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BUBBLES AND CRASHES: OPTIMISM, TREND EXTRAPOLATION AND PANIC

FRANK WESTERHOFF

Department of Economics, University of Osnabrueck, Rolandstrasse 8,
D-49069 Osnabrueck, Germany
fwesterho@oec.uni-osnabrueck.de

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The observation that large drawdowns are outliers suggests that a special mechanism may be responsible for large crashes. We develop a simple model with heterogeneous interacting agents which follow technical and fundamental trading rules to determine their orders. Although the chartists are optimistic most of the time, they panic if prices drop sharply. Our main finding is that the selling impact due to a panic attack may be so large that it directly leads to the next panic attack. Such behavior generates temporal correlation in prices, i.e., causes large drawdowns.

Keywords: Bubbles and crashes; drawdowns and drawups; technical and fundamental analysis; optimism and panic.

JEL classification code: D84, G14

1. Introduction

According to the efficient market hypothesis, prices always reflect their fundamental values [3]. A crash thus has to correspond to a really bad shock. However, thorough ex-post analysis of crashes are in many cases inconclusive as to what this dramatic piece of new information might have been [10, 13]. Inspecting major financial markets, Johansen and Sornette [7] conclude that more than 50% of the crashes occur endogenously due to market instabilities.

Johansen and Sornette [6] argue that large market drops are outliers. They define a drawdown as a persistent decrease in the price over consecutive days. For instance, three successive daily losses of 1% represent a drawdown of around 3%. The concept of drawdowns has the potential to capture correlation in price changes. For symmetric distributions of price variation, starting from a positive return, the probability to have X successive negative returns is 0.5^X. Although independence between successive returns is usually remarkably well verified, Johansen and Sornette [6] find that large drops occur more often than predicted by an exponential distribution. Extreme drawups are less pronounced.

The aim of this paper is to develop a model with heterogeneous interacting agents that mimics the aforementioned observation. Our model is inspired by the chartist-fundamentalist approach which has proven to be quite successful in explaining the stylized facts of financial markets, such as fat tails or volatility clustering. Contributions by Brock and Hommes [1], Cont and Bouchaud [2], Lux and Marchesi [8] and Farmer and Joshi [4] demonstrate that price dynamics are at least partially caused by an endogenous nonlinear law of motion. Although the news arrival process has an impact on the dynamics, it is not its sole driving force.

We consider two types of investors. While fundamentalists bet on mean reversion, chartists extrapolate past price changes into the future to predict prices. In general, the mood of chartists is optimistic. The combination of trend extrapolation and optimism stimulates bubbles. But the mood of the chartists turns into panic if the price drops more than a given percentage. Since the chartists then aggressively submit selling orders, a panic attack may last some periods during which the market crashes. Our model matches the stylized fact of drawdowns quite well.

The paper is organized as follows. In the next section, we briefly repeat the concept of drawdowns and drawups. In Sec. 3, we present a simple model of interacting heterogeneous agents and discuss its dynamic properties. The last section concludes the paper.

2. Drawdowns and Drawups

Most economists agree that prices do not always reflect their fundamental values. Detecting a bubble, however, is difficult since market fundamentals are unobservable. Taking a pragmatic view, Johansen and Sornette [5] identify a bubble as a succession of three events: (1) a preceding period of increasing prices, (2) a sharp price peak, and (3) a fast price decrease following the peak over a time interval much shorter than the accelerating period.

The first panel of Fig. 1 shows daily quotes of the Dow Jones Index between 1901 and 2000. The two panels in the second line contain two of the most famous bubbles. The left-hand side plots the evolution of the Dow Jones Index from the beginning of 1928 to the end of 1929. The right-hand side presents the same for the years 1986 and 1987. The stylized bubble pattern is clearly visible. Of course, bubbles do not always end that dramatically, they may also deflate more slowly.

Johansen and Sornette [6] define a drawdown as a persistent decrease in the price over consecutive days. A drawdown is thus the cumulative loss from the last maximum to the next minimum of the price. Symmetrically, a drawup is defined as the change between a local minimum to the following maximum. The distribution of drawdowns measures whether successive drops influence each other, i.e., whether temporal correlation exists. If prices follow a random walk, then successive price variations should be uncorrelated. Indeed, price changes are highly random. For instance, the autocorrelation function of the returns, displayed in the third panel

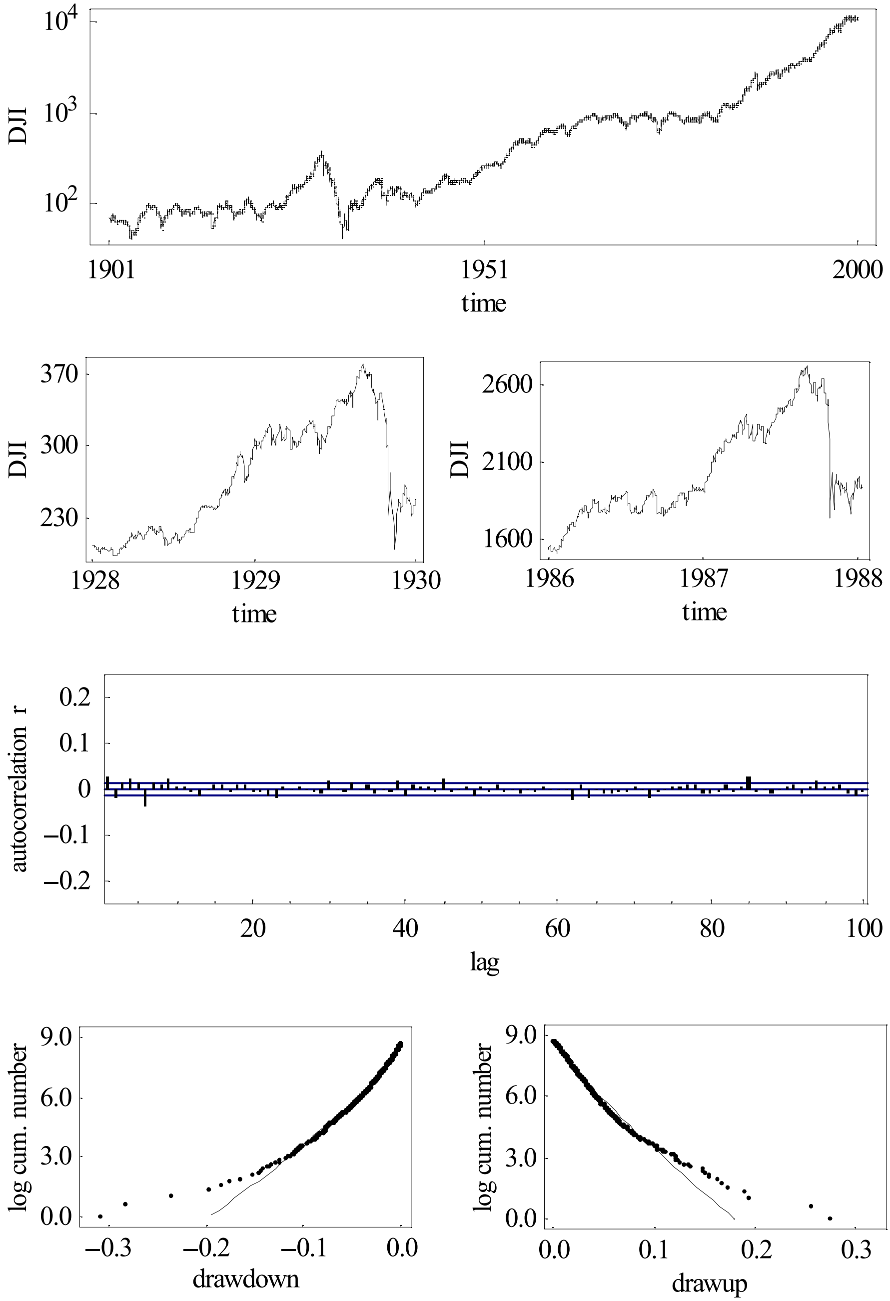

Fig. 1. The dynamics of the Dow Jones Index. The first panel shows daily quotes for the Dow Jones Index from 1901–2000 (25,034 observations). The left-hand (right-hand) side of the second panel contains the same data for the years 1928–1929 (1986–1987). The third panel displays the autocorrelation function of the returns for the first 100 lags (with 95% confidence bands). The bottom two panels present the number of times a given level of price run has been observed (left: drawdowns, right: drawups). The solid line depicts the fit of a stretched exponential distribution.

of Fig. 1, is not significant for almost all lags. But that may not be the case in extreme situations such as during a crisis.

The dotted line in the bottom left-hand (right-hand) panel of Fig. 1 shows the cumulative distribution of drawdowns (drawups). Following Johansen and Sornette [6], the solid line represents the null hypothesis taken as a stretched exponential function. To be precise, the cumulative stretched distribution is fitted as

$$\mathrm{Log}\, N(x) = \mathrm{Log}\, A - B|x|^z\,, \tag{1}$$

where x denotes drawdowns (drawups) and A stands for the total number of drawdowns (drawups). When $z < 1 (z > 1)$, $N(x)$ is a stretched exponential (super exponential). The pure exponential results for $z = 1$.

In a very extensive study incorporating quotes for various stock indices, companies and currencies, Johansen and Sornette [6, 7] ascertain that approximately 98% of the distribution of drawdowns is well represented by the null hypothesis, while the 2% largest drawdowns are outliers. Clearly, they occur at a significantly larger rate than suggested by the null hypothesis. The bottom panels illustrate this finding for the Dow Jones Index. Up to drawdowns of around 10%, the stretched exponential distribution delivers a good representation of the data. Drawdowns above 10%, however, break away from the fitted line. The evidence for drawups is similar, yet somewhat weaker.

3. The Model

We regard agents as boundedly rational in the sense of Simon [11]. Not only is information in general incomplete, but market participants also have a limited ability to analyze the available information. Although agents are not fully rational, they strive to do the right thing. As indicated by many laboratory experiments, agents display a rule-governed behavior [12, 15]. Moreover, questionnaires conducted by, e.g., Taylor and Allen [14] reveal that market professionals rely on technical and fundamental analysis to determine their orders. Since our goal is to replicate the phenomenon of bubbles and crashes within a simple setting, we limit ourselves to two types of speculators: chartists and fundamentalists.

Let us turn to the properties of the model. The fundamental value of the asset increases smoothly over time. To be precise, the log of the fundamental value F in period $t + 1$ is given as

$$F_{t+1} = F_t + \eta\,, \tag{2}$$

where η denotes a constant growth rate.

The price of the asset is determined on an order driven market. These orders reflect the information flow in the market and thus the evolution of the traders' opinions and moods. The price variation is controlled by the net order size, defined as the number of buying orders minus the number of selling orders. Prices increase if the net order size is positive and vice versa. Following Farmer and Joshi [4], the difference in the log of price S between tomorrow and today is proportional to the net order size

$$S_{t+1} = S_t + \alpha^M(\omega D_t^F + (1-\omega)D_t^C) + \delta_t\,, \tag{3}$$

where α^M is a positive scale parameter to normalize the order size. Transactions of fundamentalists D^F and chartists D^C are weighted by their market shares ω and $(1-\omega)$, respectively. The noise term δ, which is normally distributed with mean zero and constant variance σ^δ, comprises all remaining elements that may have an impact on the price but are not captured within our model (e.g., pure noise traders).

Fundamental trading first requires an estimation of the value of the asset. In practice, criteria such as price-earnings or price-dividend ratios may be used for this task. The estimation of the fundamental value of a firm is then compared with its actual price. For instance, if the price is smaller than the fundamental value, a buying opportunity is identified since the trader expects that the market will soon realize that the asset is underpriced compared to its real value. To simplify matters, agents are assumed to be able to compute the fundamental value of an asset correctly. The orders of the fundamentalists are formalized as

$$D_t^F = \alpha^F(F_t - S_t)\,, \tag{4}$$

where α^F is a positive reaction coefficient. Note that the impact of fundamentalists tends to bring the price back to its fundamental value.

Chartists derive trading signals out of past price movements. One of the most popular trading strategies is the moving average rule which extrapolates past price trends into the future [9]. Accordingly, if prices go up (down), chartists submit buying (selling) orders. The orders of chartists are written as

$$D_t^C = \alpha^C(S_t - S_{t-\lambda}) + \begin{cases} +o & S_t - S_{t-1} > -\tau \\ -\pi & S_t - S_{t-1} < -\tau \end{cases}\,, \tag{5}$$

where α^C is a positive reaction coefficient of the trend extrapolation rule. The time horizon of the technical trading rule is λ. The novel idea of (5) is represented by the regime switching expression. Chartists are optimistic most of the time, but they panic if prices decrease by more than the threshold value τ. In the optimistic regime, chartists buy the additional amount o of the asset, whereas in the pessimistic regime, they sell the additional amount π of the asset. We assume that $\pi > o$.[a]

The law of motion of the asset price, obtained by combining (2)–(5), is a nonlinear stochastic difference equation. Since the solution precludes closed analysis,

[a]Traders regularly rely on stop-loss orders in order to limit their risk. Note that such program/computer trading, which is often assigned a prominent role for amplifying a crash, is consistent with (5).

we proceed with numerical analysis. We use the following parameter setting:

$$\eta = 0.0002\,, \quad \alpha^M = 1\,, \quad \omega = 0.5\,, \quad \sigma^\delta = 0.01\,, \quad \alpha^F = 0.001\,, \quad \alpha^C = 0.005\,,$$

$$\lambda = 3\,, \quad o = 0.002\,, \quad \pi = 0.06\,, \quad \tau = 0.03\,.$$

Let us briefly interpret the main coefficients. The dynamics are calibrated to daily data. Hence, the fundamental value grows by approximately 5% per year. The groups of chartists and fundamentalists are equal in size. The technical trading rule operates with a lag of three time steps. Finally, the chartists panic if the price decreases by more than 3%.

4. Simulation Analysis

Figure 2 illustrates the dynamics of the model in the same way as Fig. 1 portrays the dynamics of the Dow Jones Index. The first panel shows again the evolution of the asset price in the time domain (25,000 observations). In addition, the straight line represents the development of the fundamental value. Due to the optimistic mood of the chartists, the asset is overvalued most of the time. The two panels in the second line of Fig. 2 exemplify the phenomenon of bubbles and crashes within our model (500 observations). In both cases, we observe a price increase over some time, followed by a sharp price decrease. Remember that this is the typical behavior of bubbles and crashes, as sketched by Johansen and Sornette [5].

The trajectory of the asset price appears as a random walk. Indeed, the autocorrelation function of the returns, plotted in the third panel of Fig. 2, suggests the absence of predictability. But predictability does exist. The bottom left-hand panel demonstrates that large drawdowns occur too frequently to be in harmony with the stretched exponential distribution. Put differently, large price decreases are not random but display transient correlation. Overall, the features of Figs. 1 and 2 appear similar.

Does our model significantly produce large drawdowns? The panels of Fig. 3 show the log cumulative distribution of 20 simulation runs. Each simulation run contains 25,000 observations. All panels are based on the same seed of random variables. The first panel is computed with the parameter setting of Sec. 3. Visual inspection reveals that all simulation runs possess the property of temporal correlation. However, large drawups are more in agreement with the stretched exponential distribution.

Within the second panel, we set α^C and o equal to zero. Still, we observe a large number of strong crashes, yet not as many as in the previous panel. The main reason for this is that selling orders generated by the technical trading rule accelerate the price decrease so that the threshold value is more likely to be crossed. If this mechanism is missing, temporal correlation is weaker. Furthermore, the optimistic behavior of the chartists tends to shift the asset price above its fundamental value. The stronger the asset is overvalued, the more selling orders are submitted by the fundamentalists.

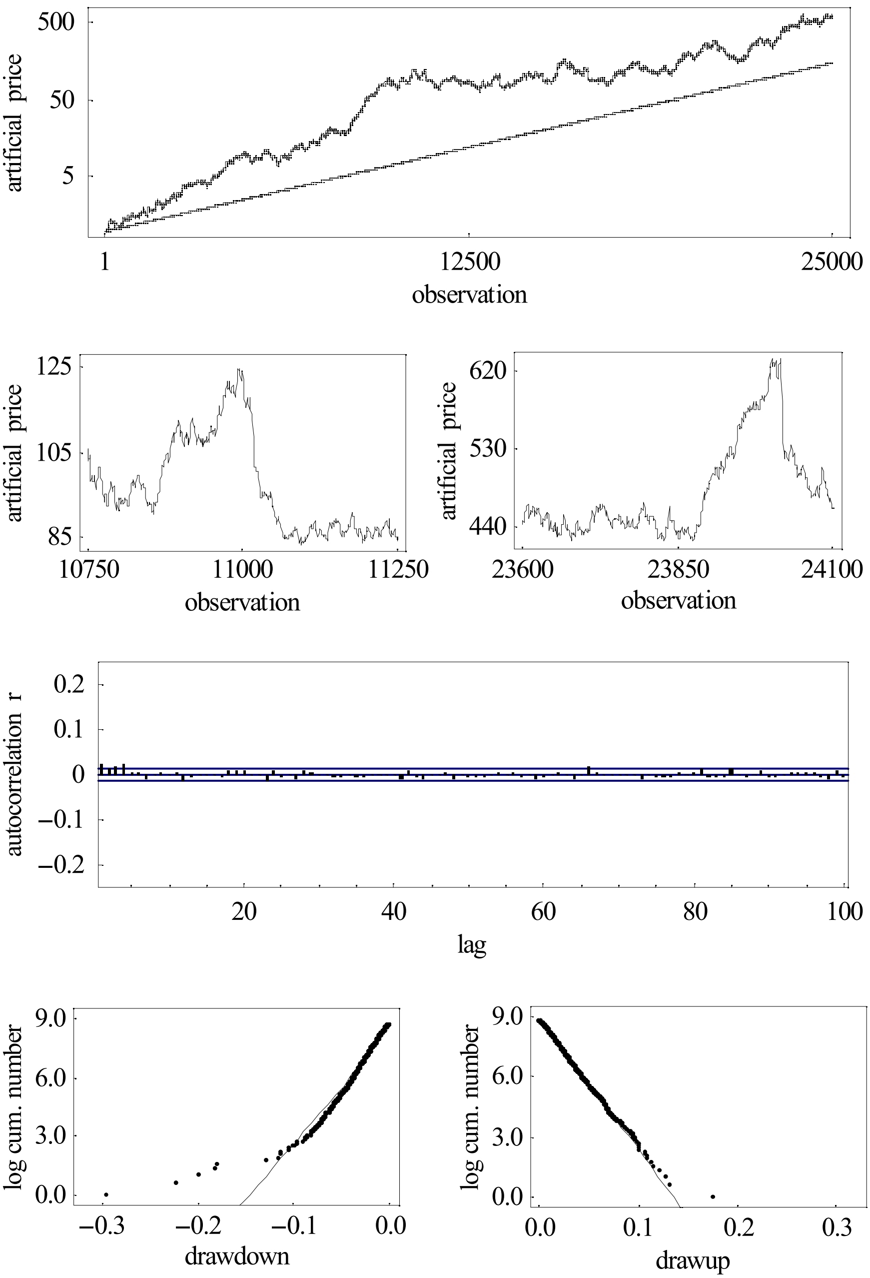

Fig. 2. The dynamics of the artificial price. The first panel shows the artificial price, where the straight line indicates the fundamental value (25,000 observations). The left-hand (right-hand) side of the second panel contains the same data for the period 10,750–11,250 (23,600–24,100). The third panel displays the autocorrelation function of the returns for the first 100 lags (with 95% confidence bands). The bottom two panels present the number of times a given level of price run has been observed (left: drawdowns, right: drawups). The solid line depicts the fit of a stretched exponential distribution. Parameter setting as in Sec. 3.

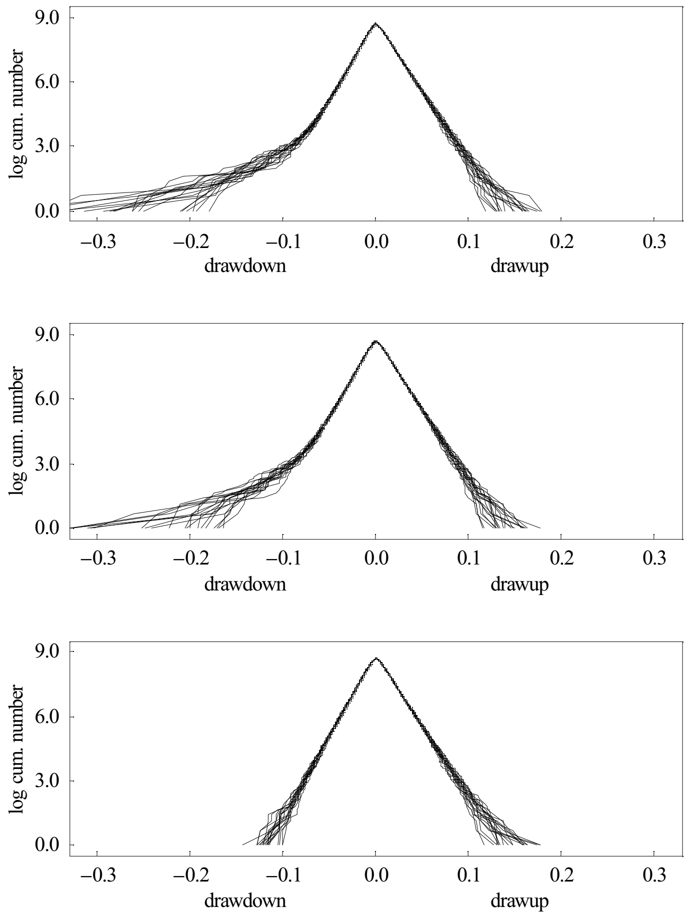

Fig. 3. Drawdowns and drawups. The panels show the number of times a given level of price run has been observed. The results are plotted for 20 simulation runs, each containing 25,000 observations. First panel: parameter setting as in Sec. 3. Second panel: parameter setting as in Sec. 3, but $\alpha^C = o = 0$. Third panel: parameter setting as in Sec. 3, but $\pi = 0$. All panels are based on the same seeds of random variables.

The time series of the third panel are again calculated with the parameter setting of Sec. 3, but now $\pi = 0$. Without panic attacks, the model does not generate drawdowns. In fact, trend extrapolation and optimism alone do not suffice to produce temporal correlation. Our model therefore suggests that panic attacks are an important mechanism for transient bursts of dependence in successive returns.

5. Conclusions

We propose a simple behavioral model of interacting heterogeneous traders to account for the fact that large drawdowns are outliers. As detected by Johansen and

Sornette [6, 7], the bulk of the drawdowns and drawups are very well fitted by the exponential model, which is a natural assumption for independently distributed price changes. But the largest price runs occur much too frequently, indicating transient correlations.

The main finding of our paper is as follows: The selling pressure due to a panic attack, triggered by a sharp price drop, may decrease the price so severely that the traders begin to panic again. Trend-extrapolating trading rules amplify this process. The dynamics of a crash are therefore at least partially endogenous.

Persistencies in the price process not only affect portfolio management — they may are also consequential to the overall stability of the financial system. A good understanding of the working of financial markets may help the construction of more stable and efficient markets. We hope that the mechanism explored in this paper adds to the body of knowledge.

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