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Speculative Behavior and Asset Price Dynamics

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This paper deals with speculative trading. Guided by empirical observations, a nonlinear deterministic asset pricing model is developed in which traders repeatedly choose between technical and fundamental analysis to determine their orders. The interaction between the trading rules produces complex dynamics. The model endogenously replicates the stylized facts of excess volatility, high trading volumes, shifts in the level of asset prices, and volatility clustering.

KEY WORDS: financial markets; nonlinear dynamics and chaos; technical and fundamental analysis.

INTRODUCTION

According to the efficient market hypothesis, asset prices always reflect the fundamental value. Price changes are completely random and solely driven by unexpected news about fundamentals. Recently, a new class of models has emerged which challenges the efficient market paradigm. The chartist-fundamentalist approach shows that the interaction between heterogeneous traders may produce endogenous price dynamics.

An early model of this genre that used catastrophe theory was due to Zeeman (1974), while the first to endogenously show chaotic dynamics in such a context was due to Day and Huang (1990). Contributions such as Arthur, Holland, LeBaron, Palmer and Tayler (1997), Brock and Hommes (1997), Farmer and Joshi (2002) and Lux and Marchesi (2000) are able to replicate the main stylized facts of financial markets in a quite remarkable

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way. To match the behavior of real asset prices closely, the dynamics are buffeted with random shocks.

Although these models are fairly elegant, they also face some criticism. For instance, LeBaron (2000) argues that due to the inherent complexity of these models it is often very difficult to pin down causalities acting inside the market. Our paper tries to alleviate such protest by following a suggestion of Mandelbrot (1997). According to Mandelbrot, a good model of price variation is one that mimics a great number of empirical regularities within a simple framework.

We therefore try to derive a simple deterministic framework which has the potential to mimic some important phenomena of financial markets. Moreover, to develop a realistic framework we describe the behavior of the agents with the help of empirical evidence. In our model, traders choose between technical and fundamental trading rules to determine their orders, thus generating a nonlinear feedback process.

Our main findings are as follows. Simulations produce equilibrium prices which hover erratically around some fundamental value without any apparent tendency to converge. The trading signals needed to keep asset prices in motion are generated by the agents themselves. Clearly, it is the activity of the traders that creates excess volatility and high trading volumes. Since predictions of technical trading rules may be more often right than wrong, the use of such heuristics is not irrational per se. In addition, the dynamics display endogenous shifts in the level and in the volatility of asset prices.

Nonlinearity is a necessary condition for chaos. Indeed, asset prices fluctuate chaotically within our setting. Note that the presence of random perturbations makes it hard to distinguish between high dimensional chaos and pure randomness. Nevertheless, Barnett and Serletis (2000) and Guastello (1995) are able to report some evidence of chaos and clear evidence of nonlinear dependence in financial markets. We argue that nonlinearities are essential for the empirical regularities discussed above.

This paper is organized as follows. First, we propose a simple nonlinear asset pricing model. We provide empirical evidence supporting our approach and discuss some simulation results. Then, we establish that the model is able to produce chaotic price movements. Afterwards, we try to relate several nonlinear phenomena to stylized facts of financial markets. Finally, we offer some conclusions and point out some extensions.

A NONLINEAR ASSET PRICING MODEL

Motivation

Psychological experiments impressively indicate that people are boundedly rational (Kahneman, Slovic & Tversky, 1986). To determine their action, they use simple heuristics. In the case of financial markets, we are luckily able to approximate these rules quite well. Let us briefly review the trading environment of the agents.

Since the mid 1980s, the daily turnover in financial markets has increased sharply. The trading volume increasingly reflects very short-term and speculative transactions. In foreign exchange markets, for example, operations of intraday traders account for 75 percent of the market volume (BIS, 2002). To derive their orders, agents rely on both technical and fundamental trading rules (Taylor & Allen, 1992).

Our model thus considers traders who are familiar with both technical and fundamental analysis. Technical trading rules rely on past movements of the asset price as an indicator of market sentiment and extrapolate these into the future, thus adding a positive feedback to the dynamics (Murphy, 1999). Fundamental trading rules are designed to exploit differences between prices and fundamentals. Since fundamentalists trade on a reduction of the mispricing they add a negative feedback to the dynamics (Moosa, 2000).

The decision to select a particular trading rule depends on expected profit opportunities, which the agents try to derive out of the condition (the mood) of the market. One often observes that fundamentalism, compared to chartism, becomes more popular the wider the spot rate deviates from its perceived fundamental value. In the language of the traders, the market becomes oversold or overbought (Murphy, 1999). In such a situation, agents believe that the chance of the asset price returning to its fundamental value increases as the mispricing rises.

Note that the selection of the rules introduces nonlinearity into the dynamics. The chartists are most influential if prices are near to fundamentals. Since the behavior of technical traders is trend extrapolating, the asset price is typically driven away from its fundamental value. However, the higher the mispricing, the more the market impact of the fundamentalists increases. Transactions of this group lead to a mean reversion until the chartists again reign over the market.

Setup

One of the most popular technical trading rules is the double crossover method, in which a buy (sell) signal is given when a short-term moving average of past asset prices crosses a long-term moving average of past asset prices from below (above). The time windows of the moving averages typically vary from trader to trader. However, for our purpose it seems appropriate to approximate the demand of chartists in period t as

$$d_t^C = \alpha^C((Log S_{t-1} - Log S_{t-2}) - 0.5(Log S_{t-1} - Log S_{t-3})), \qquad (1)$$

where S is the asset price. The first (second) term of Eq. 1 is a one-period (two-period) change reflecting the short-term (long-term) moving average. The reaction coefficient α^C is positive. Note that by Eq. 1 chartists place a market order today in response to past price changes, i.e. price changes between period t and $t-1$ are disregarded. Such a lag structure is typical for technical trading rules because only the past price movements are taken into account (Murphy, 1999).

Fundamental analysis is built on the premise that asset prices converge towards fundamentals. For simplicity, we assume that market participants correctly perceive the constant fundamental asset price F. Takagi (1991) reports that agents typically form regressive expectations like

$$E_t[S_{t+1}] = \gamma F + (1-\gamma)S_{t-1}, \tag{2}$$

where γ represents the expected adjustment speed of the asset price towards its fundamental value ($0 < \gamma < 1$). Since expectations have to be formed before trading starts, the last available data is from period $t-1$.

The demand of fundamentalists may be written as

$$d_t^F = \alpha^F(E_t[S_{t+1}] - S_t)/S_t = \alpha^F(\gamma F + (1-\gamma)S_{t-1} - S_t)/S_t. \tag{3}$$

Fundamental trading rules deliver a buy (sell) signal if the expected future asset price is above (below) the spot rate. The demand is calibrated according to α^F, with $\alpha^F > 0$. Westerhoff (2002) studies the case in which traders misperceive the fundamental value.

The selection of a trading rule depends on expected performance possibilities and has to be made before trading starts. The weight of chartists is defined as

$$m_t = \frac{1}{1 + \beta^1 + \beta^2((F - S_{t-1})/S_{t-1})^2}, \tag{4}$$

whereas that of fundamentalists is $(1 - m_t)$. The coefficient β^1 reflects the basic proportion of agents who are always fundamentalists. If, for example, $\beta^1$is 0.25, then 20 percent of agents are permanently fundamentalists whatever the situation of the market ($\beta^1 > 0$).

Nevertheless, most traders adjust their trading strategies with respect to the condition of the market. The intuition behind Eq. 4 is as follows. The more prices deviate from fundamentals, the more traders are convinced that a price correction will occur. These traders naturally prefer fundamental analysis. The coefficient β^2 indicates the popularity of fundamentalism ($\beta^2 > 0$).

The market clearing condition is given as the sum of all trading positions

$$m_t d_t^C + (1 - m_t)d_t^F = 0. \tag{5}$$

Note that only two types of traders act in the market. Since chartists trade on the basis of past price movements, their demand is perfectly inelastic. The job of market-making is thus assigned to fundamentalists. They clear the market and set new prices.

Combining Eq. 1–5 and solving for S yields

$$S_t = \frac{\gamma F + (1-\gamma) S_{t-1}}{1 - \frac{\alpha^C (0.5 Log S_{t-1} - Log S_{t-2} + 0.5 Log S_{t-3})}{\alpha^F (\beta^1 + \beta^2 ((F - S_{t-1})/S_{t-1})^2)}}, \tag{6}$$

which is a three-dimensional nonlinear deterministic difference equation. Because Eq. 6 precludes closed analysis, we simulate the dynamics to demonstrate that the underlying structure gives rise to complex price behavior, as is observed empirically.

Calibration

Unfortunately, most coefficients of our model are not directly empirically observable. To obtain a first base run, the following assumptions have been made. The reaction coefficients are equal ($\alpha^C = \alpha^F = 1$), the expected adjustment speed of prices towards fundamentals is 20 percent ($\gamma = 0.2$), and the fundamental value of the asset is $F = 100$. Survey studies (Taylor & Allen, 1992) report that 5 to 15 percent of market participants rely solely on fundamental analysis. Therefore, $\beta^1 = 0.125$ seems to be a reasonable choice. The extent of volatility is then calibrated via the popularity of fundamental analysis. For $\beta^2 = 12{,}800$, the dynamics evolve quite realistically.

Using Eq. 6 and these parameter values it should be relatively easy to replicate our results. We consider this to be one advantage of our study over more complicated contributions.

Simulations

The top chart of Fig. 1 displays a typical example of the behavior of asset prices within our model. After an initial shock, the prices circle in a complex fashion around their fundamental value. The bottom chart shows the corresponding weight of chartists. The agents often switch between technical and fundamental analysis. At no time is one of the rules ever driven out of the market.

The dynamics could be explained as follows. Technical trading rules always produce some kind of buy or sell signal and may, on the basis of a feedback process, induce a self-reinforcing run. But such a run cannot last

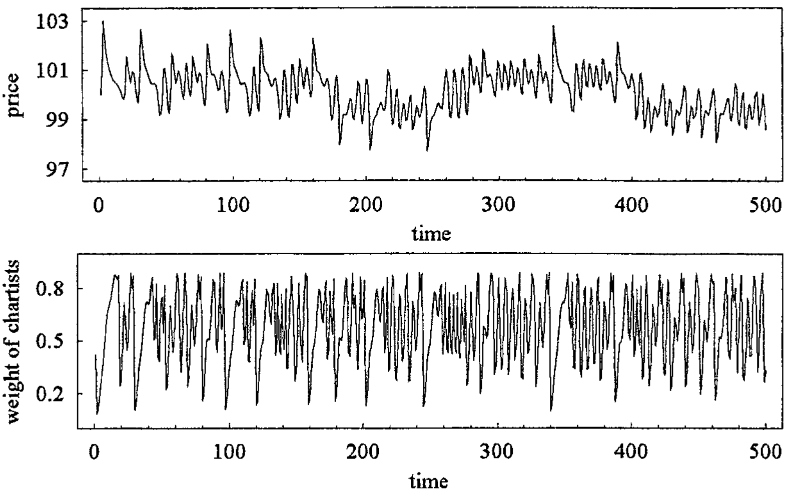

Fig. 1. Asset Price Dynamics. $F = S_1 = 100, S_2 = 101, S_3 = 103, \alpha^C = \alpha^F = 1, \gamma = 0.2, \beta^1 = 0.125, \beta^2 = 12{,}800$.

because investment rules based on fundamentals work like a center of gravity. The more prices depart from fundamentals, the stronger the influence of fundamentalists, until eventually their increasing net position triggers a mean reversion. However, this already indicates a new signal for chartists and leads directly to the next momentum. Prices are repelled by fundamentals because chartism dominates the market in this region. Heavy outliers occur when chartists have a clear trading signal and the influence of fundamentalists is low.

It is worth noting that our simple model already suffices to produce the high volatility of financial markets that is observed empirically. A single disturbance is enough to trigger price movements that do not converge to some fixed point. Excess volatility is at least partially generated by an endogenous nonlinear law of motion. The trading signals needed to keep the process going are generated by the agents themselves.

This is exactly what Black (1986) has called noise trading. Black concludes that noise in the sense of a large number of small events is essential to the existence of liquid markets. He argues that a person who wants to trade needs another person with opposite beliefs. To explain the high trading volume in financial markets it is not reasonable to assume that differences in beliefs are merely the result of different information. In our model, noise is permanently produced by the agents themselves. Even when there is no new information at all, trading volume and volatility will be high. Noise trading is trading on noise as if it were information.

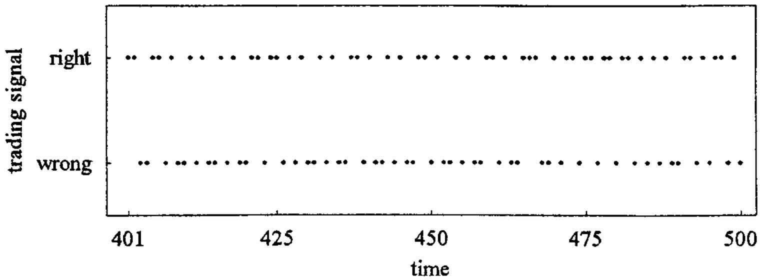

Fig. 2. Technical Analysis. $F = S_1 = 100, S_2 = 101, S_3 = 103, \alpha^C = \alpha^F = 1, \gamma = 0.2, \beta^1 = 0.125, \beta^2 = 12{,}800$.

For Heiner (1983), such behavior may not necessarily be irrational. Heiner argues that the limits to maximizing in an uncertain environment are the origin of a rule-governed behavior. For example, for every agent the specific complexity of financial markets leads to a gap between his competence to make an optimizing decision and the actual difficulty involved with this decision. Agents can do no better in the presence of complex dynamics than to follow some adaptive scheme of behavior.

Is this also true within our framework? To answer this question we have designed the following experiment. Suppose that one agent always relies on technical analysis as specified by Eq. 1. The technical trading rule delivers a right trading signal if it correctly predicts the change between S_{t+1} and S_t (else the signal is wrong). For example, the trading rule delivers a right signal if it generates a buy signal and the price rises. Figure 2 displays the outcome for trading periods 401–500. Within this sample, technical analysis produces 51 right decisions. Since the agent is more often right than wrong, his behavior cannot necessarily be called irrational. In addition, the possible profitability of technical analysis has also been demonstrated empirically (Brock, Lakonishok & LeBaron, 1992).

NONLINEAR DYNAMICS AND CHAOTIC MOTION

Nonlinear dynamic systems have the potential to produce chaotic motion. Although no commonly agreed definition of chaos exists, three important aspects regularly emerge:

First, the trajectory of a deterministic process should be highly irregular. At least some of the standard tests of randomness cannot distinguish between chaotic patterns of change and truly random behavior.

Second, the time path is sensitive to a microscopic change in the value of the initial conditions (SIC). This means that for a slightly different choice

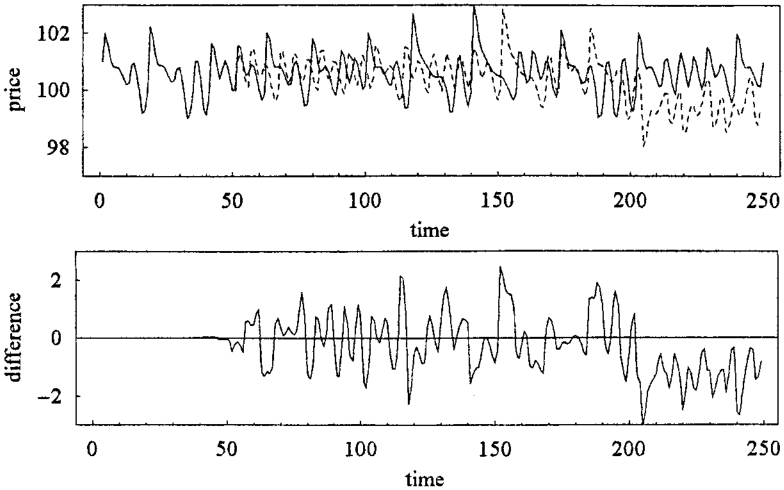

Fig. 3. Sensitivity to Initial Conditions. Solid line: $F = S_1 = 100$, $S_2 = 101$, $S_3 = 102$, $\alpha^C = \alpha^F = 1$, $\gamma = 0.2$, $\beta^1 = 0.125$, $\beta^2 = 12,800$. Dashed line: the same but $S_3 = 102.0001$.

of an initial value, the trajectory of the system may diverge relatively rapidly from the original phase space.

Third, the time path may never return to any point it has previously crossed, but displays an oscillator pattern in a bounded region. In a phase space representation, the dynamics may display a complex structure, known as a strange attractor.

A discussion of the competing definitions of chaos can be found in Rosser (2000). For econometric issues compare Dechert (1996).

Next, we illustrate what is meant by sensitivity to initial conditions and complex orbit structure. Both phenomena can be found in our time series. Figure 3 compares two simulation runs with nearly identical sets of initial conditions and parameters. The only difference is that the solid line is computed with $S_3 = 102$ and the dashed line with $S_3 = 102.0001$. Surprisingly, after about 50 periods the time series start to diverge. In the bottom chart the difference between the two time series is plotted. After some iterations, the difference grows to the same order of magnitude as the usual fluctuations. A small change in the initial conditions alters the whole future path of the asset price in quite a dramatic way. A similar picture emerges if the parameters of the model are slightly changed.

Figure 4 displays the dynamics in phase space, that is S_t is plotted against S_{t-1}. The same is carried out for the log of price changes (the returns). The top chart of Fig. 4 contains the whole attractor; the bottom chart shows a

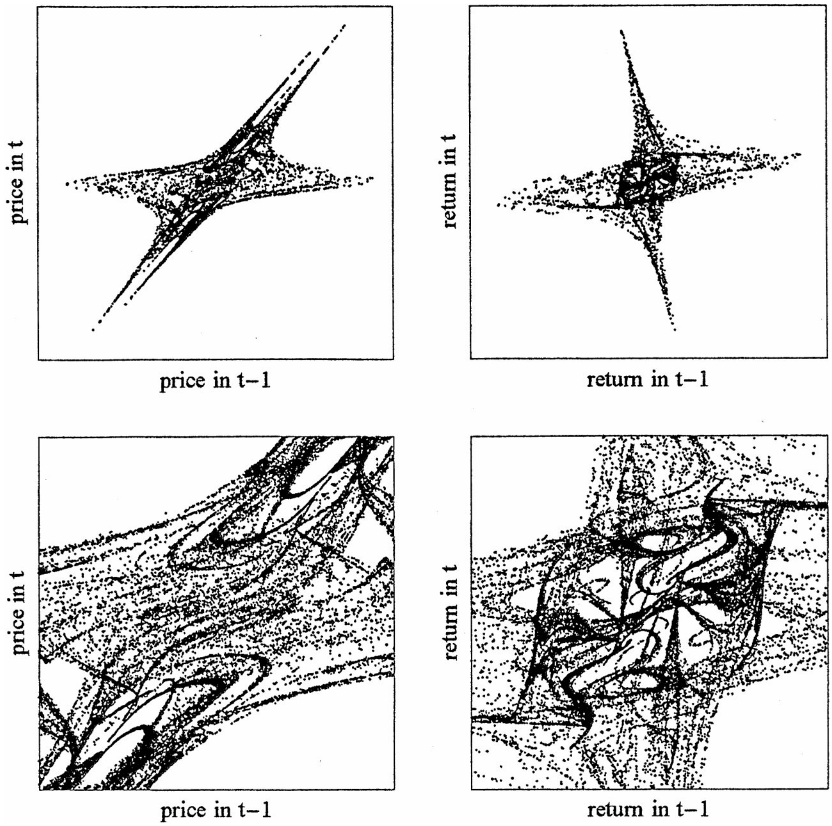

Fig. 4. Asset Prices and Returns in Phase Space. $F = S_1 = 100$, $S_2 = 101$, $S_3 = 103$, $\alpha^C = \alpha^F = 1$, $\gamma = 0.2$, $\beta^1 = 0.125$, $\beta^2 = 12{,}800$. Top: data from $t = 1{,}000 - 11{,}000$. Bottom: data from $t = 1{,}000 - 31{,}000$.

blow-up. Although the dynamics in the time domain seem rather random, a distinct structure is built up in phase space. Due to SIC, no meaningful precise forecasting is possible in the long run. However, some long-term properties can be identified, such as the structure in the phase space.

Numerical tests for chaos mainly consist of computing the largest Lyapunov exponent and the correlation dimension. The Lyapunov exponent measures how sensitively a trajectory reacts to a change in initial conditions. If the Lyapunov exponent is positive, two originally close orbits start to diverge in phase space. The correlation dimension describes the complexity of the attractor in phase space. It indicates to which degree the phase space is occupied by the attractor. The higher the dimension, the more complex the structure. An object with a noninteger or fractal dimension is evidence of a strange attractor.

We do not intend to go into the technical details of computing these statistics and refer the reader to the relevant literature (Hilborn, 2000; Kantz & Schreiber, 1999; Ott, Sauer & Yorke, 1994). However, since the Lyapunov exponent is estimated as 0.142 and the correlation dimension as 1.62 the time series of our base line simulation (Fig. 1) exhibits strong evidence of chaos.

In our model, chaos results from a nonlinear switching between different trading rules. In a related study, Kaizoji (2002) derives nonlinearities from the demand functions of chartists and fundamentalists. He is able to prove the existence of chaos mathematically.

NONLINEAR DYNAMICS AND STYLIZED FACTS

The price dynamics of financial markets is often characterized by certain stylized facts (Campbell, Lo, & MacKinlay, 1997; Guillaume, et al., 1997; Pagan, 1996). In this section, we aim to explain these universal features by relating them to some typical nonlinear phenomena. The first group of phenomena is concerned with breaks in the level of asset prices, whereas the second group involves volatility clustering.

Sudden changes in the level of asset prices are a commonly observed price pattern. Such breaks may be triggered by fundamental shocks. But, as reported by Goodhart (1988), large price movements unrelated to any item of news are also apparent. If the agents suddenly expect a fundamental shift in the economy, a regime break may occur and financial prices react immediately. Moreover, when expectations prove to be wrong, prices move back to their previous level. In such a case, the price level has changed twice although no fundamental shock has occurred.

Structural breaks can easily be modeled exogenously within our model. The top chart of Fig. 5 shows a simulation in which a single fundamental shock occurs in period 300, raising the fundamental value of the asset from $F = 100$ to $F = 104$. In the first 300 periods, the asset price fluctuates as usual around its fundamental value. Afterwards the whole dynamics are raised to the new equilibrium. In the bottom chart, the fundamental value between periods 200 and 400 is $F = 96$; otherwise it is $F = 100$. This can be interpreted as a temporary interlude of misperception of the fundamental value.

A certain characteristic observable in nonlinear dynamical systems allows us to explain such a pattern with fewer exogenous interventions. In the case of coexisting attractors, one has the puzzling feature that the repetition

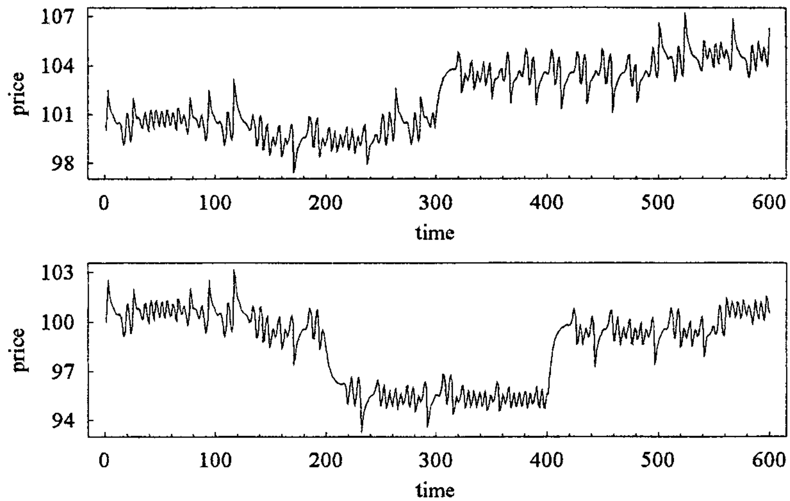

Fig. 5. Changes in the Level of Asset Prices. $S_1 = 100$, $S_2 = 101$, $S_3 = 103$, $\alpha^C = \alpha^F = 1$, $\gamma = 0.2$, $\beta^1 = 0.125$, $\beta^2 = 12{,}800$. Top: for $t > 300$ $F = 104$, otherwise $F = 100$. Bottom: for $200 < t < 400$ $F = 96$, otherwise $F = 100$.

of a simulation with the same parameters may yield a qualitatively different result. On which attractor the trajectory will settle depends on the initial conditions.

Figure 6 shows such an example. The only difference between the simulations in the top and the middle charts is that $S_3 = 101.2$ in the top chart and $S_3 = 99.6$ in the middle chart. However, the outcome is quite distinct. After some iterations, one orbit evolves to a limit cycle above the fundamental value of the asset price; the other lies below it. For clarity of exposition, a simple example of coexisting attractors is chosen. Of course, more complicated combinations of attractors exist. For instance, $\beta^1 = 0.375$ generates two quasiperiodic attractors and for $\beta^1 = 0.1475$, one finds a limit cycle combined with a chaotic attractor.

In the bottom section of Fig. 6, there seem to be fundamental breaks in the time series. For instance, between $t = 167$ and $t = 333$ the prices fluctuate around a lower level than at other times. But no fundamental shock has occurred. The asset market is only hit by exogenous noise in two trading periods, that is we have set $S_{167} = 99.5$ and $S_{333} = 101.2$. This suffices for the trajectory to change its attractor. Apparently, structural breaks in a time series may be nothing more than jumping between different attractors.

So far it is necessary to add at least some noise to mimic structural breaks. But the model has the potential to generate such phenomena

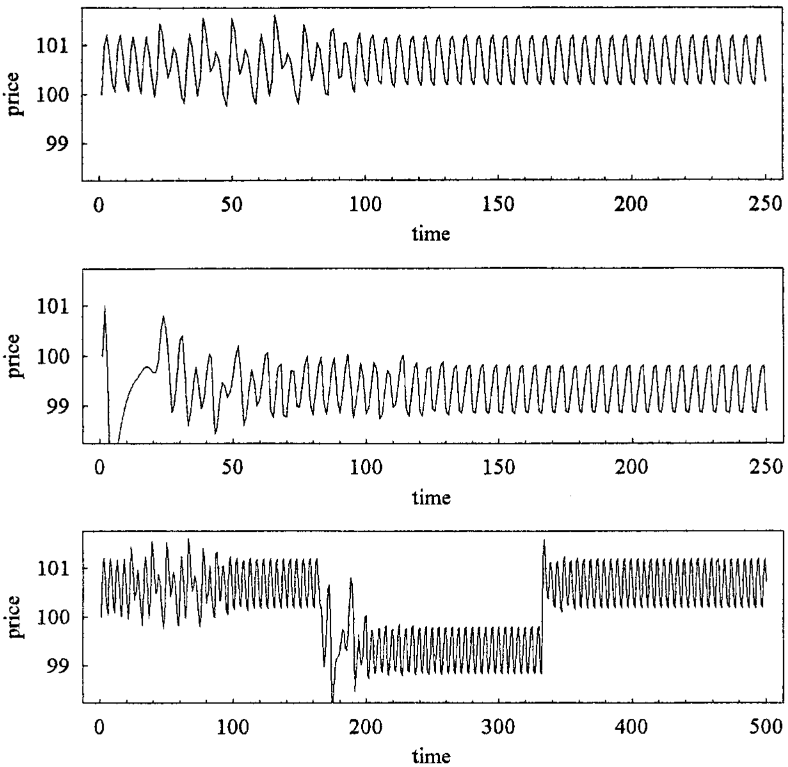

Fig. 6. Coexisting Attractors. Top: $F = S_1 = 100$, $S_2 = 101$, $S_3 = 101.2$, $\alpha^C = \alpha^F = 1$, $\beta^1 = 0.167$, $\beta^2 = 12{,}800$, $\gamma = 0.2$. Middle: the same but $S_3 = 99.6$. Bottom: the same but $S_{167} = 99.5$ and $S_{333} = 101.2$.

completely endogenously. Figure 7 provides an example. In the top section, the asset price is plotted for 30,000 periods, while the bottom section shows the time series for $t = 4,100–4,500$. For a long time period, the asset price fluctuates within a band between 100 and 104. Suddenly, without any apparent reason, the band shifts downwards for some time. Again, out of the blue, the dynamics switch back to its former region and stay there for a long time until a similar pattern repeats itself. Note that the fluctuations are driven solely endogenously; there is no noise added.

Besides changes in the level of asset prices, one also observes variations in the volatility. Volatility clustering describes the phenomenon in which periods of low volatility alternate with periods of high volatility (Mandelbrot, 1963). One way to replicate such a pattern is, of course, to exogenously adjust the parameters of the model.

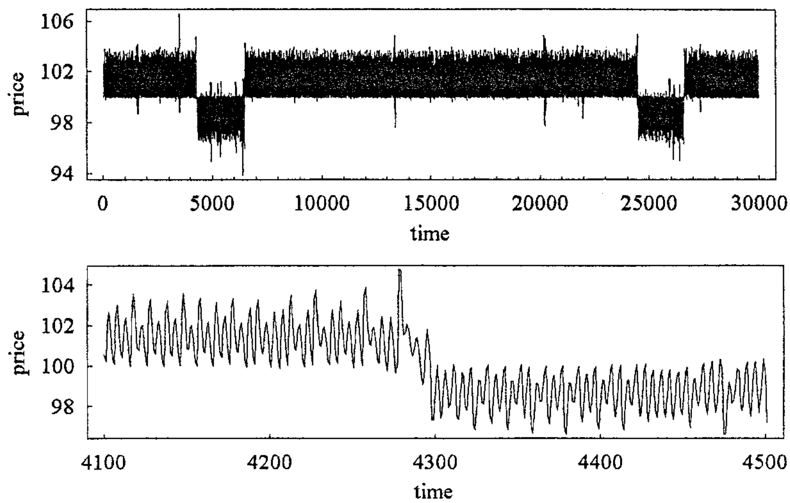

Fig. 7. Endogenous Breaks. $F = S_1 = 100$, $S_2 = 101$, $S_3 = 102.5$, $\alpha^C = \alpha^F = 1$, $\gamma = 0.2$, $\beta^1 = 0.1125$, $\beta^2 = 12{,}800$.

Figure 8 displays two examples. The following shocks are assumed. In the top section for $2,000 < t < 4,000$ $\beta^2 = 2,800$, otherwise $\beta^2 = 32,000$, and in the bottom section for $200 < t < 400$ $\beta^2 = 3,200$, otherwise $\beta^2 = 32,000$. Hence, one source of volatility clustering might be changes in the popularity of fundamental analysis. Periods of high volatility coincide with periods in which the popularity of fundamentalism is low (and vice versa). Especially in periods of high uncertainty about the fundamental condition of a market, fundamentalism is not very popular. In such a situation, a central authority may have the chance to calm down the market by creating a greater consensus about fundamentals. For example, a central bank may stabilize exchange rate fluctuations by providing better public information. But β^2 may also autonomously switch its value. Sometimes the traders tend to herd together. If a famous guru supports a certain class of trading strategies, then volatility clustering is unrelated to fundamental reasons.

Some nonlinear phenomena are also suggestive for volatility clustering: transient behavior and on-off intermittency. By definition, transient behavior disappears after some time. Therefore at first sight it might seem neither relevant nor interesting. But both impressions are wrong. Starting with some initial conditions, one has to wait a certain time until the trajectory has settled down on the attractor. During this transient phase the motion may have completely different properties to those on the attractor itself. The transient time can be extremely short (for instance, for a stable fixed point). But in

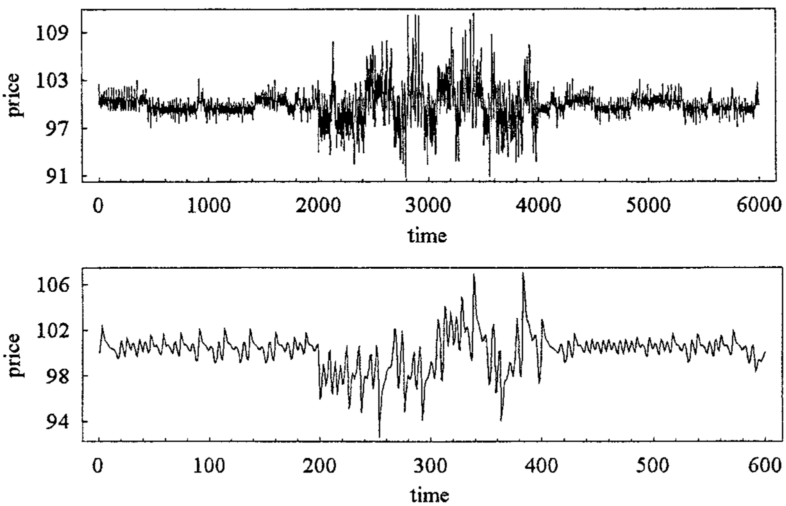

Fig. 8. Volatility Clustering. $F = S_1 = 100$, $S_2 = 101$, $S_3 = 102.5$, $\alpha^C = \alpha^F = 1$, $\gamma = 0.2$, $\beta^1 = 0.125$. Top: for $2{,}000 < t < 4{,}000$ $\beta^2 = 2{,}800$, otherwise $\beta^2 = 32{,}000$. Bottom: for $200 < t < 400$ $\beta^2 = 3{,}200$, otherwise $\beta^2 = 32{,}000$.

certain cases, transients can last a long time. The transient can be nontrivial even if the attractor itself is simple.

Figure 9 contains a simulation run in which the attractor is a limit cycle. Before the attractor is reached, the dynamics are highly irregular. The bottom part of Fig. 9 shows a transient which lasts over 500 periods. During this period the dynamics are comparatively volatile. Afterwards, the fluctuations are less pronounced. Note that a single disturbance can be enough for the system to switch back to turbulent motion. In period $t = 1,204$, the asset price is set to 102.6555. The trajectory needs around 500 periods to approach its attractor. Hence, volatility clustering might be caused by temporary shocks which trigger complex transient dynamics in an otherwise calm environment.

On-off intermittency means that the dynamics alternate between tranquil and turbulent motion in an irregular fashion. The chaotic phases can be long or they can look like short bursts. Figure 10 shows an example for three different time periods. For a certain parameter combination the behavior of the model switches back and forth between two qualitatively different kinds of motion, even though all the parameters remain constant and no external noise is present. The switching appears to occur randomly. Both the duration and the frequency of bursts of chaotic behavior are unsystematic. On-off intermittency is an endogenous source of volatility clustering.

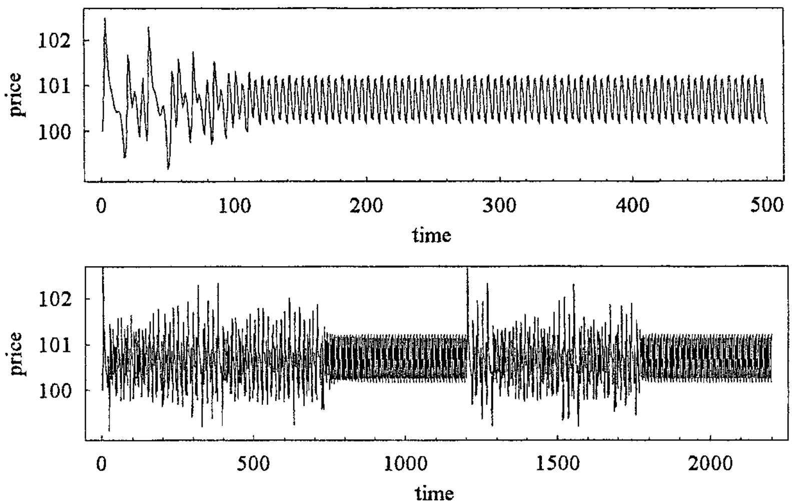

Fig. 9. Transient Behavior. Top: $F = S_1 = 100$, $S_2 = 101$, $S_3 = 102.5$, $\alpha^C = \alpha^F = 1$, $\beta^1 = 0.15875$, $\beta^2 = 12{,}800$, $\gamma = 0.2$. Bottom: the same but $S_3 = 103$ and $S_{1,204} = 102.6555$.

CONCLUSIONS

The aim of our paper is to explore the impact of speculative trading on asset price fluctuations within a simple behavioral setting. Crucial to the dynamics is the interaction between technical and fundamental trading rules. In general, technical analysis tends to destabilize the market, whereas fundamental analysis has a mean reverting effect. Since the agents tend to prefer technical trading rules when the market is not strongly mispriced, the asset price is driven away from its fundamental value. Due to this fact, one may draw the conclusion that financial markets are inherently instable.

Besides replicating the stylized fact of excess volatility and high trading volumes, we have identified several sources of changes in the level and the volatility of asset prices. Of course, a shift in the price level may be the result of a permanent fundamental shock. However, in the case of coexisting attractors, a single shock may suffice for the economy to switch its dynamical behavior. Moreover, nonlinear dynamic systems even possess the ability to endogenously mimic regime breaks. As we have seen, the asset price may fluctuate in a certain area for some time before it suddenly shifts to another region.

Volatility clustering is partly the result of changing economic conditions. For instance, a crisis may lower the willingness of the fundamentalists

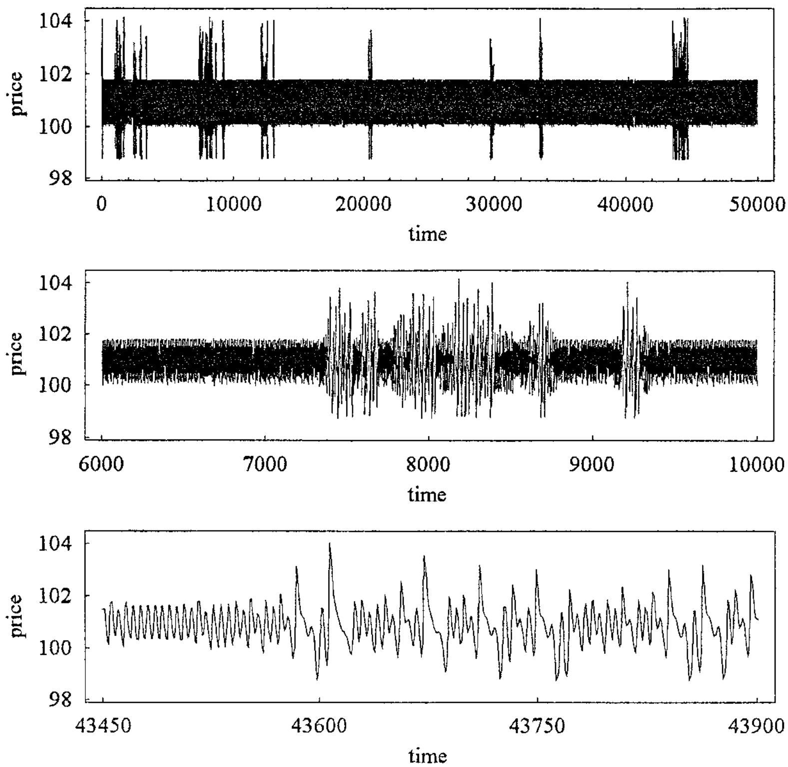

Fig. 10. On-off Intermittency. $F = S_1 = 100$, $S_2 = 101$, $S_3 = 103$, $\alpha^C = \alpha^F = 1$, $\gamma = 0.2$, $\beta^1 = 0.145$, $\beta^2 = 6{,}800$.

to take risks. But the popularity of the trading rules may also be affected by social phenomena such as herding behavior. Transient motion of a system, triggered by some external noise, may also contribute to volatility clustering. Without assuming an arbitrary shock pattern, periods of low and high volatility may be explained endogenously by on-off intermittency.

There is, of course, no question that financial markets are affected by different kinds of stochastic shocks. Extending our model in that direction allows us to mimic the time series properties of financial markets very closely (Westerhoff, 2002). Recently, efforts have been undertaken to use chartist-fundamentalist models as laboratories to study the working of policy means like trading breaks or transaction taxes. Since the dynamics are at least partially due to an endogenous nonlinear law of motion, a thorough understanding of the deterministic skeleton of such models is quite important.

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