaction coefficient denoting the strength of the trading. Accordingly, technical traders submit buying orders if prices go up and vice versa. In contrast, fundamentalists expect the price to track its fundamental value. Orders from this type of agent may be written as

$$D_t^F = b(F - P_t)\,. \tag{2}$$

Again, b is a positive reaction coefficient, and F stands for the log of the fundamental value. For instance, if the asset is overvalued, fundamentalists submit selling orders.

As usual, excess buying drives the price up and excess selling drives it down so that the price adjustment process may be formalized as

$$P_{t+1} = P_t + A_t(wD_t^C + (1-w)D_t^F)\,, \tag{3}$$

where w indicates the fraction of chartists and $(1-w)$ the fraction of fundamentalists. The novel idea is to base the degree of price adjustment A on a nonlinear function of the market depth.[23] Exploiting that given excess demand has a larger (smaller) impact on the price if the trading volume is low (high), one may write

$$A_t = \frac{c}{(|wD_t^C| + |(1-w)D_t^F|)^d}\,. \tag{4}$$

The curvature of A is captured by $d \geq 0$, while $c > 0$ is a shift parameter.

For $d = 0$, the price adjustment function is log–linear.[1,3] In that case, the law of motion of the price, derived from Eqs. (1)–(4), is a second-order linear difference equation which has a unique steady state at

$$P_{t+1} = P_t = P_{t-1} = F\,. \tag{5}$$

Rewriting Schur's stability conditions, the fixed point is stable for

$$0 < c < \begin{cases} \dfrac{1}{aw} & \text{for } w > \dfrac{b}{4a+b}\,, \\ \dfrac{2}{b(1-w)-2aw} & \text{else}\,. \end{cases} \tag{6}$$

However, we are interested in the case where $d > 0$. Combining Eqs. (1)–(4) and solving for P yields

$$P_{t+1} = P_t + c\frac{wa(P_t - P_{t-1}) + (1-w)b(F - P_t)}{(|wa(P_t - P_{t-1})| + |(1-w)b(F-P_t)|)^d}\,, \tag{7}$$

which is a two-dimensional nonlinear difference equation. Since Eq. (7) precludes closed analysis, we simulate the dynamics to demonstrate that the underlying structure gives rise to endogenous deterministic motion.

4. Some Results

Figure 1 contains three bifurcation diagrams for $0 < d < 1$ and $w = 0.7$ (top), $w = 0.5$ (central) and $w = 0.3$ (bottom). The other parameters are fixed at $a = b = c = 1$ and the log of the fundamental value is $F = 0$. We increase d in 500 steps. In each step, P is plotted from $t = 1001$–1100. Note that bifurcation diagrams are frequently used to illustrate the dynamic properties of nonlinear systems.

Figure 1 suggests that if d is small, there may exist a stable equilibrium. For instance, for $w = 0.5$, prices converge towards the fundamental value as long as d is smaller than around 0.1. If d is increased further, the fixed point becomes unstable. In addition, the range in which the fluctuations take place increases too. Note also that many different types of bifurcation occur. Our model generates the full range of possible dynamic outcomes: fixed points, limit cycles, quasi periodic motion and chaotic fluctuations. For some parameter combinations coexisting attractors emerge. Comparing the three panels indicates that the higher the fraction of chartists, the less stable the market seems to be.

To check the robustness of endogenous motion, Fig. 2 presents bifurcation diagrams for $0 < a < 2$ (top), $0 < b < 2$ (central) and $0 < c < 2$ (bottom), with the remaining parameters fixed at $a = b = c = 1$ and $d = w = 0.5$. Again, complicated movements arise. While chartism seems to destabilize the market, fundamentalism is apparently stabilizing. Naturally, a higher price adjustment destabilizes the market as well. Overall, many parameter combinations exist which trigger complicated motion.[a]

[a] To observe permanent fluctuations only small variations in A are needed. Suppose that A takes two values centered around the upper bound of the stability condition X, say $X - Y$ and $X + Y$, depending on whether trading volume is above or below a certain level Z. Such a system obviously not only produces nonconvergent but also nonexplosive fluctuations for arbitrary values of Y and Z.

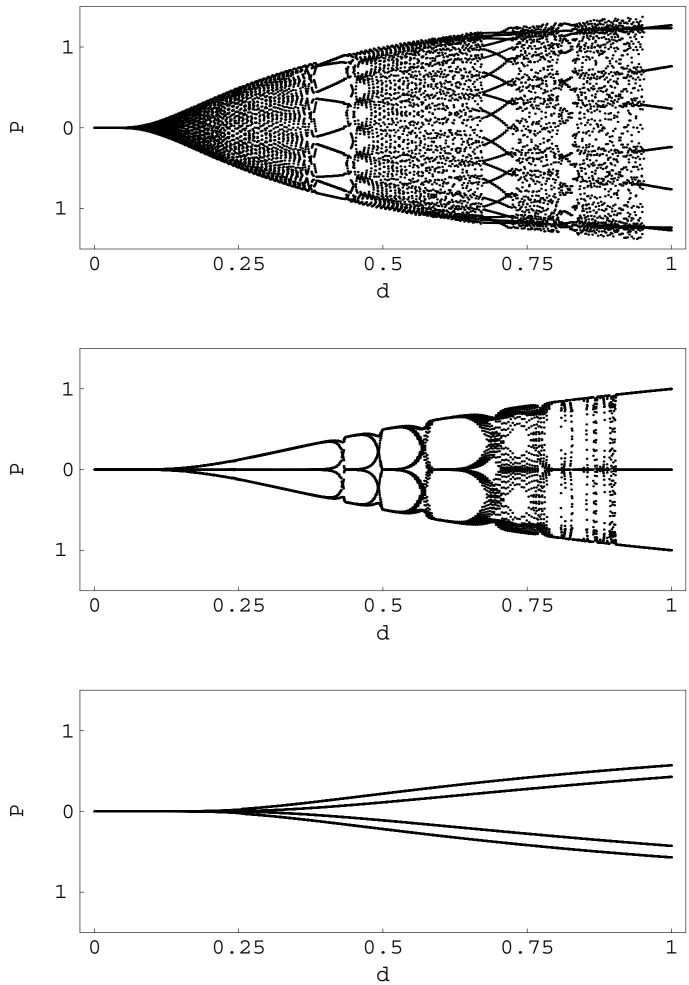

Fig. 1. Bifurcation diagrams for $0 < d < 1$ and $w = 0.7$ (top), $w = 0.5$ (central) and $w = 0.3$ (bottom). The other parameters are fixed at $a = b = c = 1$. The parameter d is increased in 500 steps. For each value of d, P is plotted from $t = 1001$–1100. The log of the fundamental value is $F = 0$.

Let us explore what drives the dynamics. Figure 3 shows the dynamics in the time domain for $a = 0.85$, $b = c = 1$, and $d = w = 0.5$. The first, second and third panels present the log of the price P, the price adjustment A and the trading volume V for 150 observations, respectively. Visual inspection reveals that the price circles around its fundamental value without any tendency to converge. Nonlinear price adjustment may thus be an endogenous engine for volatility and trading volume. Note that when trading volume drops the price adjustment increases and price movements are amplified. However, the dynamics does not explode since a higher trading volume leads again to a decrease in the price adjustment.

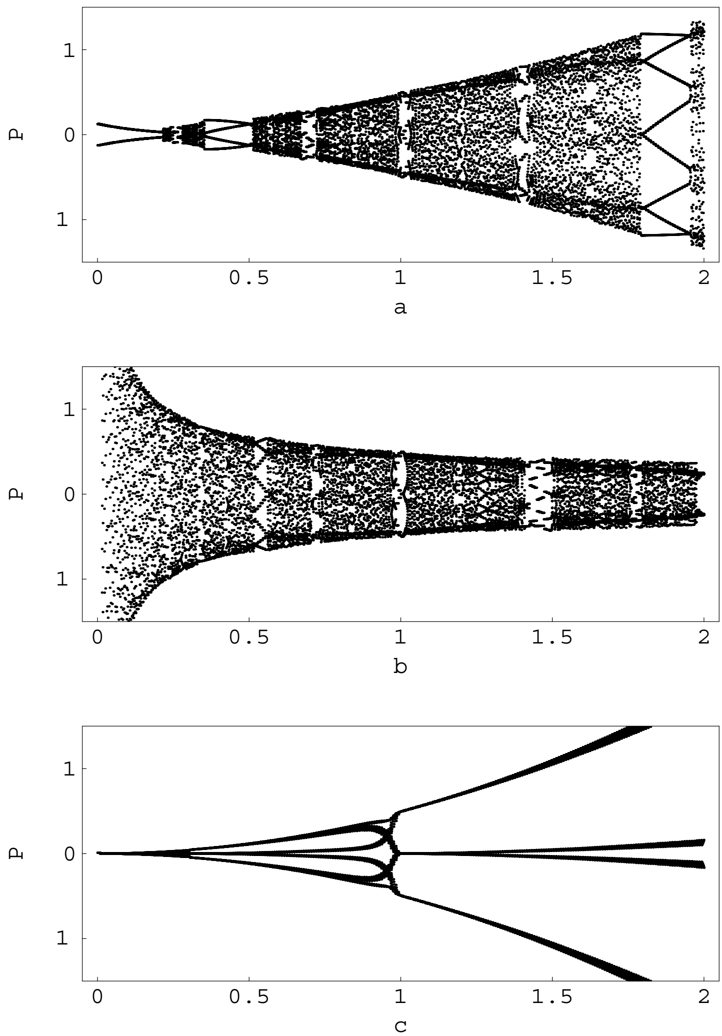

Fig. 2. Bifurcation diagrams for $0 < a < 2$ (top), $0 < b < 2$ (central) and $0 < c < 2$ (bottom), with the remaining parameters fixed at $a = b = c = 1$ and $d = w = 0.5$. The bifurcation parameters are increased in 500 steps. For each value, P is plotted from $t = 1001$–1100. The log of the fundamental value is $F = 0$.

Finally, Fig. 4 displays the price (top panel) and the trading volume (bottom panel) for 5000 observations ($a = 0.25$, $b = 1$, $c = 50$, $d = 2$ and $w = 0.5$). As can be seen, the dynamics may become quite complex. Remember that trading volume increases with increasing price changes (orders of chartists) and/or increasing deviations from fundamentals (orders of fundametalists). In a stylized way, the dynamics may thus be sketched as follows: suppose that trading volume is relatively low. Since the price adjustment A is strong, the system is unstable. As the trading becomes increasingly hectic, prices start to diverge from the fundamental value.

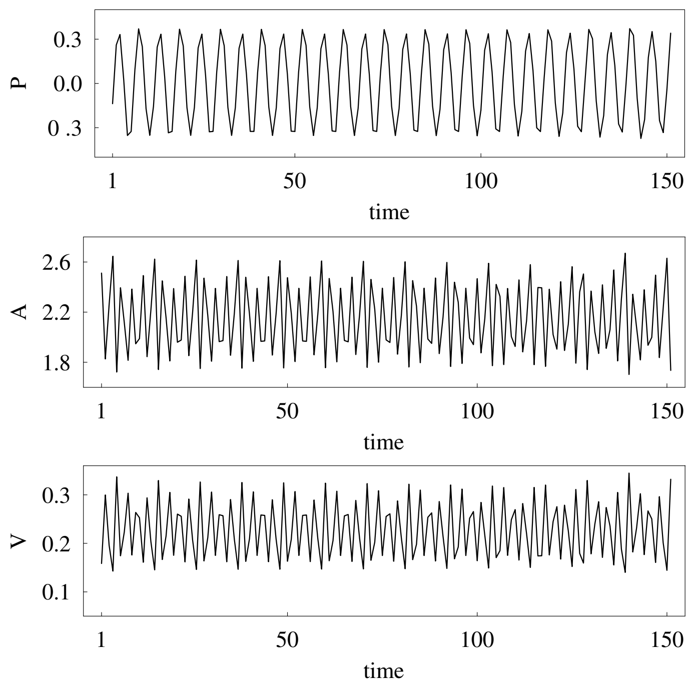

Fig. 3. The dynamics in the time domain for $a = 0.85$, $b = c = 1$, and $d = w = 0.5$. The first, second and third panels show the price P, the price adjustment A and the trading volume V for 150 observations, respectively. The log of the fundamental value is $F = 0$.

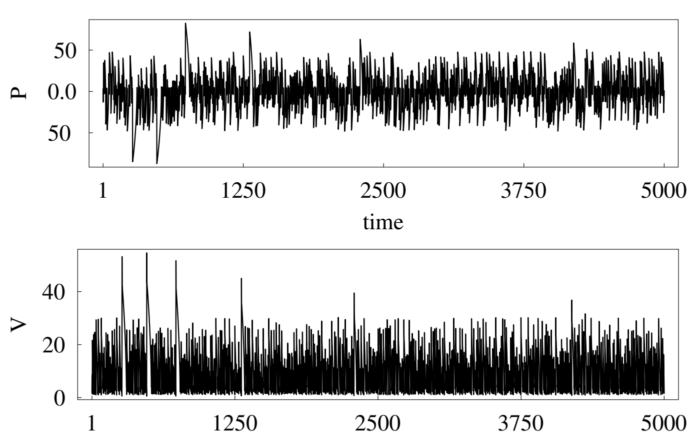

Fig. 4. The dynamics in the time domain for $a = 0.25$, $b = 1$, $c = 50$, $d = 2$ and $w = 0.5$. The first (second) panel displays the price P (the trading volume V) for 5000 observations. The log of the fundamental value is $F = 0$.

At some point, however, the trading activity has become so strong that, due to the reduction of the price adjustment A, the system becomes stable. Afterwards, a period of convergence begins until the system jumps back to the unstable regime. This process continually repeats itself but in an intricate way.

5. Conclusions

When switching between simple linear trading rules and/or relying on nonlinear strategies, interactions between heterogeneous agents may cause irregular dynamics. This note shows that changes in market depth also stimulate price changes. The reason is that if market liquidity goes down, a given order obtains a larger price impact. For a broad range of parameter combinations, erratic yet deterministic trajectories emerge since the system switches back and forth between stable and unstable regimes.

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