Taking Stock:
A Rigorous Modelling of Animal Spirits in Macroeconomics

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Abstract

This paper is a survey of the burgeoning literature that seeks to take the enigmatic concept of the animal spirits more seriously by building heterodox macrodynamic models that can capture some of its crucial aspects in a rigorous way. Two approaches are considered: the discrete choice and the transition probability approach, where individual agents face a binary decision and choose one of them with a certain probability. These assessments are adjusted upward or downward in response to what the agents observe, which leads to changes in the aggregate sentiment and the macroeconomic variables resulting from the corresponding decisions. Typical applications of the two approaches alternatively give rise to what will be called a weak and a strong form of animal spirits. On the whole, the literature included in this survey provides examples of applications of a modelling tool that demonstrates a considerable flexibility within a canonical framework.

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1 Introduction

A key issue in which heterodox macroeconomic theory differs from the orthodoxy is the notion of expectations, where it determinedly abjures the Rational Expectations Hypothesis. Instead, to emphasize its view of a constantly changing world with its fundamental uncertainty, heterodox economists frequently refer to the famous idea of the 'animal spirits'. This is a useful keyword that poses no particular problems in general conceptual discussions. However, given the enigma surrounding the expression, what can it mean when it comes to rigorous formal modelling? More often than not, authors garland their model with this word, even if there may be only loose connections to it. The present survey focusses on heterodox approaches that take the notion of the 'animal spirits' more seriously and, seeking to learn more about its economic significance, attempt to design dynamic models that are able to definitively capture some of its crucial aspects.[1]

The background of the term as it is commonly referred to is Chapter 12 of Keynes' *General Theory*, where he discusses another elementary "characteristic of human nature," namely, "that a large proportion of our positive activities depend on spontaneous optimism rather than on a mathematical expectation" (Keynes, 1936, p. 161). Although the chapter is titled "The state of long-term expectation", Keynes makes it clear that he is concerned with "the state of psychological expectation" (p. 147).[2]

It is important to note that this state does not arise out of the blue from whims and moods; it is not an imperfection or plain ignorance of human decision-makers. Ultimately, it is due to the problem that decisions resulting in consequences that reach far into the future are not only complex, but also fraught with irreducible uncertainty. "About these matters", Keynes wrote elsewhere to clarify the basic issues of the *General Theory*, "there is no scientific basis on which to form any calculable probability whatever" (Keynes, 1937, p. 114). Needless to say, this facet of Keynes' work is completely ignored by the "New-Keynesian" mainstream.

To cope with uncertainty that cannot be reduced to a mathematical risk calculus, enabling us nevertheless "to behave in a manner which saves our faces as

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[1] The term is so appealing that a number of orthodox economists also invoke it to advertise their models. In a branch of the Dynamic Stochastic General Equilibrium literature, the term is used interchangeably with sunspot equilibria and self-fulfilling prophecies. It goes without saying that the discussion in the present paper has nothing to do with these (very elaborate) refinements of rational expectations, where observations of an exogenous stochastic process induce the agents to coordinate on recurrent switches between multiple equilibria; see, for example, Farmer and Guo (1994), and Galí (1994).

[2] The actual term 'animal spirits' is mentioned in the same chapter on p. 161.

rational economic men" (*ibid.*), Keynes refers to "a variety of techniques", or "principles", which are worth quoting in full.

> "(1) We assume that the present is a much more serviceable guide to the future than a candid examination of past experience would show it to have been hitherto. In other words we largely ignore the prospect of future changes about the actual character of which we know nothing.
>
> "(2) We assume that the *existing* state of opinion as expressed in prices and the character of existing output is based on a *correct* summing up of future prospects, so that we can accept it as such unless and until something new and relevant comes into the picture.
>
> "(3) Knowing that our own individual judgment is worthless, we endeavor to fall back on the judgment of the rest of the world which is perhaps better informed. That is, we endeavor to conform with the behavior of the majority or the average. The psychology of a society of individuals each of whom is endeavoring to copy the others leads to what we may strictly term a *conventional* judgment." (Keynes, 1937, p. 114; his emphasis).[3]

The third point is reminiscent of what is currently referred to in science and the media as herding. As it runs throughout Chapter 12 of the *General Theory*, decision-makers are not very concerned with what an investment might really be worth; rather, under the influence of mass psychology, they devote their intelligences "to anticipating what average opinion expects the average opinion to be", a judgement of "the third degree" (Keynes, 1936, p. 156). Note that it is rational in such an environment "to fall back on what is, in truth, a *convention*" (*ibid.*, p. 152; Keynes' emphasis). Going with the market rather than trying to follow one's own better instincts is rational for "persons who have no special knowledge of the circumstances" (p. 153) as well as for expert professionals.

If the general phenomenon of forecasting the psychology of the market is taken for granted, then it is easily conceivable how waves of optimistic or pessimistic sentiment are generated by means of a self-exciting, possibly accelerating mechanism. Hence, any modelling of animal spirits will have to attempt to incorporate a positive feedback effect of this kind.

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[3] A general review of Keynes' concepts can be found in Minsky (1975, Chapter 3). A more roughly sketched discussion focussing on their fruitfulness for macroeconomic modelling is given by Flaschel et al. (1997, Chapter 12.2). A good survey of the role of (psychological) expectations and confidence is provided by Boyd and Blatt (1988). In the wake of the financial crisis, Akerlof and Shiller's (2009) book on *Animal Spirits* brought the keyword to the attention of a wider audience.

The second point in the citation refers to more ‘objective’ factors such as prices or output (or, it may be added, composite variables derived from them). According to the first point, it is the current values that are most relevant for the decision-maker. According to the second point, this is justified by his or her assumption that these values are the result of a correct anticipation of the future by the other, presumably smarter and, in their entirety, better informed market participants.

If one likes, it could be said that the average opinion also plays a role here, only in a more indirect way. In any case, insofar as agents believe in the objective factors mentioned above as fundamental information, they will have a bearing on the decision-making process. Regarding modelling, current output, prices and the like could therefore be treated in the traditional way as input in a behavioural function. In the present context, however, these ordinary mechanisms will have to be reconciled with the direct effects of the average opinion. It is then a straightforward idea that the ‘fundamentals’ may reinforce or keep a curb on the ‘conventional’ dynamics.

In the light of this discussion, formal modelling does not seem to be too big a problem: set up a positive feedback loop for a variable representing the ‘average opinion’ and combine it with ordinary behavioural functions. In principle, this can be, and has been, specified in various ways. The downside of this creativity is that it makes it hard to compare the merits and demerits of different models, even if one is under the impression that they invoke similar ideas and effects. Before progressing too far to concrete modelling, it is therefore useful to develop building blocks, or to have reference to existing blocks, which can serve as a canonical schema.

Indeed, modelling what may be interpreted as animal spirits is no longer virgin territory. Promising work has been performed over the last ten years that can be subdivided into three categories (further details later). Before discussing them one by one, we set up a unifying frame of reference which makes it easier to site a model. As a result, it will also be evident that the models in the literature have more in common than it may seem at first sight. In particular, it is not by chance that they have similar dynamic properties.

The work we focus on is all the more appealing since it provides a microfoundation of macroeconomic behaviour, albeit, of course, a rather stylized one. At the outset, the literature refers to a large population of agents who, for simplicity, face a binary decision. For example, they may choose between optimism and pessimism, or between extrapolative and static expectations about prices or demand. Individual agents do this with certain probabilities and then take a decision. The central point is that probabilities endogenously change in the course of time. They adjust upward or downward in reaction to agents’ observations, which may include output, prices as well as the aforementioned ‘average opinion’. As a consequence, agents switch between two attitudes or two strategies. Their decisions vary correspondingly, as does the macroeconomic outcome resulting from them.

By the law of large numbers, this can all be cast in terms of aggregate variables, where one such variable represents the current population mix. The relationships between them form an ordinary and well-defined macrodynamic system specified in discrete or continuous time, as the case may be. The animal spirits and their variations, or that of the average opinion, play a crucial role as the dynamic properties are basically determined by the switching mechanism.

Owing to the increasing and indiscriminate use of the emotive term 'animal spirits', causing it to become an empty phrase, in the course of our presentation we will distinguish between a weak and a strong form of animal spirits in macrodynamics. We will refer to a weak form if a model is able to generate waves of, say, an optimistic and pessimistic attitude, or waves of applying a forecast rule 1 as opposed to a forecast rule 2. A prominent argument for this behaviour is that the first rule has proven to be more successful in the recent past. A strong form of animal spirits is said to exist if agents also rush toward an attitude, strategy, or so on, simply because it is being applied at the time by the majority of agents. In other words, this will be the case if there is a component of herding in the dynamics because individual agents believe that the majority will probably be better informed and smarter than they themselves. To give a first overview, the weak form of animal spirits will typically be found in macro models employing the discrete choice approach, whereas models in which we identify the strong form typically choose the transition probability approach. However, this division has mainly historical rather than logical reasons.

The remainder of this survey is organized as follows. The next section introduces the two approaches of discrete choice and the transition probabilities. It also points out that they are more closely related than it may appear at first sight and then sets up an abstract two-dimensional model that allows us to study the dynamic effects that they possibly produce. In this way, it can be demonstrated that it is the two approaches themselves and their inherent nonlinearities that, with little additional effort, are conducive to the persistent cyclical behaviour emphasized by most of the literature.

Section 3 surveys an early literature that begins roughly ten years ago. Section 4 is concerned with a class of models that are concerned with heterogeneous rule-of-thumb expectations within the New-Keynesian three-equation model (but without its rational expectations). This work evaluates the fitness of the two expectation rules by means of the discrete choice probabilities. It is also noteworthy because orthodox economists have shown an interest in it and given it attention. Section 5 discusses models with an explicit role for herding, which, as stated, is a field for the transition probability approach (and where we will also reason about the distinction between animal spirits in a weak and strong form).

While the modelling outlined so far is conceptually attractive for capturing a sentiment dynamics, it would also be desirable to have some empirical support for it. Section 6 is devoted to this issue. Besides some references to laboratory experiments, it covers work that investigates whether the dynamics of certain business survey indices can be explained by a suitable application of (mainly) the transition probability approach. On the other hand, it presents work that takes a model from Section 4 or 5 and seeks to estimate it in its entirety. Here, the sentiment variable is treated as unobservable and only its implications for the dynamics of the other, observable macro variables are taken into account. Section 7 concludes.

2 The general framework

The models we shall survey are concerned with a large population of agents who have to choose between two alternatives. In principle, their options can be almost anything: strategies, rules of thumb to form expectations, diffuse beliefs. In fact, this is a first feature in which the models may differ. For concreteness, let us refer in the following general introduction to two attitudes that agents may entertain and call them optimism and pessimism, identified by a plus and minus sign, respectively. Individual agents choose them, or alternatively switch from one to the other, on the basis of probabilities. They are the same for all agents in the population in the first case, and for all agents in each of the two groups in the second case.

It has been indicated that probabilities vary endogenously over time. This idea is captured by treating them as functions of something else in the model. This ‘something else’ can be one macroscopic variable or several such variables. In the latter case, the variables are combined in one auxiliary variable, most conveniently by way of weighted additive or subtractive operations. Again, the variables can be almost anything in principle; their choice is thus a second feature for categorizing the models.

Mathematically, we introduce an auxiliary variable, or index, which is in turn a function of one or several macroeconomic variables. Regarding the probabilities, we deal with two approaches: the *discrete choice approach* (DCA) and the *transition probability approach* (TPA). In the applications we consider, they typically differ in the interpretation of the auxiliary variable and the type of variables entering this function. However, both approaches could easily work with setting up the same auxiliary variable for their probabilities.

2.1 The discrete choice approach

As a rule, the discrete choice approach is formulated in discrete time. At the beginning of period t, each individual agent is optimistic with probability π_t^+ and pessimistic with probability $\pi_t^- = 1 - \pi_t^+$. The probabilities are not constant, but change with two variables $U^+ = U_{t-1}^+$, $U^- = U_{t-1}^-$ which, in the applications, are often interpreted as the success or fitness of the two attitudes.[4] As the dating indicates, the latter are determined by the values of a set of variables from the previous or possibly also earlier periods. Due to the law of large numbers, the shares of optimists and pessimists in period t, n_t^+ and n_t^-, are identical to the probabilities, that is,

$$n_t^+ \;=\; \pi_t^+ \;=\; \pi^+(U_{t-1}^+)\,, \qquad n_t^- \;=\; \pi_t^- \;=\; \pi^-(U_{t-1}^-) \;=\; 1-\pi^+(U_{t-1}^+) \tag{1}$$

A priori there is a large variety of possibilities to conceive of functions $\pi^+(\cdot)$, $\pi^-(\cdot)$. In macroeconomics, there is currently one dominating specification that relates π^+, π^- to U^+, U^-. It derives from the multinomial logit (or 'Gibbs') probabilities. Going back to these roots, standard references for an extensive discussion are Manski and McFadden (1981), and Anderson et al. (1993). For the ordinary macroeconomist, it suffices to know the gist as it has become more broadly known with two influential papers by Brock and Hommes (1997, 1998). They applied the specification to the speculative price dynamics of a risky asset on a financial market, while it took around ten more years for it to migrate to the field of macroeconomics. With respect to a positive coefficient $\beta > 0$, the formula reads:

$$\begin{aligned} \pi^+(U_{t-1}^+) &= \frac{\exp(\beta U_{t-1}^+)}{\exp(\beta U_{t-1}^+) + \exp(\beta U_{t-1}^-)} = \frac{1}{1+\exp[\beta(U_{t-1}^- - U_{t-1}^+)]} \\ \pi^-(U_{t-1}^-) &= \frac{\exp(\beta U_{t-1}^-)}{\exp(\beta U_{t-1}^+) + \exp(\beta U_{t-1}^-)} = \frac{1}{1+\exp[\beta(U_{t-1}^+ - U_{t-1}^-)]} \end{aligned} \tag{2}$$

($\exp(\cdot)$ being the exponential function).[5] Given the scale of the fitness expressions, the parameter β in (2) is commonly known as the *intensity of choice*. Occasionally, reference is made to $1/\beta$ as the propensity to err. For values of β close to

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[4] Although this is not done in the typical applications, U^+, U^- could also take account of direct social interactions (similar to the transition probability approach in the next subsection). Brock and Durlauf (2001) is an often cited paper that discusses such effects at a level logically prior to the probabilities π_t^+, π_t^-.

[5] An extension of (2) to more than two (but still a finite number of) options is obvious. A generalization to a continuous space of options, or 'beliefs', is also possible; see Diks and van der Weide (2005) for such a continuous choice model. For example, agents may have a prediction rule that is parameterized by a scalar or vector θ, which they are free to choose.

zero, the two probabilities π^+, π^- would nearly be equal, whereas for $\beta \to \infty$ they tend to zero or one, so that almost all of the agents would either be optimistic or pessimistic.[6] The second equals sign follows from dividing the numerator and denominator by the numerator. It makes clear that what matters is the difference in the fitness.

Equations (1) and (2) are the basis of the animal spirits models employing the discrete choice approach. The next stage is, of course, to determine the fitnesses U^+, U^-, another salient feature for characterizing different models. Before going into detail about this further below, we should put the approach as such into perspective by highlighting two problems that are rarely mentioned. First, there is the issue of discrete time. It may be argued that (1), (2) could also be part of a continuous-time model if the lag in (1) is eliminated, that is, if one stipulates $n_t^+ = \pi^+(U_t^+)$. This is true under the condition that the fitnesses do not depend on n_t themselves. Otherwise (and quite likely), because of the nonlinearity in (2), the population share would be given by a nontrivial implicit equation with n_t on the left-hand and right-hand side, which could only be solved numerically.

The second problem is of a conceptual nature. It becomes most obvious in a situation where the population shares of the optimists and pessimists are roughly equal and remain constant over time. Here, the individual agents would nevertheless switch in each and every period with a probability of one-half.[7] This requires the model builder to specify the length of the period . If the period is not too long then, for psychological and many other reasons, the agents in the model would change their mind (much) more often than most people in the real world (and also in academia). This would somewhat undermine the microfoundation of this modelling, even though the invariance of the macroscopic outcome n_t^+, n_t^- may make perfect sense.

Apart from being meaningful in itself, both problems can be satisfactorily solved by taking up an idea by Hommes et al. (2005). They suppose that in each period not all agents but only a fraction of them think about a possible switch, a modification which they call discrete choice with asynchronous updating. Thus, let μ be the fixed probability *per unit of time* that an individual agent reconsiders his attitude, which then may or may not lead to a change. Correspondingly, $\Delta t\, \mu$ is his probability of operating a random mechanism for π_t^+ and π_t^- between t and Δt, while over this interval he will unconditionally stick to the attitude he already had

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[6] A remarkable alternative is the proposal by Chiarella and Di Guilmi (2015), who invoke the concept of maximum entropy inference in order to model the intensity of choice as an endogenous variable. It depends on the values of U_{t-1}^+, U_{t-1}^- and can also become negative, which requires these fitnesses to be positive.

[7] Hence, for example, the probability that an agent will maintain his attitude over only four consecutive periods is as low as $(1/2)^4 = 6.67$ per cent.

at time t with a probability of $(1-\Delta t\,\mu)$. From this, the population shares at the macroscopic level at $t+\Delta t$ result like

$$\begin{array}{rcl} n^{+}_{t+\Delta t} &=& (1-\Delta t\,\mu)\,n^{+}_{t} \,+\, \Delta t\,\mu\,\pi^{+}(U^{+}_{t}) \;=\; n^{+}_{t} \,+\, \Delta t\,\mu\,[\,\pi^{+}(U^{+}_{t}) - n^{+}_{t}\,] \\ n^{-}_{t+\Delta t} &=& (1-\Delta t\,\mu)\,n^{-}_{t} \,+\, \Delta t\,\mu\,\pi^{-}(U^{-}_{t}) \;=\; n^{-}_{t} \,+\, \Delta t\,\mu\,[\,\pi^{-}(U^{-}_{t}) - n^{-}_{t}\,] \end{array} \tag{3}$$

It goes without saying that these expressions reduce to (1) if the probability $\Delta t\,\mu$ is equal to one. Treating μ as a fixed parameter and going to the limit in (3), $\Delta t \to 0$, gives rise to a differential equation for the changes in n^{+}. It actually occurs in other fields of science, especially and closest to economics, in evolutionary game theory, where this form is usually called *logit dynamics*.[8] At least in situations where one or both reasons indicated above are relevant to the discrete choice approach, the continuous-time version of (3) with $\Delta t \to 0$ may be preferred over the formulation (1), (2) in discrete time.

With a view to the transition probability approach in the next subsection, it is useful to consider the special case of symmetrical fitness values, in the sense that the gains of one attitude are the losses of the other, $U^{-} = -U^{+}$. To this end, we introduce the notation $s = U^{+}$ and call s the switching index. Furthermore, instead of the population shares we study the changes in their difference $x := n^{+} - n^{-}$ (which can attain values between ± 1). Subtracting the population shares in (3) and making the adjustment period Δt infinitesimally small, a differential equation in x is obtained: $\dot{x} = \mu\,\{\,[\exp(\beta s) - \exp(-\beta s)]/[\exp(\beta s) + \exp(-\beta s)] \,-\, x\}$. The fraction of the two square brackets is identical to a well-established function of its own, the hyperbolic tangent (tanh), so that we can compactly write,

$$\dot{x} \;=\; \mu\,[\tanh(\beta s) - x] \tag{4}$$

The function $x \mapsto \tanh(x)$ is defined on the entire real line; it is strictly increasing everywhere with $\tanh(0) = 0$ and derivative $\tanh'(0) = 1$ at this point; and it asymptotically tends to ± 1 as $x \to \pm\infty$. This also immediately shows that x cannot leave the open interval $(-1, +1)$.

2.2 The transition probability approach

The transition probability approach goes back to a quite mathematical book on quantitative sociology by Weidlich and Haag (1983). It was introduced into eco-

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[8] In this framework, a differential equation such as (in the present notation) $\dot{n}^{+} = \mu\,[\pi^{+} - n^{+}]$, which we obtain from (3) with $\Delta t \to 0$, can also be derived by making reference to a special case of the concept of a so-called revision protocol; see Lahkar and Sandholm (2008, p. 577) or, with a broader background, Ochea (2010, Chapter 2.2), who set $\mu = 1$.

nomics by Lux (1995) in a seminal paper on a speculative asset price dynamics.[9] It took a while before, with Franke (2008a, 2012a), macroeconomic theory became aware of it.[10] The main reason for this delay was that Weidlich and Haag as well as Lux started out with concepts from statistical mechanics (see also footnote 15 below), an apparatus that ordinary economists are quite unfamiliar with. The following presentation makes use of the work of Franke, which can do without this probabilistic theory and sets up a regular macrodynamic adjustment equation.[11]

In contrast to the discrete choice approach, it is now relevant whether an agent is optimistic or pessimistic at present. The probability that an optimist will remain optimistic and that of a pessimist becoming an optimist will generally be different. Accordingly, the basic concept are the probabilities of switching from one attitude to the other, that is, transition probabilities. Thus, at time t, let p_t^{-+} be the probability *per unit of time* that a pessimistic agent will switch to optimism (which is the same for all pessimists), and let p_t^{+-} be the probability of an opposite change. More exactly, in a discrete-time framework, $\Delta t\, p_t^{-+}$ and $\Delta t\, p_t^{+-}$ are the probabilities that these switches will occur within the time interval $[t, t+\Delta t)$.[12]

In the present setting, we refer directly to the difference $x = n^+ - n^-$ of the two population shares. It is this variable that we shall call the aggregate *sentiment* of the population (average opinion, state of confidence, or just animal spirits are some alternative expressions). In terms of this sentiment, the shares of optimists and pessimists are given by $n^+ = (1+x)/2$ and $n^- = (1-x)/2$.[13] With a large population, changes in the two groups are given by their size multiplied by the transition probabilities. Accordingly, the share of optimists decreases by $\Delta t\, p_t^{+-}\,(1+x_t)/2$ due to the agents leaving this group, and it increases by $\Delta t\, p_t^{-+}\,(1-x_t)/2$ due to the pessimists who have just joined it. With signs reversed, the same holds true for the population share of pessimistic agents. The net effect on x is described by a deterministic adjustment equation.[14] We express this for a specific length Δt of the

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[9] Kirman (1993) is a slightly earlier and equally famous paper with a nice story about ants and two food sources between which they have to choose. It shares the same spirit as Lux (1995), but is specified differently and, as it emerged over time, somewhat less conveniently.

[10] To be fair, as shortly discussed at the beginning of Section 3, there are some earlier (but now practically forgotten) examples.

[11] The price for this simpler treatment is a loss of some information, but this would only become relevant if one wanted to take a higher, probabilistic point of view.

[12] These probabilities are required to be less than one, but not necessarily p^{-+}, p^{+-} themselves.

[13] Since $n^+ = n^+/2 + n^+/2 = (1-n^-)/2 + n^+/2 = (1+n^+ - n^-)/2 = (1+x)/2$. The second relationship follows analogously.

[14] Franke (2008a,b) gives a rigorous mathematical argument that includes a finite population size and the intrinsic noise which will thus be present. It is a more direct procedure than the treatment in statistical mechanics, which first sets up the Fokker-Planck equation and then derives the stochastic so-called Langevin equation from it, which in turn reduces to eq. (5) below as the popula-

adjustment period as well as for the limiting case when Δt shrinks to zero, which yields an ordinary difference and differential equation, respectively:[15]

$$\begin{aligned} x_{t+\Delta t} &= x_t + \Delta t\,[\,(1-x_t)\,p_t^{-+} - (1+x_t)\,p_t^{+-}\,] \\ \dot{x} &= (1-x)\,p^{-+} - (1+x)\,p^{+-} \end{aligned} \tag{5}$$

Similar to the discrete choice approach, the transition probabilities are functions of an index variable. Here, however, as indicated in the derivation of eq. (4), the same index enters p^{-+} and p^{+-}. That is, calling it a switching index and denoting it by the letter s, p^{-+} is supposed to be an increasing function and p^{+-} a decreasing function of s. We adopt this new notation because the type of arguments upon which this index depends typically differs to those of the functions U^+ and U^- in (1). In particular, s may positively depend on the sentiment variable x itself, thus introducing a mechanism that can represent a contagion effect, or 'herding'.

Regarding the specification in which the switching index influences the transition probabilities, Weidlich and Haag (1983) introduced the natural assumption that the *relative* changes of p^{-+} and p^{+-} in response to the changes in s are linear and symmetrical. As a consequence, the function of the transition probabilities is proportional to the exponential function $\exp(s)$. Analogously to the intensity of choice in (2), the switching index may furthermore be multiplied by a coefficient $\beta > 0$. In this way, we arrive at the following functional form,[16]

$$p_t^{-+} = p^{-+}(s_t) = \nu \exp(\beta s_t)\,, \qquad p_t^{+-} = p^{+-}(s_t) = \nu \exp(-\beta s_t) \tag{6}$$

Technically speaking, ν is a positive integration constant. In a modelling context it can, however, be similarly interpreted to β as a parameter that measures how strongly agents react to variations in the switching index. Weidlich and Haag (1983, p. 41) therefore call ν a *flexibility parameter*. Since the only difference between β and ν is that one has a linear and the other has a nonlinear effect on the probabilities, one of them may seem dispensable. In fact, we know of no example that works with

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tion becomes infinitely large. The intellectual copyright, however, is with Alfarano and Lux (2007, Appendices A1 and A2).

[15] At first sight, eq. (5) seems to be identical to eq. (2) in Lux (1995, p. 884). A subtle difference, however, is that here in (5), the variable x represents the *actual* value of the sentiment index of an infinitely large population, whereas in Lux's presentation, x is its *expected* value with respect to the stochastic system with a finite population. As indicated by Lux (p. 895) himself, his eq. (5) constitutes a quasi-deterministic dynamics. Its interpretation is, however, somewhat problematic.

[16] It corresponds to eq. (3) in Lux (1995, p. 885), the right-hand side of which reads $\nu \exp(\pm\alpha x)$ and can be regarded as a special case of the present equation (2) with $\beta = \alpha$, $s = \alpha x$ and α representing the strength of infection or herd behaviour (a coefficient that we will employ as well below and designate ϕ_x).

$\beta \neq 1$ in (6). We maintain this coefficient for pedagogical reasons, because it will emphasize the correspondence with the discrete choice approach below.

Substituting (6) for the probabilities in (5) yields $\dot{x} = \nu\,[(1-x)\exp(\beta s) - (1+x)\exp(-\beta s)] = 2\nu\,\{\,[\exp(\beta s) - \exp(-\beta s)]/2 - x\,[\exp(\beta s) + \exp(-\beta s)]/2\,\}$. Making use of the definition of the hyperbolic sine and cosine (sinh and cosh), the curly brackets are equal to $\{\sinh(\beta s) - x\cosh(\beta s)\}$. Since the hyperbolic tangent is defined as $\tanh = \sinh/\cosh$, eq. (5) becomes

$$\begin{aligned} x_{t+\Delta t} &= x_t + \Delta t\, 2\nu\,[\tanh(\beta s_t) - x_t]\cosh(\beta s_t) \\ \dot{x} &= 2\nu\,[\tanh(\beta s) - x]\cosh(\beta s) \end{aligned} \tag{7}$$

A comparison of equations (4) and (7) reveals a close connection between the transition probability approach and the continuous-time modification of the discrete choice approach.[17] If we consider identical switching indices and $\mu = 2\nu$, then the two equations describe almost the same adjustments of the sentiment variable (because the hyperbolic cosine is a strictly positive function). More specifically, if these equations are integrated into a higher-dimensional dynamic system, (4) and (7) produce the same isoclines $\dot{x} = 0$, so that the phase diagrams with x as one of two variables will be qualitatively identical. When, moreover, these systems have an equilibrium with a balanced sentiment $x = 0$ from $s = 0$, it will be locally stable with respect to (7) if and only if it is locally stable with respect to (4).[18]

2.3 Basic dynamic tendencies

A central feature of the models we consider are persistent fluctuations. This is true irrespective of whether they employ the discrete choice or transition probability approach. With the formulations in (4) and (7), we can argue that there is a deeper reason for this behaviour, namely, the nonlinearity brought about by the hyperbolic tangent in these adjustments. Making this statement also for the discrete choice models, we follow the intuition that basic properties of a system using (4) can also be found in its discrete-time counterpart (2), (3) (albeit possibly with somewhat different parameter values).

To reveal the potential inherent in (4) and (7), we combine the sentiment equation with a simple dynamic law for a second variable y. Presently, a precise

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[17] This relationship with the suitable reformulation of the adjustment equations was established in Franke (2014).

[18] This holds true since $\cosh(\beta s) = \cosh(0) = 1$ in such a case. It does not necessarily apply for other equilibria, because some entries in the Jacobian matrix derived from (7) will be ‘distorted’ by the factor $\cosh(\beta s) > 1$. Nevertheless the phenomenon that an equilibrium is stable under the adjustments (4) and unstable under (7), or *vice versa*, will occur for only a narrow and special range of parameter values.

economic meaning of x and y is of no concern, simply let them be two abstract variables. Forgoing any further nonlinearity, we posit a linear equation for the changes in y with a negative autofeedback and a positive cross-effect. Regarding x let us, for concreteness, work with the logit dynamics (4) and put $\mu = \beta = 1$. Thus, consider the following two-dimensional system in continuous time:

$$\begin{aligned} \dot{x} &= \tanh[\,s(x,y)\,] - x \\ \dot{y} &= \eta_x x - \eta_y y \\ s(x,y) &= \phi_x x - \phi_y y \end{aligned} \tag{8}$$

We fix $\phi_y = 1.80$, $\eta_x = \eta_y = 1.00$ and study the changes in the system's global behaviour under variations of the remaining coefficient ϕ_x. A deeper analysis of the resulting bifurcation phenomena when the dynamics changes from one regime to another is given in Franke (2014). Here it suffices to view four selected values of ϕ_x and the corresponding phase diagrams in the (x,y)-plane.

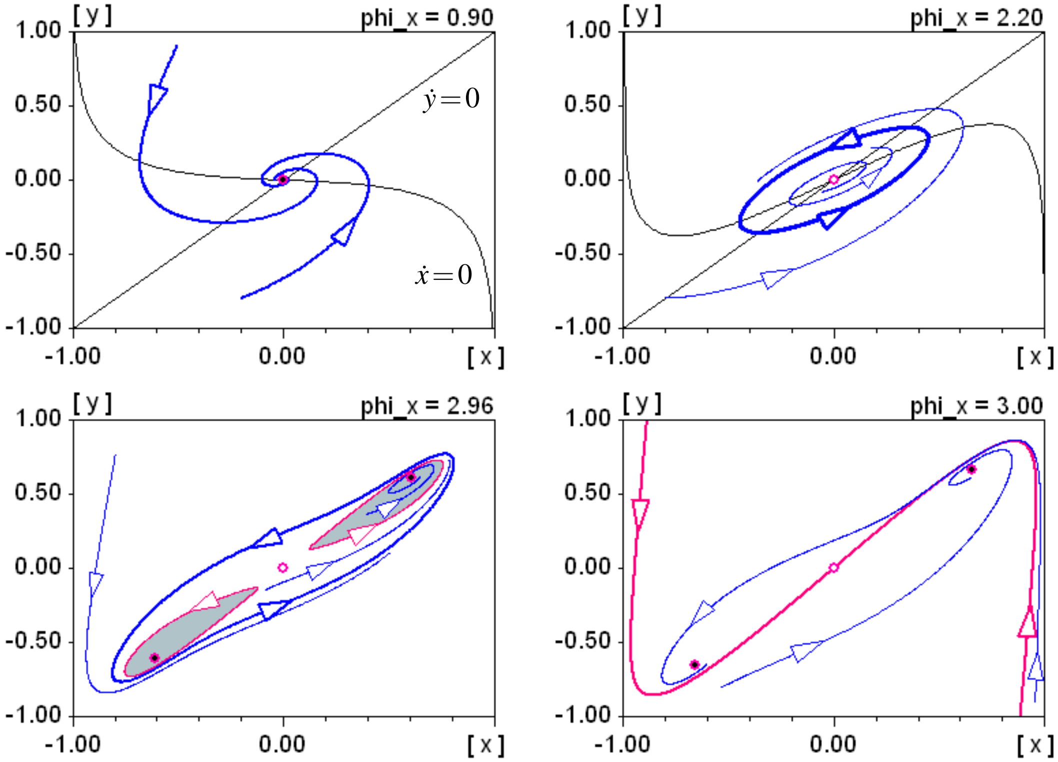

Figure 1: Phase diagrams of (8) for four different regimes.

Since tanh has a positive derivative everywhere, positive values of ϕ_x represent a positive, i.e. destabilizing feedback in the sentiment adjustments. By contrast, $\phi_y > 0$ together with $\eta_x > 0$ establishes a negative feedback loop for the sentiment variable: an increase in x raises y and the resulting decrease in the switching index lowers (the change in) x. The stabilizing effect will be dominant if ϕ_x is sufficiently small relative to ϕ_y. This is the case for $\phi_x = 0.90$, which is shown in the top-left diagram of Figure 1. The two thin solid (black) lines depict the isoclines of the two variables; the straight line is the locus of $\dot{y} = 0$ and the curved line is $\dot{x} = 0$. Their point of intersection at $(x^o, y^o) = (0,0)$ is the equilibrium point of system (8). Convergence towards it takes place in a cyclical manner.

The equilibrium (x^o, y^o) and the $\dot{y} = 0$ isocline are, of course, not affected by the changes in ϕ_x. On the other hand, increasing values of this parameter shift the isocline $\dot{x} = 0$ downward to the left of the equilibrium and upward to the right of it. The counterclockwise motions are maintained, but at our second value $\phi_x = 2.20$, they locally spiral outward, that is, the equilibrium has become unstable. Nevertheless, further away from the equilibrium the centripetal forces prove dominant and generate spirals pointing inward. As a consequence, there must be one orbit in between that neither spirals inward nor outward. Such a closed orbit is indeed unique and constitutes a limit cycle that globally attracts all trajectories, wherever they start from (except the equilibrium point itself). This situation is shown in the top-right panel of Figure 1.

If ϕ_x increases sufficiently, the shifts of the $\dot{x} = 0$ isocline are so pronounced that it cuts the straight line at two (but only two) additional points (x^1, y^1) and (x^2, y^2). One lies in the lower-left corner and the other symmetrically in the upper-right corner of the phase diagram. First, over a small range of ϕ_x, these outer equilibria are unstable, after that, for all ϕ_x above a certain threshold, they are always locally stable. The latter case is illustrated in the bottom-left panel of Figure 1, where the parameter has increased to $\phi_x = 2.96$ (the isoclines are not shown here, so as not to overload the diagram).

The two shaded areas are the basins of attraction of (x^1, y^1) and (x^2, y^2), each surrounded by a repelling limit cycle. Remarkably, the stable limit cycle from $\phi_x = 2.20$ has survived these changes; it has become wider, encompasses the two outer equilibria together with their basins of attraction, and attracts all motions that do not start there.

The extreme equilibria move toward the limits of the domain of the sentiment variable, $x = \pm 1$, as ϕ_x increases. They do this faster than the big limit cycle widens. Eventually, therefore, the outer boundaries of the basins of attraction touch the big cycle, so to speak. This is the moment when this orbit disappears, and with it all cyclical motions. The bottom-right panel of Figure 1 for $\phi_x = 3.00$ demonstrates that then the trajectories either converge to the saddle point (x^o, y^o) in the middle, if they happen to start on its stable arm, or they converge to one of the other two equilibria.

To sum up, whether the obvious, the 'natural' equilibrium (x^o, y^o) is stable or unstable, system (8) shows a broad scope for cyclical trajectories. Furthermore, whether there are additional outer equilibria or not, there is also broad scope for self-sustaining cyclical behaviour, that is, oscillations that do not explode and, even in the absence of exogenous shocks, do not die out, either.

3 An early generation of models

Already soon after the publication of Weidlich and Haag's book (1983) in which their transition probability approach was advanced, attempts were made to utilize this concept for macroeconomic modelling. Examples that we know of are Kraft et al. (1986), Haag et al. (1987), Weise and Kraft (1988), and Weidlich and Braun (1992). However, these contributions received vurtually no attention in the research community. Apart from the dominance of mainstream economics and the papers' reference to the unfamiliar apparatus of statistical mechanics, two further reasons seem to be responsible for this neglect. The economic topics addressed by these authors were somewhat detached, or 'exotic', and ordinary readers soon became overwhelmed by a lot of specification details, so that they could no longer appreciate the essence of the basic approach and its potential.

Let us therefore begin our survey with Kirman's (1993) seminal paper about a biological phenomenon published in an economic journal. The story he tells can nevertheless be immediately understood by any non-specialist. It is about a population of ants that can live on two permanently identical food sources, the question being how the ants are distributed between the two in the long run. While intuitively it may seem that they would be split evenly, in experiments the ants were typically observed to stabilize in a very unbalanced situation: a sizeable majority exploits one source and the rest the other, but eventually, once in a while, reswitching between the two sources occurs.

These repeated finding suggest that one should look for a simple model to explain this majority building. Kirman's paper is a fascinating and convincing proposal in this direction. He slices time into short periods, where in each period two ants meet at random. In such an encounter, the first ant is converted to the second ant's food source with a given probability; with another (small) probability, it changes sources independently. Kirman is able to compute the long-run distribution between the two sources and thus prove that, over a certain range of parameter values, the experimentally observed behaviour does indeed evolve. At the heart of the result is the mechanism of herding (contagion, mimicking, recruitment are synonymous expressions). This means that the more ants feed on the first source, the higher the probability of an ant from the other type being converted; and the lower the probability of a reverse change.

The constituent part of Kirman's approach is the concept of transition probabilities. In fact, his model is quite similar in kind to the transition probability approach presented in Section 2.2 before one considers the limit of an infinite population, $N \to \infty$, when the switching index s is an increasing function of the majority index x (which was referred to there as the agents' sentiment). The only essential difference is that, with Kirman's specification, contrary to what happens in our equation (5), the intrinsic noise resulting from the ants'/agents' individual random choices does not disappear as N becomes large (which may or may not be an attractive feature for model building).

Having understood Kirman's model and its functioning, it is not a very far-fetched idea to incorporate its herding mechanism into a simple economic framework, expecting the salient properties to carry over and give rise to persistent cyclical behaviour. As far as we know, the first example of such a strategy is Westerhoff and Hohnisch (2007).[19] They consider a population of $N = 100$ agents within the setting of the Keynesian textbook multiplier. Fixing investment, they distinguish between optimistic and pessimistic agents who, respectively, have a higher and lower marginal propensity to consume. Hence output increases linearly with the number of optimistic consumers. In the random meetings, a pessimist is converted to optimism with a probability that is higher when output has increased recently than when it has decreased, and *vice versa* for an optimist becoming a pessimist.

With suitable parameter values, this is in fact all that is needed to generate the desired persistent cyclical fluctuations. To see this, consider the phase of an expansion. Then the probability of switching from pessimism to optimism is higher than the opposite change, which reinforces the upswing. This process will, however, slow down as the number of remaining pessimists and potential converts declines. On the other hand, consumers can also change their attitude independently. Even though this may occur with a small probability only, this effect will eventually dominate the herding toward optimism (at the latest, when the entire population has turned optimistic). *Via* the multiplier, the resulting decrease in the number of optimists reduces output, which in turn increases the overall probability of switching from optimism to pessimism. In this way, a turnaround is obtained and

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[19] Before, Kirman's model was successfully utilized to model speculation processes on financial markets (Kirman, 1991). Having outlined the close relationship of Kirman's model to the transition probability approach in Section 2.2, it would have also been possible in principle to try his specification for macroeconomic modelling.

the economy begins to enter a contraction.[20]

Westerhoff and Hohnisch (2010) introduce fiscal policy rules into this model. They point out that a fiscal stimulus does not only have a direct effect on economic activity *via* the Keynesian multiplier, but that the increase in national income also affects the agents' sentiment and thus reinforces the initial effect. Hohnisch and Westerhoff (2008) triplicate Westerhoff's first model, so to speak, by postulating the same economy for three different countries. The national cycles are then seen to be synchronized if agents' transition probabilities depend on economic performance at home and in the foreign countries.

In another series of papers, Westerhoff (2006a, 2006b, 2008) and Lines and Westerhoff (2006) introduce heterogeneous expectations into Samuelson's (1939) multiplier-accelerator model.[21] Instead of the usual dependence of investment on the change in output most recently observed, in this case investment increases with the *expected* change in that variable. A fraction of the agents adopt extrapolative expectations to predict output; the others rely on regressive expectations (that is, they expect output to gradually return to its equilibrium level). The other idea behind this approach is that the agents are aware of the fact that it is impossible to maintain an upward or downward motion forever. For this reason, the more current output deviates from its equilibrium, the less convincing the extrapolative expectations appear to them. While nowadays one would perhaps apply the discrete choice approach to determine the population shares on the basis of this argument, these papers use another, straightforward functional specification.[22]

The fluctuations which are indeed obtained in this way are easy to explain once it has been noted that the extrapolative expectations, when taken on their own, are destabilizing, whereas the regressive expectations are stabilizing. The former dominate in a vicinity of the equilibrium, which drives the economy away from it. As the 'misalignment' increases, the agents become more prudent and the regressive expectations gain in weight. This puts a curb on the divergent tendencies and sooner or later reverses the path of the economy, causing it to return to more moderate output levels.

This mechanism is common to all the aforementioned papers; they merely differ with regard to a number of minor specification details. Furthermore, Wegener et al. (2009) apply the idea of interacting extrapolative and regressive expectations to Metzler's (1941) model of an inventory cycle; unsurprisingly, as we know by

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[20] Westerhoff (2010) considers a similar economy but, differing to our focus in this survey, places his agents on a square lattice, which allows him to study what emerges from their local interactions.

[21] In contrast to the models inspired by Kirman, these and all other models considered in the rest of this section are purely deterministic.

[22] From our present point of view, this function might be called *ad hoc*, but it serves its purpose equally well.

now, it works out quite the same. In spite of being elementary, these results demonstrate that we have here a fairly straightforward device that may prove useful for generating persistent fluctuations, also in less pedagogical, more ambitious model settings.

Westerhoff (2006c) also studies extrapolative *versus* regressive expectations, but it seems the first macroeconomic paper that refers to Brock and Hommes (1997, 1998) and employs the discrete choice approach to model the competition between two rules or attitudes. Within the Keynesian textbook setting, Westerhoff concentrates on consumption demand. He treats investment as being fixed and assumes that consumption is proportional to expected output (that is, to national income). Instead of the previous misalignment argument to determine the shares of the two forecast rules, they are judged by their relative success. Thus, the fitness which enters the discrete choice probabilities is given by minus the most recent squared prediction error. In addition, regressive expectations are supposed to be more costly than their counterpart, which is expressed by subtracting a positive constant number from their performance measure.

Westerhoff chooses a parametrization such that if all agents adopt extrapolative expectations, divergence is not monotonic but occurs in a cyclical manner. While prediction errors are similar near the equilibrium, around the turning points, before and after them, it is the regressive expectations that are more successful, even after accounting for their cost. Their existence clearly depresses the next turning point to come, but it is not entirely obvious what exactly prevents an ever increasing amplitude; indeed, in the simulations, the fluctuations always happen to be bounded. This difficulty is illustrated by the cyclical patterns of the time series, which can be rather complex. In any case, they do not look like a regular sine wave. It is even more remarkable that, although the trajectories converge to a periodic or quasi-periodic limit cycle, such an attractor is not unique. Moreover, also the equilibrium point itself is a local attractor. The example of Figure 2 shows that, apart from the latter, the economy may converge to no less than four different non-degenerate periodic motions, depending on the initial conditions out of equilibrium.[23]

In this economic environment, Westerhoff (2006c) subsequently studies the scope for a stabilizing fiscal policy. The nonlinear, possibly complicated dynamics provides a serious challenge to government. Common ideas such as trend-offsetting or level-adjusting interventions turn out to be a mixed blessing. If the policy-makers choose the wrong intensity—and here even tiny differences may matter—it can happen that output fluctuations are amplified rather than dampened.

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[23] In other simulations, Westerhoff obtains periodic orbits with larger variations in the amplitude from one (intermediate) cycle to another.

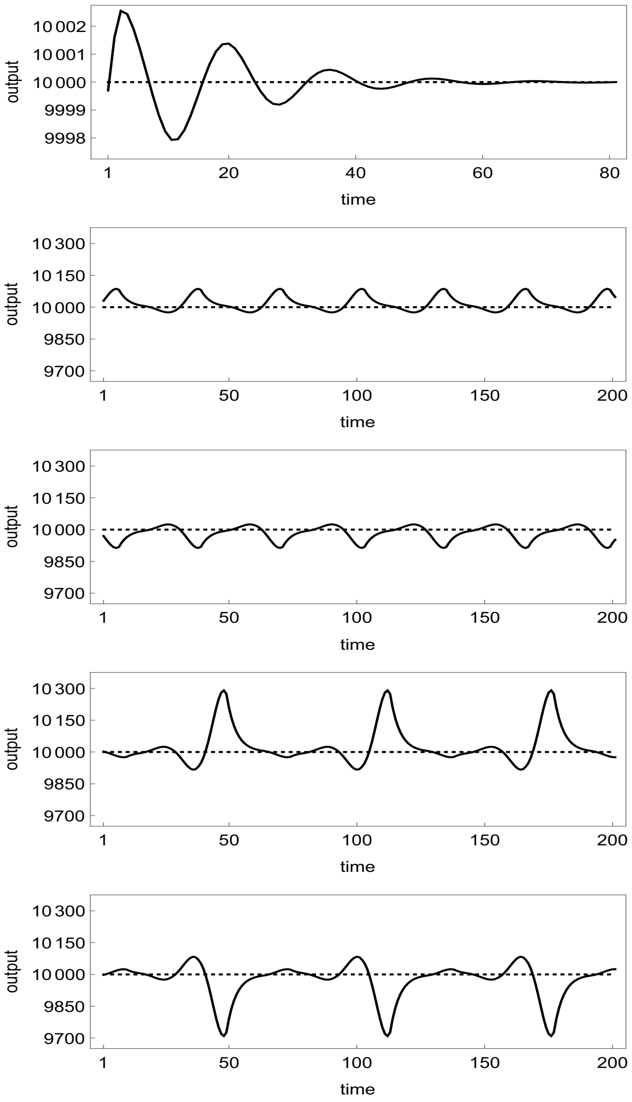

Figure 2: Five different, locally attracting periodic motions of the Westerhoff (2006c) model.

Regarding the expectation formation, Wegener and Westerhoff (2012) test the same idea in a Metzlerian framework with inventory cycles. The selection of the two forecast rules are more refined in this case by allowing for some memory in the fitness measure and asynchronous updating. These modifications have more satisfactory interpretations and lead to smoother adjustments.

In a framework with a limited IS-LM flavour, Lines and Westerhoff (2010, 2012) consider the same type of expectations with respect to inflation in an expectation-augmented price Phillips curve (keeping the rest as simple as possible).[24] Once again, multiple locally attracting limit cycles are possible. In addition, it turns out that wider regions of the parameter space yield a chaotic dynamic behaviour in the sense of a strange attractor (its existence can be rigorously proven by computing a certain mathematical indicator). The authors are also concerned with policy issues—monetary policy in this case. That is, in a Taylor-like manner, the central bank may raise or lower the growth rate of the quantity of money in response to current inflation and output growth. In particular, sufficiently strong reactions to the latter are seen to provide potential for stabilization.

4 Heterogeneity and animal spirits in the New-Keynesian framework

4.1 De Grauwe's modelling approach

Given that the New-Keynesian theory is the ruling paradigm in macroeconomics, Paul De Grauwe had a simple but ingenious idea to challenge it: accept the three basic log-linearized equations for output, inflation and the interest rate of that approach, but discard its underlying representative agents and rational expectations. This means that, instead, he introduces different groups of agents with heterogeneous forms of bounded rationality, as it is called.[25] Expectations have to be formed for the output gap (the percentage deviations of output from its equilibrium trend level) and for the rate of inflation in the next period. For each variable, agents can choose between two rules of thumbs where, as specified by the discrete choice approach, switching between them occurs according to their forecasting performance. De Grauwe speaks of 'animal spirits' insofar as such a model is able to generate

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[24] Lines and Westerhoff (2010) assume costly rational expectations rather than regressive expectations. The dynamic properties are nevertheless fairly similar. On this occasion, we may also mention an interesting alternative selection mechanism that was put forward in (almost) the same model by Da Silveira and Lima (2014), which they call satisficing evolutionary dynamics. It states that an individual agent will only choose the more efficient rule if the performance differential exceeds a certain threshold, where these thresholds are randomly distributed across the population of agents. The authors prove that this leads to a locally stable equilibrium, a result that, however, may also be due in part to the fact that the extrapolative expectations are replaced with adaptive expectations. Furthermore, there was no exploration of the global dynamics.

[25] To be fair, Brazier et al. (2008) pursued a similar idea in an overlapping-generations model with money growth and expectations about inflation. This paper proved, however, to be less influential than De Grauwe's work.

waves of optimistic and pessimistic forecasts, notions that are excluded from the New-Keynesian world by construction.[26]

The following three-equation model is taken from De Grauwe (2008a), which is the first in a series of similar versions that have subsequently been studied in De Grauwe (2010, 2011, 2012a,b). The term 'three-equation' refers to the three laws that determine the output gap y, the rate of inflation π, and the nominal rate of interest i set by the central bank. The symbols $\pi^\star$ and $i^\star$ denote the central bank's target rates of inflation and interest, which are known and taken into account by the agents in the private sector. All parameters are positive where, more specifically, a_y, b_y are weighting coefficients between 0 and 1. E_t^{agg} are the aggregated expectations of the heterogeneous agents using information up to the beginning of the present period t. They are substituted for the mathematical expectation operator E_t, the aforementioned rational expectations. Then, the three equations are:

$$\begin{aligned} y_t &= a_y E_t^{agg} y_{t+1} + (1-a_y)\, y_{t-1} + a_i \left[i_t - E_t^{agg} \pi_{t+1} - (i^\star - \pi^\star)\right] + \varepsilon_{y,t} \quad (9) \\ \pi_t &= b_\pi E_t^{agg} \pi_{t+1} + (1-b_\pi)\, \pi_{t-1} + b_y\, y_t + \varepsilon_{\pi,t} \quad (10) \\ i_t &= c_i\, i_{t-1} + (1-c_i)\, i^\star + c_\pi (\pi_t - \pi^\star) + c_y\, y_t + \varepsilon_{i,t} \quad (11) \end{aligned}$$

Equation (9) for the output gap is usually referred to as an IS equation, here in hybrid form, which means that the expectation term is combined with a one-period lag of the same variable. The Phillips curve in (10), likewise in hybrid form, is viewed as representing the supply side of the economy. Equation (11) is a Taylor rule with interest rate smoothing, that is, it contains the lagged interest rate on the right-hand side.[27] The terms $\varepsilon_{y,t}$, $\varepsilon_{\pi,t}$, $\varepsilon_{i,t}$ are white noise disturbances, interpreted as demand, supply and monetary policy shocks, respectively. Qualitatively little would change if some serial correlation were allowed for them.

The aggregate expectations in these equations are convex combinations of two (extremely) simple forecasting rules. With respect to the output gap, De Grauwe considers optimistic and pessimist forecasters, predicting a fixed positive and negative value of y, respectively. With respect to the inflation rate, he distinguishes between agents who believe in the central bank's target and so-called extrapolators, who predict that next period's inflation will be last period's inflation.[28] Accord-

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[26] It has already been indicated in footnote 1 that the special branch of 'sunspot equilibria' within the DSGE literature makes reference to 'animal spirits', too, but that these concepts are fundamentally different from the mechanisms in the present models. As the term 'sunspots' suggests, the waves generated there have an exogenous source, while De Grauwe emphasizes their endogenous origin in his approach.

[27] De Grauwe mostly simplifies his equations by putting $i^\star = \pi^\star = 0$.

[28] It would be more appropriate to call the latter naive expectations; cf. De Grauwe and Macchiarelli (2015, p. 97).

ingly, with $g > 0$ as a positive constant, n^{opt} as the share of optimistic agents regarding output, and n^{tar} as the share of central bank believers regarding inflation, expectations are given by

$$\begin{array}{rclcrcl} E_t^{opt} y_{t+1} & = & g\,, & & E_t^{pess} y_{t+1} & = & -g \\ E_t^{tar} \pi_{t+1}\,, & = & \pi^\star & & E_t^{ext} \pi_{t+1} & = & \pi_{t-1} \\ E_t^{agg} y_{t+1} & = & \multicolumn{5}{l}{n_t^{opt} E_t^{opt} y_{t+1} \;+\; (1-n_t^{opt})\, E_t^{pess} y_{t+1}} \\ E_t^{agg} \pi_{t+1} & = & \multicolumn{5}{l}{n_t^{tar} E_t^{tar} \pi_{t+1} \;+\; (1-n_t^{tar})\, E_t^{ext} \pi_{t+1}} \end{array} \tag{12}$$

In other papers, De Grauwe alternatively stipulates so-called fundamental and extrapolative output forecasters, $E_t^{fun} y_{t+1} = 0$ and $E_t^{ext} y_{t+1} = y_{t-1}$. However, the dynamic properties of his model are not essentially affected by such a respecification.

The populations shares of the heterogeneous agents are determined by the suitably adjusted discrete choice equations (1), (2). Denoting the measures of fitness that apply here by U^{opt}, U^{pess}, U^{tar}, U^{ext}, we have

$$\begin{array}{rcl} n_t^{opt} & = & \dfrac{\exp(\beta U_{t-1}^{opt})}{\exp(\beta U_{t-1}^{opt}) \;+\; \exp(\beta U_{t-1}^{pess})} \\ n_t^{tar} & = & \dfrac{\exp(\beta U_{t-1}^{tar})}{\exp(\beta U_{t-1}^{tar}) \;+\; \exp(\beta U_{t-1}^{ext})} \end{array} \tag{13}$$

Conforming to the principle that better forecasts attract a higher share of agents, fitness is defined by the negative (infinite) sum of the past squared prediction errors, where the past is discounted with geometrically declining weights. Hence, with a so-called memory coefficient $0 < \rho < 1$, superscripts $A = opt, pess, tar, ext$ and variables $z = y, \pi$ in obvious assignment,

$$\begin{array}{rcl} U_t^A & = & -\sum_{k=1}^{\infty} \omega_k \,(z_{t-k} \;-\; E_{t-k-1}^A\, z_{t-k})^2\,, \qquad \omega_k = (1-\rho)\,\rho^k \\ & = & -\rho\,\{\,(1-\rho)(z_{t-1} - E_{t-2}^A\, z_{t-1})^2 \;+\; U_{t-1}^A\,\} \end{array} \tag{14}$$

This specification of the weights ω_k makes sure that they add up to unity. The second expression in (14) is an elementary mathematical reformulation. It allows a recursive determination of the fitness, which is more convenient and more precisely computable than an approximation of an infinite series.

Equation (14) completes the model. De Grauwe makes no explicit reference to an equilibrium of the economy (or possibly several of them?) and does not attempt to characterize its stability or instability. He proceeds directly to numerical simulations and then discusses what economic sense can be made of what we see. Depending on the specific focus in his papers, additional computer experiments with some modifications may follow.

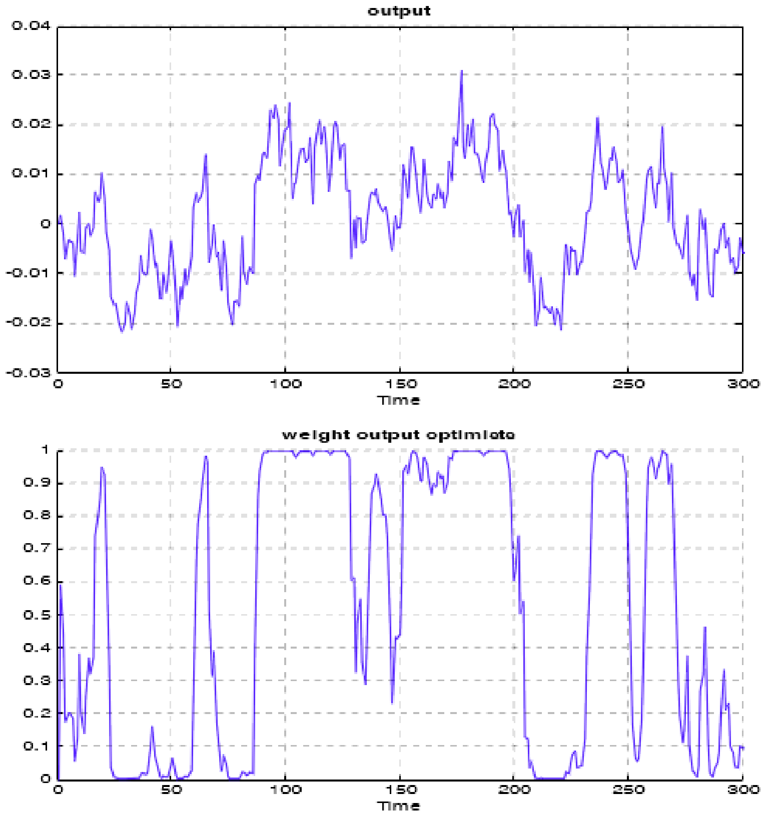

Figure 3: A representative simulation run of model (9) – (14).

A representative simulation run for the present model and similar models is shown in Figure 3. This example, reproduced from De Grauwe (2008a, p. 24), plots the time series of the output gap (upper panel) and the share of optimistic forecasters (lower panel). The underlying time unit is one month, i.e. the diagram covers a period of 25 years. The strong raggedness of the output series is indicative of the stochastic shocks that De Grauwe assumes. In fact, the deterministic core of the model is stable and converges to a state with $y = 0$, $\pi = \pi^\star$, $i = i^\star$. Without checking any stability conditions or eigen-values, this can be inferred from various diagrams of impulse-response functions in De Grauwe's work.

The fluctuations in Figure 3 are therefore not self-sustaining. De Grauwe nevertheless emphasizes that his model generates endogenous waves of optimism and pessimism. This characterization may be clarified by a longer quote from De Grauwe (2010, p. 12):

> "These endogenously generated cycles in output are made possible by a self-fulfilling mechanism that can be described as follows. A series of random shocks creates the possibility that one of the two forecasting rules, say the extrapolating one, delivers a higher payoff, i.e. a lower mean squared forecast error (MSFE). This attracts agents that were using the fundamentalist rule. If the successful extrapolation happens to be a positive extrapolation, more agents will start extrapolating the positive output gap. The 'contagion-effect' leads to an increasing use of the optimistic extrapolation of the output-gap, which in turn stimulates aggregate demand. Optimism is therefore self-fulfilling. A boom is created. At some point, negative stochastic shocks and/or the reaction of the central bank through the Taylor rule make a dent in the MSFE of the optimistic forecasts. Fundamentalist forecasts may become attractive again, but it is equally possible that pessimistic extrapolation becomes attractive and therefore fashionable again. The economy turns around.
>
> These waves of optimism and pessimism can be understood to be searching (learning) mechanisms of agents who do not fully understand the underlying model but are continuously searching for the truth. An essential characteristic of this searching mechanism is that it leads to systematic correlation in beliefs (e.g. optimistic extrapolations or pessimistic extrapolations). This systematic correlation is at the core of the booms and busts created in the model."

Thus, in certain stages of a longer cycle, the optimistic expectations are superior, which increases the share of optimistic agents and enables output to rise, which in turn reinforces the optimistic attitude. This mechanism is evidenced by the co-movements of y_t and n_t^{opt} in Figure 3 and conforms to the positive feedback loop highlighted in a comment on the small and stylized system (8) above.[29] A stabilizing counter-effect is not as clearly recognizable. De Grauwe only alludes to the central bank's reactions in the Taylor rule, when positive output gaps and inflation rates above their target (which will more or less move together) lead to both higher nominal and real interest rates. This is a channel that puts a curb on y_t in the IS equation. In addition, a suitable sequence of random shocks may occasionally work in the same direction and initiate a turnaround.

The New-Keynesian theory is proud of its "microfoundations". Within the framework of the representative agents and rational expectations, they derive the macroeconomic IS equation (9) and the Phillips curve (10) as log-linear approximations to the optimal decision rules of intertemporal optimization problems. As

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[29] De Grauwe (2010, p. 14) reports that the correlation between the fraction of optimists and the output gap is as high as 0.86. This requires the intensity of choice β to be sufficiently high and the memory coefficient ρ to be less than one (though only slightly so); see *ibid.*, pp. 14f.

these two assumptions have now been dropped, the question arises of the theoretical justification of (9) and (10). Two answers can be given.

First, Branch and McGough (2009) are able to derive these equations invoking two groups of individually boundedly rational agents, provided that their expectation formation satisfies a set of seven axioms.[30] The authors point out that the axioms are not only necessary for the aggregation result, but some of them could also be considered rather restrictive; see, especially, Branch and McGough (2009, p. 1043). Furthermore, it may not appear very convincing that the agents are fairly limited in their forecasts, and yet they endeavour to maximize their objective function over an infinite time horizon and are smart enough to compute the corresponding first-order Euler conditions.

Acknowledging these problems, the second answer is that the equations make good economic sense even without a firm theoretical basis. Thus, one is willing to pay a price for the convenient tractability obtained, arguing that more consistent attempts might be undertaken in the future. In fact, De Grauwe's approach also succeeded in gaining the attention of New-Keynesian theorists and a certain appreciation by the more open-minded proponents. This is indeed one of the rare occasions where orthodox and heterodox economists are able and willing to discuss issues by starting out from a common basis.

Branch and McGough (2010) consider a similar version to eqs (9) – (11) where, besides naive expectations, they still admit rational expectations. However, the latter are more costly, meaning that they may be outperformed by boundedly rational agents in tranquil times, in spite of their systematic forecast errors. For greater clarity, the economy is studied in a deterministic setting (hence rational expectations amount to perfect foresight). The authors are interested in the stationary points of this dynamics: in general there are multiple equilibria and the questions is which are stable/unstable, and what are the population shares prevailing in them.

Branch and McGough's analysis provides a serious challenge for the rational expectations hypothesis. Its recommendation to monetary policy is to guarantee determinacy in models of this type (this essentially amounts to the Taylor principle, according to which the interest rate has to rise more than one-for-one with inflation). Branch and McGough illustrate that, in their framework, the central bank may unwittingly destabilize the economy by generating complex ('chaotic') dynamics with inefficiently high inflation and output volatility, even if all agents are initially rational. The authors emphasize that these outcomes are not limited to unusual calibrations or *a priori* poor policy choices; the basic reason is rather the dual

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[30] To be exact, the equations they obtain do not contain the lagged endogenous variable on the right-hand side. A model in a similar spirit but with more specific assumptions is studied by Massaro (2013).

attracting and repelling nature of the steady state values of output and inflation.

Anufriev et al. (2013) abstract from output and limit themselves to a version of (10) with only expected inflation on the right-hand side. Since there is no interest rate smoothing in their Taylor rule ($c_1 = 0$) and, of course, no output gap either, the inflation rate is the only dynamic variable. These simplifications allow the authors to consider greater variety in the formation of expectations and to study their effects almost in a vacuum. In this case, too, the main question is whether, in the absence of random shocks, the system will converge to the rational expectations equilibrium. This is possible but not guaranteed because, again, certain ecologies of forecasting rules can lead to multiple equilibria, where some are stable and give rise to intrinsic heterogeneity.

Maintaining the (stochastic) equations (9), (10) (but without the lagged variables on the right-hand side) and considering different dating assumptions in the Taylor rule (likewise without interest rate smoothing), Branch and Evans (2011) obtain similar results, broadly speaking. They place particular interest in a possible regime-switching of the output and inflation variances (an important empirical issue for the US economy), and in the implications of heterogeneity for optimal monetary policy.

Dräger (2016) examines the interplay between fully rational (but costly) and boundedly rational (but costless) expectations in a subvariant of the New-Keynesian approach, which is characterized by a so-called rational inattentiveness of agents. As a result of this concept, entering the model equations for quarter t are not only contemporary but also past expectations about the variables in quarter $t+1$. The author's main concern is with the model's ability to match certain summary statistics and, in particular, the empirically observed persistence in the data. Not the least due to the flexible degree of inattention, which is brought about by the agents' switching between full and bounded rationality (in contrast to the case where all agents are fully rational, when the degree is fixed), the model turns out to be superior to the more orthodox model variants.[31]

4.2 Modifications and extensions

The attractiveness of De Grauwe's modelling strategy is also shown by a number of papers that take his three-equation model as a point of departure and combine it with a financial sector. To be specific, this means that a financial variable is added to eq. (9), (10) or (11), and that the real economy also feeds back on financial markets

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[31] The two precursory working papers Dräger (2010, 2011) may help generate a better understanding of the more elaborate parts of this analysis and of the conditions that may give rise to the superior results with the flexible-degree version.

via the output gap or the inflation rate. It is here a typical conjecture, which then needs to be tested, that a financial sector tends to destabilize the original model in some sense; for example, output or inflation may become more volatile.

An early extension of this kind is the integration of a stock market in De Grauwe (2008b). He assumes that an increase in stock prices has a positive influence on output in the IS equation and a negative influence on inflation in the Phillips curve (the latter because this reduces marginal costs). In addition, it is of special interest that the central bank can try to lean against the wind by including a positive effect of stock market booms in its interest rate reaction function. The stock prices are determined, in turn, by expected dividends discounted by the central bank's interest rate plus a constant markup. The actual dividends are a constant fraction of nominal GDP, i.e. their forecasts are closely linked to the agents' forecasts of output and inflation.

In a later paper, De Grauwe and Macchiarelli (2015) include a banking sector in the baseline model. In this case, the negative spread between the loan rate and the central bank's short-term interest rate enters the IS equation in order to capture the cost of bank loans. Along the lines of the financial accelerator by Bernanke et al. (1999), banks are assumed to reduce this spread as firms' equity increases which, by hypothesis, moves in step with their loan demand. Besides y_t, π_t, i_t, the model contains private savings and the borrowing-lending spread as two additional dynamic variables. In the final sections of the paper, the model is extended by introducing variable share prices and determining them analogously to De Grauwe (2008b).

De Grauwe and Gerba (2015a) is a very comprehensive contribution that starts out from De Grauwe and Macchiarelli (2015), but specifies a richer structure of the financial sector, which also finds its way into the IS equation. One consequence of the extension is that capital now shows up as another dynamic variable, and that new types of shocks are considered.[32] Once again, the discrete choice version is contrasted to the world with rational expectations. In a follow-up paper, De Grauwe and Gerba (2015b) introduce a bank-based corporate financing friction and evaluate the relative contribution of that friction to the effectiveness of monetary policy. On the whole, it is impressive work, but, given the long list of numerical parameters to set, readers have to place their trust in it.

Lengnick and Wohltmann (2013) and, in a more elaborated version, (2016) choose a different approach to add a stock market to the baseline model.[33] There are

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[32] There are lots of microfoundation details in the first section. Unexperienced readers should not be deterred by this, but may proceed to Section 3.5, which provides a familiar, though more colourful picture.

[33] In eqs (9) and (10) they consider three types of expectations: targeting, naive and extrapolative expectations proper.

two channels through which stock prices affect the real side of the economy. One is a negative influence in the Phillips curve, which is interpreted as an effect on marginal cost, the other is the difference between stock price and goods price inflation in the IS equation, which may increase output. The modelling of the stock market, on the other hand, is borrowed from the burgeoning literature on agent-based speculative demands for a risky asset. Such a market is populated by fundamentalist traders and trend chasers who switch between these strategies analogously to (13) and (14). The market is now additionally influenced by the real sector through the assumption that the fundamental value of the shares is proportional to the output gap. Furthermore, besides speculators, there is a stock demand by optimizing private households, which increases with output and decreases with the interest rate and higher real stock prices.

While in the simulations the authors maintain the usual quarter as the length of the adjustment period in (9) – (11) for the real sector, they specify financial transactions on a daily basis and use time aggregates for their feedback on the quarterly equations. Even in isolation and without random shocks, the stock market dynamics is known for its potential to generate endogenous booms and busts. The spill-over effects can now cause a higher volatility in the real sector. For example, it can modify the original effects of a given shock in the impulse-response functions and make them hard to predict.[34] One particular concern of the two papers is a possible stabilization through monetary policy, another is a taxation of the financial transactions or profits. An important issue is whether a policy that is effective under rational expectations can also be expected to be so in an environment with heterogeneous and boundedly rational agents.

Scheffknecht and Geiger's (2011) modelling is in a similar spirit (including the different time scales for the real and financial sector), but limits itself to one channel from the stock market to the three-equation baseline specification. To this end, the authors add a risk premium ζ_t (i.e. the spread between a credit rate and i_t) to the short-term real interest rate in (9). The transmission is a positive impact of the change in stock prices on ζ_t, besides effects from y_t, i_t and the volatilities (i.e. variances) of y_t, π_t, i_t on this variable.

A new element is an explicit consideration of momentum traders' balance sheets (but only of theirs, for simplicity). They are made up of the value of the shares they hold and money, which features as cash if it is positive and debt if it is negative. This brings the leverage ratios of these traders into play, which may

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[34] The reference for the authors' impulse-response functions (IRFs) is not an equilibrium position, but an entire stochastic simulation run. Subsequently, the model is run a second time with the same random shocks, except for one shock in the initial period. The IRFs are then the difference in the variables from these two runs.

constrain them in their asset demands. Although the latter extension is not free of inconsistencies, these are ideas worth considering.[35]

5 Herding and objective determinants of investment

Apart from the models inspired by Kirman (1993), the models discussed so far were concerned with expectations about an economic variable in the next period. Here, a phenomenon to which an expression like 'animal spirits' may apply occurs when the agents rush toward one of the two forecast rules. However, this behaviour is based on objective factors, normally publicly available statistics. Most prominently, they contrast expected with realized values and then evaluate the forecast performance of the rules.

In the present section, we emphasize that the success of decisions involving a longer time horizon, in particular, cannot be judged from such a good or bad prediction, or from corresponding profits in the next quarter. It takes several years to know whether an investment in fixed capital, for example, was worth undertaking. Furthermore, decisions of that kind must, realistically, take more than one dimension into account. As a consequence, expectations are multi-faceted and far more diffuse in nature. In these situations, the third paragraph of the Keynes quotation in the introductory section becomes relevant, where he points out that "we endeavor to conform with the behavior of the majority or the average", which "leads to what we may strictly term a *conventional* judgment." In other words, central elements are concepts such as a (business or consumer) sentiment or climate, or a general state of confidence. In the language of tough business men, it is not only their skills, but also their gut feelings that make them so successful.

Therefore, as an alternative to the usual focus on next-period expectations of a specific macroeconomic variable, we may formulate the following axiom: *long-term decisions of the agents are based on sentiment*, where, as indicated by Keynes, with agents' orientation toward the behaviour of the majority, this expression may also connote herding. In terms of 'animal spirits', we propose that in the models under consideration so far we have animal spirits in a weak sense, whereas in the context outlined above we have animal spirits in a strong sense; animal spirits proper, so to speak.

The discrete choice and transition probability approaches can also be used to model animal spirits in the strong sense. Crucial for this is specifying arguments with which the probabilities are supposed to vary, that is, specifying what was called

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[35] Two aspects are: (1) There is nobody in the model from which momentum traders could borrow, and whose balance sheet would be affected, too. (2) Neither the direct nor the indirect cost of borrowing shows up in the fitness function of momentum traders.

the fitness function or switching index, respectively. Such arguments may neglect an evaluation of short-term expectations, and they should provide a role for herding or contagion. The latter can be achieved conveniently by including a majority index, such that the more agents adhere to one of the attitudes, the higher *ceteris paribus* the probability that agents will choose it or switch to it.

In the following, we present a series of papers that follow this strategy. What they all have in common is that they pursue the transition probability approach, and that their sentiment variable refers to the fixed investment decisions of firms. The models are thus concerned with a business sentiment. This variable is key to the dynamics because, acting *via* the Keynesian multiplier, investment and its variations are the driving force of the economy; other components of aggregate demand play a passive role. Also, all of these models are growth models, a feature that makes them economically more satisfactory than most of the (otherwise meritorious) models described in the previous sections, which are stationary in the long run.

Let us therefore begin by specifying investment and the goods markets. Individual firms have two (net) investment options. These options are given by a lower growth rate of the capital stock g_{min}, at which firms invest if they are pessimistic, and a higher growth rate g_{max}, corresponding to an optimistic view of the world. Let g^o be the mean value of the two, $g^o = (g_{min} + g_{max})/2$, and x the sentiment of the firms as it was defined in Section 2.2, i.e. the difference between optimistic and pessimistic firms scaled by their total number. Hence the aggregate capital growth rate is given by[36]

$$g = g(x) \quad = \quad g^o \, + \, \beta_{gx} x \, , \qquad \text{where } \beta_{gx} := (g_{max} - g_{min})/2 \tag{15}$$

Being in a growth framework, economic activity is represented by the output-capital ratio u, which can also be referred to as (capital) utilization. Franke (2008a, 2012a) models the other components of demand such that, supposing continuous market clearing, IS utilization is a linear function of (only) the business sentiment,

$$u = u(x) \quad = \quad g(x) \, / \, \sigma \, + \, \beta_u \tag{16}$$

where σ is the marginal aggregate propensity to save and β_u a certain positive, structurally well-defined constant. A consistency condition can (but need not) ensure that a balanced sentiment $x = 0$ prevails in a steady state position.

For one part, the specification of the switching index includes the sentiment variable x, which can capture herding. The choice and influence of a second variable revolves around the rest of the economy. Franke (2008a) combines the sentiment dynamics with a Goodwinian struggle between capitalists and workers for the

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[36] Recall from Section 2.2 that the shares of optimists and pessimists are $n^+ = (1+x)/2$, $n^- = (1-x)/2$.

distribution of income. It is summarized in a real wage Phillips curve depending, in particular, on utilization $u = u(x)$ from (16). In this way, the wage share v becomes the second dynamic variable besides x. With a few simple manipulations, its changes can be described by

$$\dot{v} \;=\; \beta_v \, v \, (1-v) \, x \tag{17}$$

($\beta_v > 0$ another suitable constant). Regarding the sentiment, the idea is that *ceteris paribus* the firms tend to be more optimistic when the profit share increases (the wage share decreases). With two coefficients $\phi_x, \phi_v > 0$ and the equilibrium wage share v^o, the switching index is thus of the form,[37]

$$s = s(x,v) \;=\; \phi_x \, x \,-\, \phi_v \, (v - v^o) \tag{18}$$

As derived in Section 2.2, eq. (7), the sentiment adjustments read as follows (with $\beta = 2\nu = 1$),

$$\dot{x} \;=\; \{\tanh[s(x,v)] - x\} \cosh[s(x,v)] \tag{19}$$

To sum up, taking account of (18), the economy is reduced to two differential equations in the sentiment x and the wage share v. It could be characterized as a microfounded Goodwinian model that, besides the innovation of the notion of business sentiment, includes a variable output-capital ratio and an investment function (the latter two features are absent in Goodwin's original model from 1967).

It may be observed that eqs (17) – (19) have the same structure as system (8), apart from the slight distortions by $\cosh(\cdot)$ in (19) and the multiplication of x by $v(1-v)$ in (17). Therefore, depending on ϕ_x, the isocline $\dot{x} = 0$ resembles the two upper panels of Figure 1, whereas the other isocline $\dot{v} = 0$ becomes a vertical line at $x=0$. The latter rules out the multiple equilibria in the other two panels of Figure 1.

The dynamic properties are as described in the discussion of (8): the (unique) steady state is locally and globally stable if the herding coefficient ϕ_x is less than unity. Otherwise it is repelling, where the reflecting boundaries $x = \pm 1$ and the multiplicative factor $v(1-v)$ in (17) ensure that the trajectories remain within a compact set. Hence (by the Poincaré-Bendixson theorem) all trajectories must converge to a closed orbit. Numerically, by all appearances, it is unique. Accordingly, if (and only if) herding is sufficiently strong, the economy enters a uniquely determined periodic motion in the long run. Regarding income distribution, it features

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[37] In addition, utilization may have a positive effect on the sentiment. By virtue of (16), however, this influence can be subsumed within the variable x.

the well-known Goodwinian topics, regarding the sentiment, phases of optimism give way periodically to phases of pessimism and *vice versa*.

Franke (2012a) specifies the same demand side (15) and (16). Its other elements are:

- A central bank adopting a Taylor rule to set the rate of interest; that is, the interest rate increases in response to larger deviations of utilization from normal and larger deviations of the inflation rate from the bank's target.
- A price Phillips curve with a so-called inflation climate π^c taking the role of its expectation term.
- An adjustment equation for the inflation climate, which is a weighted average of adaptive expectations and regressive expectations. The latter means that agents trust the central bank to bring inflation back to target (correspondingly, the weight of these expectations can be interpreted as the central bank's credibility).[38]

In spite of its structural richness, the economy can be reduced to two differential equations in the sentiment x and the inflation climate π^c. As a matter of fact, identifying π^c with y, the system has the same form as eq. (8) in Section 2 (apart from the cosh term). Therefore, because of its business sentiment and, again, if herding is strong enough, the model can be viewed as an Old-Keynesian version of the interplay of output, inflation and monetary policy, or (in a somewhat risky formulation) of the macroeconomic consensus.[39]

For situations in which agents carry an asset forward in time, there is a problem with the transition probability and discrete choice approach alike, which should not be concealed. It arises from the fact that, with the switching between high and low growth rates in the investment decisions, the capital stocks of individual firms change from one period to another (in absolute and relative terms). On the other hand, the definition of the aggregate capital growth rate in (15) together with the macroscopic adjustment equation for the sentiment x implicity presupposes that the groups of optimistic and pessimistic firms always have the same distribution of capital stocks. As a consequence, these equations are only an approximation. Apart from the size of the approximation errors, acknowledging this feature leads to the question of whether the errors may also accumulate in the course of time.

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[38] Speaking of 'climate' might suggest that its changes are alternatively modelled by invoking transition probabilities a second time. As an interesting and somewhat puzzling aside, it can be noted that a higher credibility of the central bank tends to be destabilizing rather than stabilizing.

[39] While heterodox theory should have an affinity to the model's 'Old-Keynesian' elements, a number of heterodox economists will not endorse it either, because the macroeconomic consensus is an emotive word to them (although the observation in the previous footnote may perhaps placate these sceptics somewhat).

Yanovski (2014) goes back to the micro level of the transition probability approach to inquire into this problem. Considering a finite population, he models each firm and its probability calculus individually and also keeps track of the capital stocks resulting from these decisions (modelling that requires a few additional specifications to be made for the micro level). In short, the author finds that the approximation problem does not appear to be very serious. Of course, every macro model that uses the transition probability or discrete choice approach must be reviewed separately, but this first result is encouraging.

Interestingly, Yanovski discovers another problem, which concerns the size distribution of capital stocks: it tends to be increasingly dispersed over time. The result that some firms become bigger and bigger may or may not be attractive. Yanovski subsequently tries several specification details that may entail a bounded width of the size distribution in the long run. To them it is crucial to relax the assumption of uniform transition probabilities, and that additional, firm-specific arguments are proposed to enter them. Within a parsimonious framework, these discussions can provide a better understanding of the relationships between micro and macro.[40]

Going back to macrodynamics, Lojak (2015) adds a financial side to the monetary policy and output-inflation nexus in Franke (2012a). Besides the different saving propensities for workers and rentiers households, which yield a more involved IS equation for goods market clearing, it makes the firms' financing of fixed investment explicit. This work distinguishes between internal sources, i.e. the retained earnings of the firms, and external sources, i.e. their borrowing from the rentiers (possibly with commercial banks as intermediates). In this way, a third dynamic variable is introduced into the model, the firms' debt-to-capital ratio. It feeds back on the real sector by a negative effect of higher indebtedness on the switching index for the business sentiment x (which again demonstrates the flexibility of this concept).

Motivated by the discussion of Minskian themes in other macro models, the author concentrates on cyclical scenarios and here, in particular, on the comovements of the debt-asset ratio. While it is usually taken for granted that it lags capital utilization, the author shows that this is by no means obvious. This finding is an example of the need to carefully reconsid the dynamic features of a real-financial interaction.

In a follow-up paper, Lojak (2016) fixes the inflation rate for simplicity and drops the assumption of a constant markup on the central bank's short-term interest

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[40] The discussion can thus be more transparent than in the so-called bottom-up models with many heterogeneous agents (far more than two), which are creative but less 'canonical' than the approach under consideration here.

rate to determine the loan rate. Instead, the markup is now supposed to increase with the debt-asset ratio, d. This straightforward extension gives rise to additional strong nonlinearities. Most amazingly, in the original cyclical scenario of the two-dimensional (x,d) dynamics a second equilibrium with lower utilization and higher indebtedness comes into being (but not three as in Figure 1). It is also characterized by a locally stable limit cycle around it. The limit cycle around the 'normal' equilibrium is maintained, so that two co-existing cyclical regimes are obtained. Not all of the phenomena that one can here observe are as yet fully understood, which shows that it is work in progress and a fruitful field for further investigations. In particular, future research may consider the lending of commercial banks in finer detail, and animal spirits may then play a role in this sector as well.

6 Reality checks

Even if the discrete choice and transition probabilities are reckoned to be a conceptually attractive approach for capturing a sentiment dynamics, these specifications would gain in significance if it can be demonstrated that they are compatible with what is observed in reality, or inferred from it. A straightforward attempt to learn about people's decision-making are controlled experiments with human subjects in the laboratory. Regarding empirical testing in the usual sense, there are two different ways to try, a direct and a more indirect way. The first method treats a sentiment adjustment equation such as (3) or (5) as a single-equation estimation, where the variable x_t in (5) or the population shares n_t^+, n_t^- in (3) are proxied by an economic survey. In fact, several such surveys provide so-called sentiment or climate indices. The second method considers a model as a whole and seeks to estimate its parameters in one effort. Here, however, x or n^+, n^- remain unobserved variables, that is, only 'normal' macroeconomic variables such as output, inflation, etc. are included as empirical data. These three types of a reality check are considered in the following subsections.

6.1 Evidence from the lab

Self-inspection is not necessarily the best method to find out how people arrive at their decisions. A more systematic way that approaches people directly in this matter are laboratory experiments. To begin with, they indeed provide ample evidence that the subjects use similarly simple heuristics to those considered in the models that we have presented; see Assenza et al. (2014a) for a comprehensive literature survey. A more specific point is whether the distribution of different rules within a population and its changes over time could be explained by the discrete choice or transition probability approach. Several experiments at CeNDEF (University of Amsterdam) allow a positive answer with respect to the former. In the setting of a New-Keynesian three-equation model, Assenza et al. (2014b) find four qualitatively different macro patterns emerging out of a self-organizing process where one of four forecasting rules tends to become dominant in the consecutive rounds of an experiment. The authors demonstrate that this is quite in accordance with the discrete choice principle.[41]

In another study by Anufriev et al. (2016), where the series to be forecasted are exogenously generated prior to the experiments and the subjects have to choose between a small numbers of alternatives given to them, a discrete choice model can in most cases be successfully fitted to the subjects' predictions. In particular, the experimenters can make inference about the intensity of choice, although different treatments yield different values. For all of these studies, however, it has to be taken into account that a full understanding of the results requires the reader to get involved in a lot of details.

6.2 Empirical single-equation estimations

With respect to inference from empirical data, let us first consider surveys collecting information about the expectations or sentiment of a certain group in the economy. As far as we know, the first empirical test of this kind is Branch (2004). He is concerned with the Michigan survey where private households are asked on a monthly basis for their expectations about future inflation. For his analysis, the author equips the respondents with three virtual predictor rules: naive (i.e. static) expectations, adaptive expectations, and the relatively sophisticated expectations obtained from a vector autoregression that besides inflation includes unemployment, money growth and an interest rate. The fitness of these rules derives from the squared forecast errors and a specific cost term (which has to be re-interpreted after the estimations).

The model thus set up is estimated by maximum likelihood. The estimate of the intensity of choice is significantly positive, such that all three rule are relevant (even the naive expectations) and their fractions exhibit nonnegligible fluctuations over time. It is also shown that this model is markedly superior to two alternatives that assume the forecasts are normally distributed around their constant or time-varying mean values across the respondents.

Branch (2007) is a follow-up paper using the same data. Here the forecast rules entering the discrete choice model are more elaborate than in his earlier paper. They are actually based on explanations from a special branch of the New-

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[41] Two similar experimental studies for a financial market environment are Anufriev and Hommes (2012a, b).

Keynesian literature (which uses the concept of limited information flows as it was developed in Mankiw and Reis, 2002). Thus, heterodox economists will probably not be very convinced by this theory. Nevertheless, the heterogeneity and switching mechanism introduced into the original New-Keynesian model with its homogeneous agents prove to be essential as this version provides a better fit of the data.

Several other business and consumer surveys lend themselves for testing theoretical approaches with binary decisions, because they already ask whether respondents are 'optimistic' or 'pessimistic' concerning the changes of a variable or the entire economy. To accommodate the possibility that a third, neutral assessment is also usually allowed for, it is assumed that neutral subjects can be assigned half and half to the optimistic and pessimistic camp. Franke (2008b) is concerned with two leading German surveys conducted by the Ifo Institute (Ifo Business Climate Index) and the Center for European Research (ZEW Index for Economic Sentiment), both of which are available at monthly intervals.

The respondents are business people and financial analysts, respectively. Because they are asked about the future prospects of the economy, the aggregate outcome can be viewed as a general sentiment prevailing in these groups. Given the theoretical literature discussed above, this suggests testing the transition probability approach with a herding component included. Franke formulates the corresponding sentiment changes in discrete time and extends eq. (5) and its switching index somewhat beyond what has been considered so far:

$$\begin{aligned} x_t &= x_{t-1} + \nu\,[\,(1-x_{t-1})\exp(s_{t-1}) - (1+x_{t-1})\exp(-s_{t-1})\,] + \varepsilon_{x,t} \\ s_{t-1} &= \phi_o + \phi_x x_{t-1} + \phi_{\Delta x}\,\Delta_{\tau_x} x_{t-1} + \phi_y y_{t-1} + \phi_{\Delta y}\,\Delta_{\tau_y} y_{t-1} \\ \Delta_{\tau_z} z_t &= (z_t - z_{t-\tau_z})/\tau_z \qquad \text{for } z = x, y \end{aligned} \tag{20}$$

where x_t is the Ifo or ZEW index, respectively, and y_t is the detrended log series of industrial production (the output gap, in per cent). Compared to previous discussions, three generalizations are allowed for in the switching index. (i) The coefficient ϕ_o measures a possible predisposition to optimism (if it is positive) or pessimism (if it is negative). (ii) In addition to the levels of x and y, first differences of the two variables can account for momentum effects. In particular, herding has two aspects: joining the majority (represented by ϕ_x), and immediate reactions to changes in the composition of the sentiment, which Franke (2008b, p. 314) calls the moving-flock effect. (iii) There may be lags Δ_τ ($\tau = 1, 2, \dots$) in the first differences.

The intrinsic noise from the probabilistic decisions of a finite number of agents is neglected. Instead, the stochastic term $\varepsilon_{x,t}$ represents random forces from outside the theoretical framework, i.e. extrinsic noise. Thus, (20) can be estimated by nonlinear least squares (NLS), with $\varepsilon_{x,t}$ as its residuals.

In the estimations of (20), a number of different cases were explored. Skipping the details and turning directly to the most efficient version where all of the remaining coefficients were well identified, a herding mechanism was indeed revealed for both indices. The majority effect, however, was of secondary importance and could be justifiably dismissed from the model (i.e. $\phi_x = 0$), so that herding was best represented by the moving-flock effect.

The arrival of new information on economic activity also plays a role. Relevant for both indices is again the momentum effect, while the level effect can be discarded for one index. Remarkably, the coefficient ϕ_y is negative when it is included (even in the version where $\phi_{\Delta y}$ is set equal to zero). A possible interpretations is that subjects in a boom already anticipate the subsequent downturn. Since it may not appear entirely convincing, the negative ϕ_y could perhaps be better viewed as a mitigation of the procyclical herding effect.

The finding of a strong role for the moving-flock effect is a challenge for theoretical modelling because incorporating it into our continuous-time framework would easily spoil a model's otherwise relatively simple mathematical structure. It would also affect its dynamic properties to some extent. The somewhat inconvenient features are discussed and demonstrated in Franke (2008a, pp. 249ff), but the issue has not been taken up in the following literature.

Subsequent to these results, Franke (2008b) considers two extensions of the estimation approach (20). First, he tests for cross-effects between the two indices, where he finds that the changes in the Ifo index (though not the levels) influence the respondents of the ZEW index, but not the other way around. This makes sense, given the specific composition of the two groups. The second extension tests for an omitted variable of unknown origin. This can be achieved by adding a stochastic variable z_t to the switching index in (20) and supposing, for simplicity, that its motions are governed by a first-order autoregressive process. Such a specification allows an estimation by maximum likelihood together with the Kalman filter, which serves to recover the changes in z_t. Again, an improvement is found for one index but not the other.

Lux (2009) uses the ZEW survey to estimate the transition probability approach with an alternative and more elaborate method. To this end, he goes back to the micro level and invokes the statistical mechanics apparatus, basically in the form of the Fokker-Planck equation in continuous time. In this way, he is able to derive the conditional transitional probability densities of the sentiment variable x_t between two months and thus compute, and maximize, a likelihood function. The main conceptual difference in this treatment from the NLS estimation is that it makes no reference to extrinsic noise. Instead, it includes the intrinsic noise, so that it can also determine the finite number of 'autonomous' subjects in the sample.

The results are largely compatible with the references made about Franke

(2008b). The likelihood estimation is potentially superior because it seeks to exploit more information, albeit at the cost of considerably higher computational effort. Ideally, NLS may be employed at a first stage to identify promising specifications, which then form the basis for more precise conclusions at a second stage.

In sum, it can be concluded from the two investigations by Franke and Lux that the transition probability approach is a powerful explanation for the ups and downs in the expectation formation of the respondents in the two surveys, which does not need to rely on unobservable information shocks. For this good result, however, the specifications in the switching index are slightly more involved than in our theoretical discussions.

Ghonghadze (2016) recently conducted an NLS estimation in the spirit of eq. (20) on a survey of senior loan officers regarding their bank lending practices. The respondents were asked whether they raised *versus* lowered the spreads between loan rates and banks' costs of funds, i.e. a tightening *versus* an easing of lending terms. This work, too, finds evidence of social interactions within this group, albeit with a view to certain macroeconomic indicators.

Still being concerned with a single-equation estimation, Cornea-Madeira et al. (2015) is a contribution that tests the discrete choice approach by referring to empirical macroeconomic data. Their testing ground is the New-Keynesian Phillips curve, where regarding the expectations entering it the agents can choose between naive forecasts and forecasts derived from an ambitious vector autoregression (VAR) that in addition to inflation takes account of the output gap and the rate of change of unit labour costs and of the labour share.[42] Again, the fitness of the two rules is determined by the past forecast errors. Since the population shares constituting the aggregate expectations can be ultimately expressed as functions of these macroeconomic variables, the Phillips curve can be estimated by NLS.

There are two structural parameters to be estimated, the slope coefficient for the marginal costs and the intensity of choice in the switching mechanism. Both of them have the correct positive sign at the usual significance levels. Overall, the predicted inflation path tracks the behaviour of actual inflation fairly well. The population share of the naive agents varies considerably over time, although it exhibits a high persistence. Interestingly, their fraction is relatively high or low during certain historical episodes. On average, the simplistic rule is adopted by no less than 67% of the agents.

The authors furthermore test this model against a number of alternatives, two of which are closely related to versions from the New-Keynesian estimation liter-

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[42] The latter is included because the authors have marginal costs featuring in the Phillips curve. On the basis of econometric arguments this treatment is finally reduced to a four-lag bivariate VAR in the output gap and the changes in the labour share.

ature. All of them are rejected at a 95% confidence level. Also robustness checks regarding alternative specifications of the VAR forecasting model and different empirical measures of marginal costs are undertaken. They show no need for any qualification of the previous conclusions. In sum, these results are a strong point in favour of the agents' heterogeneity plus a discrete choice switching mechanism.

6.3 System estimations

A second estimation approach is concerned with an entire model into which a sentiment dynamics has been integrated. Accordingly, it seeks to estimate the parameters of the latter together with the parameters in the rest of the model, that is, it is about testing a joint hypothesis. While in the New-Keynesian mainstream literature the dominant and widely applied system estimation method is Bayesian likelihood estimation, it is fraught with two difficulties: the model must have been linearized and it does not admit unobservable variables. Otherwise the mathematical and computational effort increases, and prohibitively so for non-specialists.

An alternative method can easily cope with these problems. It departs from the truism that no model can capture all aspects of the real world and that every model is built for a specific purpose. Therefore, a model is good if it fulfills its purpose; failures in other directions can and need to be tolerated. In the present context, a 'purpose' is given by a number of properties, often referred to as 'stylized facts', that can be filtered from reality and that researchers wish to be displayed in their models. Generally, these properties are quantified as summary statistics, which in econometric language are also called 'moments'.

A model will be unable to reproduce the desired empirical moments perfectly. Estimation means searching for parameter values such that the moments generated by the model come as close as possible to their empirical counterparts. Since a model usually has to be simulated to obtain these moments, one speaks of the *method of simulated moments*.

The crucial point is, of course, the choice of moments, which a number of econometric critics brand as arbitrary. It can, however, be argued that this feature is a virtue rather than a vice, because it requires researchers to be explicit about their priorities; readers can then decide whether they share or accept them. While in the present wording, a likelihood function can be said to take 'all moments' into account, this concept presupposes that the model is correctly specified. If, on the other hand, it is recognized that a model (by definition) is an abstract approximation to reality, then a discussion of the choice of the moments underlying an estimation appears to be more transparent.[43]

The moments can, moreover, provide a useful diagnosis for a model. By finding out that some moments are relatively well matched and that others are not, one learns more about the merits and demerits of a model than knowing that an objective function (be it a likelihood or something else) has been optimized, and perhaps having a technical econometric measure to summarize the goodness-of-fit.

Franke (2012b, 2016) applies the method of simulated moments to several discrete-time versions of Franke's (2012a) 'Old-Keynesian' model of the macroeconomic consensus with its three observable variables: quarterly output, inflation and the interest rate (all of them as percentage deviations from trend). The empirical moments employed are the variances and cross-covariances of these variables in the US economy with lags up to eight quarters, which gives a total of 78 statistics. Their informational value is, however, lower since the moments are not independent.

One of the model versions considered is a deterministic system; one has stochastic demand, cost push and monetary policy shocks added in the corresponding equations; a third version additionally allows two of them to take effect in the switching index and the adjustments of the inflation climate. Thus nine parameters are to be estimated in the first and fourteen parameters in the third version. Such a large number relative to the number of 'effective' moments requires great care in the minimization of the objective function that measures the distance between simulated and empirical moments, because it will typically have multiple local minima or extended valleys. The latter phenomenon will also deteriorate the precision of the parameter estimates (unless one chooses to fix some of them on the basis of other arguments or priorities). However, these problems would apply equally well to likelihood methods.

With respect to the so-called period of the Great Moderation (1982 – 2007), the thin (blue) solid line in Figure 4 illustrates that already the deterministic model achieves a respectable matching of the covariance profiles (recall that lags higher than eight quarters are not included in the estimation). The shaded areas indicate that the great majority of the model-generated moments lie inside the 95% confidence band of the empirical moments. The match is even better in the full stochastic version; see the bold (red) solid lines in the diagram.[44] However, a certain price must be paid for this improvement: the influence of the stochastic dynamics becomes so strong that the herding coefficient in the switching index (the analogue of ϕ_x in (18)) is relatively low and would imply stability of the steady state in the

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[43] It sometimes seems that the following fact is well-known, but is subsequently merely put aside: "maximum likelihood does the 'right' efficient thing if the model is true. It does not necessarily do the 'reasonable' thing for 'approximate' models" (Cochrane, 2001, p. 293).

[44] This estimation adds three further moments to the aforementioned 78 second moments. They characterize the raggedness in the time series of the output gap and the rates of inflation and interest.

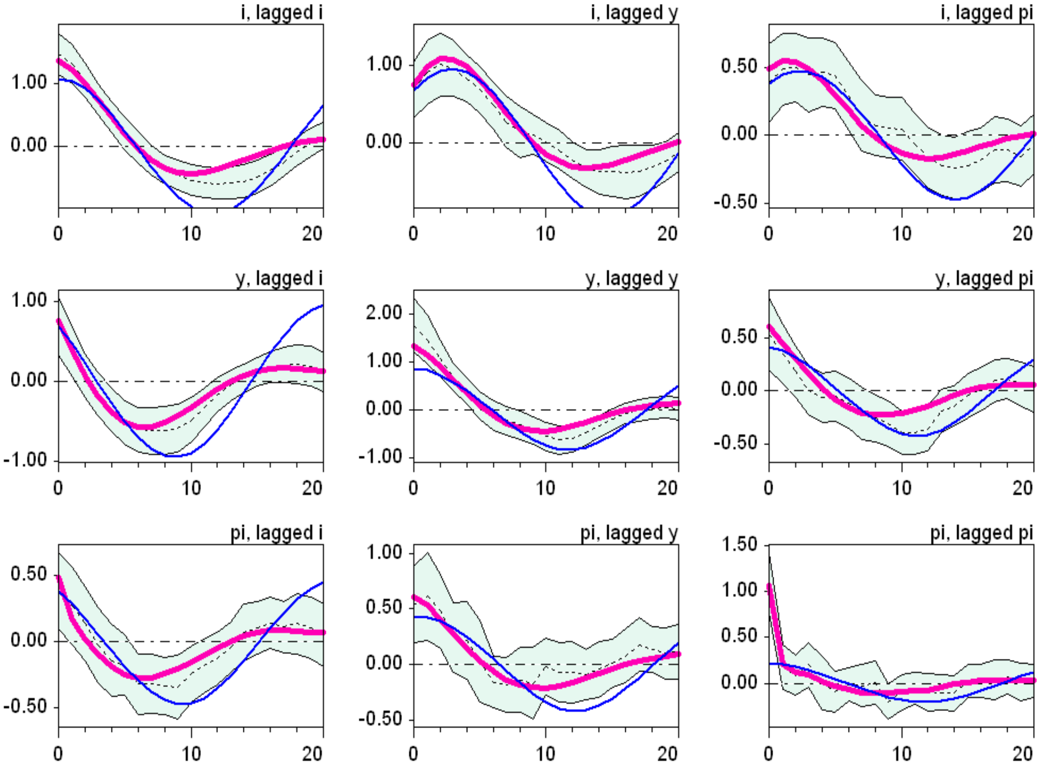

Figure 4: Auto- and cross-covariance profiles of the estimated "Old-Keynesian" model in Franke (2012b).

Note: y, pi, i designate the gaps of output, inflation and the interest rate, respectively. Shaded areas are the bootstrapped 95% confidence bands around the empirical moments (dotted lines). Other lines are explained in the text.

deterministic core of the model. In other words, it is the random shocks that keep the system in motion. Regarding the period of the Great Inflation (1960 – 1979), the coefficient is even close to zero.

In short, the model achieves a good or almost excellent matching of the chosen moments, which can also be quantitatively established in econometric terms. On the other hand, this result does not confirm the endogeneity of the cyclical behaviour of the economy. With the experience of the single-equation estimations above, one may now wonder about the possible presence of the so-called moving-flock effect, a question that has not yet been addressed in any master or PhD thesis.

An additional matter of concern in Franke's (2012b, 2016) contribution is the competitiveness of his model *versus* a corresponding (hybrid) New-Keynesian model (eqs (9) – (11) with rational expectations in place of the boundedly rational expectations E^{agg}). Broadly speaking, both models are found to be equally successful at reproducing the autocovariances of their three state variables. If one is more ambitious, however, it has to be noted that the autocovariances and the raggedness of the inflation rate can hardly be reconciled in the New-Keynesian model: either it produces a good match of the former and a bad match of the latter, or *vice versa*. In the Old-Keynesian model, by contrast, the two types of moments are largely compatible, that is, a good match of one type may exist alongside at least an acceptable match of the other type. In this sense, the Old-Keynesian model may claim to do a better job.

Using Euro Area data and the same 78 second moments as Franke, the authors Jang and Sacht (2016a) estimate De Grauwe's model (with shocks added) by the method of simulated moments. Regarding the coefficients in the discrete choice switching mechanism, they argue in favour of exogenously fixing a moderate intensity of choice $\beta = 1$ in (13) and a zero memory coefficient $\rho = 0$ in (14).[45] They furthermore provide detailed information about several other parameters. Especially the estimates of the monetary policy coefficients make good economic sense. At least to the naked eye, the matching itself is of a similar quality as in Figure 4, and slightly superior to the the matching of the model's rational expectations version.[46]

Jang and Sacht (2016b) continue this line of research. In this article, they consider four forecast rules (two of which with slightly more 'momentum' than before) and alternatively include two, three or all four in the model. Together with the rational expectations, they arrange a horse race for six model versions. With respect to US data and the Euro Area data, it is found in both cases that, when matching the same 78 moments as before, the interaction between the two new (and more dynamic) rules is almost as successful as all four rules together. In fact, insignificant values for the parameters characterizing the other two rules indicate that these are virtually ineffective. This finding supplies new knowledge in the search for efficient and parsimonious specifications within the De Grauwe framework. Besides,

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[45] Setting $\beta = 1$ is significantly superior to higher values such as 10 or 100, and insignificantly inferior to $\beta = 0.10$. The latter would, however, undermine the philosophy of the model as the decision between the two forecast rules is then close to tossing a coin.

[46] Referring to the J-test, the authors conclude that they cannot reject the null hypothesis according to which the model-generated moments may also have been obtained from the real-world data generation process. Such an argument presupposes an optimal weighting matrix in setting up the objective moment distance function, whereas, as mentioned by the authors (p. 89), this assumption is not satisfied in their treatment. Nevertheless, this observation does not necessarily mean that their conclusion is wrong, only that other methods have to be tried to address this issue; for example, repeated re-estimations of the model as discussed in Franke (2012b) and Brenneisen (2015) (see also below).

the rational expectations perform the worst, which is explained by the assumption of ruling out the lagged variables (i.e. the hybrid case) in eqs (9) and (10).

Daring to provide a brief summary of these results, at least from a higher point of view and as a preliminary conclusion, it seems that the model approaches by Franke and De Grauwe receive similarly good and satisfactory empirical support from estimations by the method of simulated moments.

Finally, Brenneisen (2015) starts out from a recent paper by Liu and Minford (2014), who compare a purely forward-looking New-Keynesian model to a (very) simple version of De Grauwe's model. Estimating them by indirect inference, these authors conclude that the former model outperforms the latter. Brenneisen points out a number of unclear points in Liu and Minford's presentation and then, instead, applies the method of simulated moments to the two models—whereby he arrives at the reverse conclusion. A more differentiated picture is obtained with respect to moderate extensions of the models.

Very informative in Brenneisen's contribution is a methodological issue regarding the precision of the parameter estimates. Distributions of the parameters can be computed by repeatedly re-estimating a model on the bootstrapped empirical moments, which is a sound but extremely time-consuming procedure. The author successfully tests a new econometric proposal for approximating these distributions by simulations of another random distribution, which can do without the re-estimations completely. This alternative device is relatively easy to implement and could prove useful in future applications of the method of simulated moments.

7 Conclusion

With respect to macroeconomics, this survey was concerned with recent attempts to translate aspects of the famous notion of the animal spirits into formal and rigorous modelling. To cope with the 'wilderness of bounded rationality', two approaches with a stylized microfoundation were addressed: the discrete choice approach and the transition probability approach. Before discussing their applications in the literature, it was revealed that they are more closely related than it might seem at first glance, and that they are well suited to endogenously generate persistent cyclical behaviour.

The literature presented here shows a considerable flexibility in building macrodynamic models from these tools, or incorporating them into existing models. As a first guideline for finding one's around the many models, a distinction was offered between animal spirits in a weak sense and in a strong sense. The first expression merely points to a rush toward one of two attitudes, strategies or similar alternatives;

the second involves an element of herding because, in a world of irreducible uncertainty, people think the majority knows better. The first variety is typically found in models that work with the discrete choice approach. The second can primarily be classified in models applying the transition probability approach. However, this role allocation is based more on historical reasons rather than on a compelling inner logic.

Besides being theoretically attractive, the two approaches were also shown to have some empirical support. Overall, the material and ideas discussed can provide fruitful stimulations for future research that seeks to capture the formation of business or consumer sentiment, or of a general state of confidence, in a serious and structured way. Maybe this survey comes at a good time, before further modelling work in this direction abounds.

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