Make or Buy: Exploiting the value-added chain for a gainful division of labor between North and South

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Summary

1. **Introduction**: Fragmentation, globalization, and the role of value-added chains (v-a-c) in economics.

2. **The modelling of v-a-cs**: Results accounting with a broken v-a-c, gains from vertical trading in an endowment model, alternative geometric methods of v-a-c modelling, Pareto efficiency in v-a-cs.

3. **Fragmentation of a linear v-a-c**: Trade gains and hinterland effects in a sliced linear v-a-c, imputed material price in a linear v-a-c, a theorem on out (global) sourcing.

4. **Convexity in v-a-c models**: General convexity vs. mixed technologies in v-a-cs, vertical trading and trade gains in a convex v-a-c, the interaction of factor markets with flexible technologies, trade and wages with a broken v-a-c, output and value-added productivity and factoral t-o-t, how to model short-term operating credit.

5. **An almost Ricardian v-a-c model with world-wide production**: The North (center) with mixed technologies as a material importer, the material-exporting South (periphery) with mixed technologies, a Pareto-efficient world trading system integrating North and South in world-wide (shared) production.

6. **The vertical North-South model with convex v-a-cs**: World supply of material, efficiency in vertical world trading, trade gains from opening to vertical trade and specialization, some results from considering the world trading system: variation in the domestic resource base (labor supply), variation in the supply of the specific factor in the input tier (land), i.e., technical progress in material production.

7. **Conclusions and qualifications**

I am indebted to R. Jones with whom I discussed the theory of input (vertical) trade many years ago in Rochester and in recent years in Konstanz, München and Geneva. He made accessible to me an early draft of “Globalization and the Theory of Input Trade”, MIT Press 2000. Many thanks to W. Meinig and H. Mallad from FAW for sharing their most valuable insights and enthusiasm into the v-a-c of the German Automobile Industry. My understanding of make or buy in the car-making business was sharpened, too, by Ph. C. Nell. I also acknowledge both very useful discussion and just talk on the topic of this paper and help by H. Schmid. I blame only myself for the remaining errors.
1. **Introduction**

This paper attempts an evaluation of chances and consequences from the fragmentation or, alternatively, the slicing of the national value-added chain (v-a-c). During the recent wave of globalization we have increasingly observed international or shared production schemes, where national v-a-cs are partially dismembered and put together or reengineered using chain links located in foreign countries. This way the new v-a-c mutated to become international or transborder. We are less interested here why this happens, but we would like to present a class of minimal models where this can be observed, and where we can study the consequences. To get the full picture we analyse a world-trading system with vertical trading between two countries. In particular the v-a-c of the South (periphery) is reengineered to fuse with the broken v-a-c of the North (center), because we want to demonstrate that a vertical division of labor is gainful to both the North and the South. The basic incentive to do so is the saving of resources or, put in the jargon of business economists, the cost-cutting effects of outsourcing (global sourcing) strategies. While this appears just right from the view of the theory of optimal factor allocation or, alternatively, the improvement of shareholder value, the more general economic consequences of this sort of economizing have to be considered as well. The paper addresses three problems: (i) the distribution of vertical trade gains between countries, (ii) the internal distribution of the trade gain within a participating country, (iii) the fragility of increased interdependency. The first topic has a long tradition in trade theory but in our context there are the gains of vertical trade between final goods and raw materials or intermediates of special interest which, in our view, are not given enough theoretical treatment. In the field of international political economics the economic relations between DCs and LDCs are fraught with long-standing scenarios of opposing schools which argue that either the North or the South gains at the cost of the other partner. We suggest instead that North and South should try to exploit the value-added-chain, i.e. should try to find a useful place within an internationally organised v-a-c. We like to demonstrate this by using well-known analytical tools of the theory of international trade and applying them to v-a-c trading.

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1 A prototype model of vertical trading which was developed within a monetary-approach framework is in Schmid (1976).
2 In business economics there exists a long debate about the optimal vertical depth of a v-a-c within a company (optimale Produktionsstiefe). There the make-or-buy decision is about core competence and how it affects performance. More recently not just the pure cost-cutting effects are discussed but the broader approach of institutional economics involving transaction costs has become useful to obtain a more complete picture of the out(global)sourcing decision. We could call this the trickle-down effect if there exists one.
3 It could be as well vertical trade between intermediates of different stages in a v-a-c.
4 There are some notable exceptions, namely Jones (2000, chapt. 7), Jones-Kierzkows (1999), Deardoff (1998), and Dixit-Grossman (1982).
6 There are isomorphisms in the geometry or algebra of v-a-c-trading models and more traditional trading models. These can be used but should not obscure the different economic thinking of input trade in particular.
We try to cope with the second topic not only by employing the assumption of a representative household, we rather look at households as the owners of unevenly distributed factor endowments. Then it is possible to study the fate of winners and losers from a strategy of globalization and to restate the most important theorem of trade theory: After adding up gains and losses the economy as a whole must gain from globalization. Within the simple v-a-c framework we here propose there are only landlords sitting in the input tier and workers being mobile between both stages of production. It is important to note that in our v-a-c framework with only one final good we do not run into problems of interpersonal utility comparisons.

Thirdly, within a v-a-c trading model international trade becomes more sophisticated in its impact on the rest of the economy. Thus we can show that labor productivity is directly and profoundly influenced by the factoral t-o-t. As wages must follow productivity in the long run, wages and the rental rate of capital are influenced directly by input trade, i.e., commodity or raw-material trade. Since today the factoral t-o-t normally are beyond the control of national policy makers there is an added degree of fragility to the texture of an open economy.

A detailed plan of the paper is given in our summary. Putting the value-added chain at the center of our modelling effort in Section 2 we describe a class of models we would like to name the minimal v-a-c model of the open economy. We design a 1(final good) x 1(primary factor) x 1(intermediate) national v-a-c and discuss the gains from vertical v-a-c trading if this v-a-c is fragmented in section 3. We also derive a basic theorem on global sourcing and discuss the magnification of trade gains via the hinterland effect. Section 4 analyses a class of v-a-c models with different technologies (smoothness) in the input and output tier and presents the general case of a convex v-a-c model. Sections 5 and 6 explore the idea of reengineering v-a-cs in two-country models of North (center) and South (periphery). Section 5 presents an almost Ricardian framework where complete specialization occurs in material production between North and South. Section 6 presents the recombination of v-a-cs in a general model with flexible production. We discuss Pareto-efficiency and distribution of trade gains between labor and land owners (oligarchs) within each country. Section 7 concludes with a perspective on v-a-c modelling in trade theory under full employment and more general in Keynesian macroeconomics.

2. Modelling the value-added chain (v-a-c)

A very simple setting in which we can analyse the importance of v-a-c production and vertical trading with imported pure intermediates (material) is illustrated in Figure 1. The open economy is completely specialized in the production of commodity Q which can be domestically absorbed or exported. The production of Q is achieved in a two-stage value-added chain using only domestic

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8 It would be more realistic to have labor and capital used in both stages of production as in Batra’s (1973) discussion of pure raw materials or Dixit-Grossman (1982) continuum of stages model.
9 The recent book by J. Stiglitz (2002) strongly recommends that we should give up designing and reforming economic systems with just one-dimensional objective functions in favor of multi-purpose designs or strategies.
10 This input-trade channel is very different from the popular current debate on trade and income disparities. While this debate seems to exonerate trade, the relation between input trade and wages is much more serious from the perspective of our v-a-c models.
labor, L, which is trapped at home, and land, T, which is specific to the material-producing input tier. In addition to the local production of material the country may import some material which is paid for by exporting some entities of the final good. This minimal 1x1x1 structure combines the ideas of both vertical trading and a sliced value-added chain. To confine ourselves to the minimalism of one-ness, note, that we employ only domestic labor as a primary factor of production and no capital; thus we move within the range of the labor-value theory of commodity pricing.

The algebraic formulation of a v-a-c model must begin with a bit of national accounting slightly adjusted to a fragmented value-added chain. In Figure 2a we illustrate the process of aggregation showing the following statement.

Real income of national (domestic) factors of production, Y, is less (more) than output, Q, if material is imported (exported) in an economy with a broken value-added chain. This is a general property of any shared production process.

\[ Y = Q (\cdot) - p_N \left[ N - Q (\cdot) \right] \]  \hspace{1cm} (1)

In (1) we introduce the two standard linear-homogeneous neoclassical production functions of the processing sector and the primary production sector. \( L_N, L_0 \) are labor inputs and \( N \) is the material input in the output tier. \( p_N \) represents the real material price.

Output tier:
\[ Q = F \left( L_Q, N \right) \]  \hspace{1cm} (2)

Input tier:
\[ Q_N = G \left( L_N, T \right) \]

If we define both the real wage, \( w \), and the real land rental rate, \( r_T \), in terms of final goods, we can apply Euler’s theorem to (1) to obtain domestic income as it accrues to domestic factors of production.

\[ Y = w \left( L_Q + L_N \right) + r_T \bar{T} = wL + r_T \bar{T} \]  \hspace{1cm} (3)

From Figure 2a it is obvious, too, that the v-a-c model determines not only the difference between output and value added but also the sectoral shares of value added created at the input and output

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11 The structure may serve as a starting point for many useful extensions of which only two are mentioned here; (i) the structure of a raw-material exporter is obtained (and will be discussed later) if we reverse the trading pattern; (ii) if we add the import of a second final good. We obtain (a) hybrid Mundell-Fleming structure as in Schmid (2001) in a model with a Keynesian modus operandi.

12 In textbooks of macroeconomics and international economics nearly all national-accounting procedures assume a closed v-a-c where the value of output equals the value of national income. In reality of course, national accounting never does so.
tier, respectively, i.e., $\bar{Y}_N$ and $\bar{Y}_Q$. Note that the accounts in Figure 2a illustrates the v-a-c of a material-importing economy in nominal terms, whereas accounts in Figure 2b presents the v-a-c of a material exporter in real, i.e., final-goods units, $Y_N$, $Y_Q$.

2.1 A preliminary consideration: Vertical trade in an endowment model

As a preliminary step it is useful to discuss the idea of vertical trading in a stripped-down version of a model with intermediates where the input tier does not produce material from other inputs, like labor and land, but appears just as a material endowment $\bar{N}$. In this setting the resource base of the economy appears as point $E$ in Figure 3. The coordinates of $E$ are the endowments of material, $\bar{N}$, and domestic labor, $\bar{L}$. The isoquants in $N$, $L$ space illustrate the processing technology $F(\cdot)$. In autarchy the economy reaches the isoquant $Q^* = Y^*$ just by utilizing the material and labor endowments $E$ at the implicit factor-price relation $tg_f = (W/P_N)^f = w^f/p_N^f$. Suppose the world market offers a lower material price, $p_N^f < p_N^s$, then the economy would like to substitute domestic labor for the cheaper material input. At full employment of domestic labor this is viable at point $P$ if an amount of $Z = N - \bar{N} > 0$ of material is imported. The ingenuity of the diagram which we have adopted from H.G. Johnson’s discussion of labor trade consists in a geometric representation of the gains from vertical trading in material (commodities) and final goods. At world-market prices $tg_f$ the isoquant through point $P$ represents the level of produced output. This output, however, has been manufactured in a shared production process where domestically and foreign-owned factors of production have been utilized. Namely, an amount of material, $Z = N - \bar{N} > 0$, was imported for which the domestic economy has to pay for, $p_N [N - \bar{N}]$. Only after paying the foreign factor owners for their contribution to this shared (world-wide) production scheme we obtain the payments for the domestic factors which are equivalent to domestic income (or national value added). The output equivalent to domestic income is represented by the isoquant $Y^f$ which is the isoquant in point $C$ tangent at the isocost line running through point $E$. The difference between $Q^f$ and $Y^f$ equals the export of final goods necessary to pay for the imported material at balanced trade. Obviously, for the economy it is useful to trade vertically as we reach a higher isoquant $Y^f$ in free trade than in autarchy, i.e., $Y^f > Y^*$. Moreover, the trade gain rises, the more material is imported at even lower world-market prices for material. We can

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13 The two-country endowment model of vertical trading is discussed more extensively in Schmid (1999).
14 In Schmid (1976), (1985), (1999), (2001) the diagram has been repeatedly adopted for an analysis of factor trade: raw materials like oil, intermediates or primary factors.
15 In Schmid (1976), (1985), (1999), (2001) the diagram has been repeatedly adopted for an analysis of factor trade: raw materials like oil, intermediates or primary factors.
16 The original diagram in Figure 3 is attributed to H.G. Johnson in Caves-Frankel-Jones (1990, 5th ed., p. 196) in their discussion of mobile labor showing labor and capital at the axes.
demonstrate the importance of this process by writing the output version of the goods market taken from inspection of Figure 1.

\[ Q = C + I + X \]  

(4)

where \( C + I \) is the domestic absorption and \( X \) is the final-goods exports. If in (4) we subtract the value of imported material from both sides we obtain

\[ Q - p_N \left[ N - \bar{N} \right] = C + I + X - p_N \left[ N - \bar{N} \right] \]

(5)

and rewriting (5) by using (1) yields the income version of the goods-market equilibrium.

\[ Y = C + I + X - p_N \left[ N - \bar{N} \right] \]

(6)

It is obvious from (6) that at balanced trade income must equal absorption. It follows that absorption, i.e., welfare today and (via investment) tomorrow must rise one-to-one if \( Y \) can be increased at the supply side by vertical trading. Recalling that value added can be expressed from the view of income accruing to domestic factors of production

\[ Y = w \bar{L} + p_N \bar{N} \]

(7)

the important part of our basic insight on vertical trade gains can be seen from (7). Given the factor endowments \( \bar{L} \) and \( \bar{N} \), \( Y \) must unambiguously rise if \( p_N \) declines, thus the real wage must have increased. Therefore \( Y \) unambiguously increases although \( p_N \) and \( w \) move in opposite directions.\(^{17}\)

The increase in \( Y \) is completely due to vertical trade gains and therefore constitutes a channel of events not covered by the more traditional trade theory without input trade. It is a consequence of factor trade only. With respect to internal income distribution it is apparent that the owners of material (commodities) lose and the wage earners gain if the economy opens to vertical trade and if we give up the representative household agent.\(^{18}\) As usual, the gains of the winning group are larger than the losses of the losing group. These results are very robust as they are obtained without any recourse to utility considerations. They are derived solely from the productive system at the supply side of the economy. The importance of these results suggests another derivation given in Figures 4a and 4b. This derivation of vertical trade gains in due to Schmid (1985) and (1999). Figure 4a portrays the trade triangle of vertical trading in Q, N space. In autarchy the economy just employs \( \bar{N} \)

\(^{17}\) The rise in \( w \) is independently assured by the factor-price frontier in the processing tier.

\(^{18}\) Note that a sharp distinction has to be made between the welfare effects of trade opening and the comparative statics of a world trading system.
at the autarchic real-material price $p^*_N$. This situation would be captured by point A. At the lower world-market price $p^*_N < p^*_D$, the economy produces at point C by importing material DB. At balanced trade it must give up BC in output units to pay for imported material, but after this deduction the output left for absorption is given by point D. Obviously, the trade gains appear as the distance AD. Without further discussion the reader is referred to Figure 4b where the vertical-trade gains appear as the area e because the winners gain more, i.e., $b+e$ than the losses of the losers b.\footnote{Figure 4b is an adaptation of the more familiar diagram representing labor trade or capital mobility. The geometric isomorphism is due to the fact that we are dealing with “input trade”.} Without further discussion the reader can reconstruct the situation in Figures 5a, b for the case of a material-exporting economy.

### 2.2. Alternatives to modelling a v-a-c.

Given the simple v-a-c structure illustrated in Figure 1 and Figure 2 there are many different ways to geometrically and algebraically capture this class of models.\footnote{Jones (2000, chapt. 5) covers another class of v-a-c models where trade is vertical in the sense that one raw material is exported at a lower stage against another material imported at a higher stage. This vertical trade involves middle products only, i.e., we have vertical intermediate trade. We prefer, however, our own structure, because most DC economies make a living by importing something at a low stage of the v-a-c and by exporting something at the high end, i.e., the final good.} As portrayed in Figure 6 the choice is between fixed coefficients and smooth technology at the input or output tier, respectively. Following Schmid (1985) we speak of a linear (convex) v-a-c if the economy has completely fixed (flexible) coefficients in both tiers. These extreme cases are shown in the top and the bottom diagram in Figure 6. We speak of mixed technologies if substitution is allowed in the output tier ($M_{T_0}$ model) or input tier ($M_{T_1}$ model) only. In Figure 6, all v-a-c models capture the highly realistic idea of final goods being created in at least two stages of production. Furthermore, we assume an endowment of one primary factor, $L$, which must be used in both stages. Starting from point E looking in a leftward direction this endowment is used to produce the material output (middle product) according to a production function which is shown hatched and may or may not exhibit decreasing returns.\footnote{If we assume land as a specific factor in the input tier the decreasing-return property follows quite naturally.} The material output, $Q^*$, is given at the vertical axis. It is utilized in the output tier, thus the isoquants $Q^* = Y^*$ illustrate the production of the final goods in the output tier. With respect to the origin, the axis show the inputs of material N and the primary factor $L_{Q}$. As has been said before, the processing technology may or may not exhibit smooth production. Obviously, the v-a-c equilibrium is portrayed at point A. In this equilibrium the labor force is allocated between the two stages of production and in autarchy the produced material is completely utilized in processing. Thus point A represents a closed v-a-c which at the same time turns out to be an efficient v-a-c.\footnote{It is well-known that in reality most v-a-cs are complicated networks where self-interested links of the chain fight for their individual advantage, although they have to rely upon each other. V-a-c management (Wertschöpfungspartnerschaft) turns out to be the secret behind many success stories in modern economies.} The v-a-c could produce anywhere in input space but only point A is productive-efficient. We already know that in the present simple setting productive efficiency is both necessary and sufficient for consumptive
efficiency. Thus point A represents overall efficiency of our economy. It is also important to realize that the closed v-a-c determines a shadow value for the real material price $p_N = p^*_N$ and a shadow value for the real wage (in output units).\textsuperscript{23} These shadow factor prices cannot be seen in Figure 6 directly, where only the relative factor price $t_g = w / p_N$ appears. Next we turn to the calculation of these imputed factor prices in a linear v-a-c.

3. **Fragmentation of a linear v-a-c**

The case where technology is linear in both tiers of the economy allows an algebraic sketch of the autarchic equilibrium. Prices are determined by the two factor-price frontiers.

\begin{align*}
\text{Output tier:} & \quad 1 = a_L w + a_N p_N \quad (8) \\
\text{Input tier:} & \quad p_N = a_{LN} w + a_{TN} r_T \quad (9)
\end{align*}

We assume in (9) that land being necessary to produce material receives the real land rent $r_T$.\textsuperscript{24} The right-hand diagram in Figure 7 demonstrates how we can find the real factor prices $w$, $p_N$ from the two factor-price frontiers. The solutions of (8) and (9) are

\begin{align*}
w^n &= \frac{1}{a_N a_{LN} + a_L} \quad \text{and} \quad p^*_N = \frac{a_{LN}}{a_N a_{LN} + a_L} \quad (10)
\end{align*}

The left-hand diagram in Figure 7 repeats the geometry of the general equilibrium of a linear v-a-c from Figure 6. From (10) it follows that in autarchy the relative factor price must coincide with the slope of the input-tier production function $tg_\alpha$.

\begin{align*}
\frac{w}{p_N} = \frac{w^*}{p^*_N} = 1 / a_{LN} = tg_\alpha
\end{align*}

Thus in autarchy costs are covered in the input tier without profits or losses.

The interpretation of our algebraic result is straightforward.

1. The real shadow price of material (imputed price) is determined by the ratio of directly and indirectly utilized labor requirements.\textsuperscript{25}

\textsuperscript{23} The reader should realize that any inefficient organization of the v-a-c must severely damage the labor productivity of our economy.
\textsuperscript{24} The symmetry of the two production tiers commands the presence of land in the input tier apart from a touch of realism if the input tier is interpreted as a raw-material producing sector where natural resources are extracted from soil. If we put $T = 0$ a strict marginal productivity rule holds in autarchy, $p_N = a_{LN} w$, and profits or losses appear only when trade takes place. We assume in autarchy $r_T = 0$. 


(2) The real wage equals the indirect labor productivity expressing the simple principle that the closed v-a-c as a whole determines the real wage of our economy.\textsuperscript{26} In conclusion, the imputed real material price is in accordance with what a business company would obtain from a properly conducted results accounting.

At the heart of the make-or-buy decision of a company is the comparison between the imputed material price, derived from the internal production process, and the outside market price for material. The outsourcing decision follows a simple rule that buying from the market is gainful if it can take place at the same quality and reliability at a price lower than resulting from the internal production process. Thus if the world market offers a lower material price than obtained in autarchy, \( p_s^i < p_s \), our economy should gain if it out(glo)alsources, material production, i.e., if it shuts off the input tier and imports the necessary material from the world market and relocates the domestic labor resources completely into the output tier. In what follows, we will test this basic idea, i.e., we prove a basic theorem on global sourcing.

The proof can be given first algebraically considering the quantity equations in Figure 8 which are repeated here for convenience.

We assume a flexible domestic labor market working under full employment.

\[
\text{Labor market: } Q_N + L_N = L \tag{11}
\]

The material market is integrated into the world-commodity market where the economy procures any amount it wishes to import, \( Z > 0 \), at the given world-market price for material, \( p_s^i < p_s \).

\[
\text{Material market: } N - Q_N = Z > 0 \tag{12}
\]

Using fixed-coefficients production functions in the input and output tier we can solve from (11), (12) for final-goods output, \( Q \), and material output, \( Q_N \).

\[
Q = \frac{L + a_L N Z}{\Delta} \quad Q_N = \frac{a_N L - a_L Z}{\Delta} \tag{13}
\]

with \( \Delta = a_N a_{LN} + a_L > 0 \) and \( Z \geq 0 \)

\textsuperscript{25} Obviously, this simple pricing rule is due to our strict labor-value theory excluding capital as a second primary factor of production.

\textsuperscript{26} We will see later, how the real wage is determined in a fragmented v-a-c.
Using (10) in (13) we can express \( Q \), showing the autarchic factor prices.

\[
Q = w \bar{L} + p_N Z \quad \text{with } Z \geq 0
\]  

(14)

The economic interpretation of (13) is clear and simple: Given the labor supply \( \bar{L} \), the import of material raises (lowers) \( Q \) (\( Q_N \)) because the domestic production of material is substituted by imported material and labor resources are shifted from the input into the output tier. If we are interested in the well-being of our economy the solution for \( Y \), however, is more important than \( Q \). Recall that national income (value added) is smaller than output in a shared production process, because we have to pay the foreigners first.

\[
Y = Q - p_N Z
\]

Substituting from (14) we get

\[
Y = \frac{\bar{L}}{\Delta} + \left[ \frac{a_{LN}}{\Delta} - p_N^* \right] Z
\]  

(15)

with \( \frac{a_{LN}}{\Delta} = p_N^* \) and \( Z \geq 0 \)

Equation (15) proves our basic theorem on out(global)sourcing:

Global-sourcing increases income (value added) if and only if \( p_N^f < p_N^* \). If \( Z \) rose at autarchic prices, i.e., \( p_N^f = p_N^* \), the structure of the economy would be changed but value added remained constant. The magnification of \( Y \) is equivalent to the vertical trade gains which can be separated in the familiar way: (i) There is a terms-of-trade effect which raises \( Y \) if and only if initially \( Z > 0 \). (ii) If \( p_N^f < p_N^* \) initially, specialization (structural adjustment) magnifies \( Y \) even more until the input tier is completely wiped out.

By inspection of the left-hand diagram in Figure 7 we illustrate geometrically our theorem in global-sourcing. (1) If we lower the relative factor price below the autarchic level, i.e., \( \gamma > \gamma_0 \), then national income does not rise if this has been done in autarchy. National income, however, rises if the same change occurs with trade taking place initially. Furthermore, from Figure 7 it can be seen, that any importing at autarchic prices does change the structure of the economy but does not create any increase in value added.

The complete specialization result is a reflex of our fixed-coefficients assumption. It is obvious from the left-hand diagram in Figure 7 that opening to vertical trade creates losses in the input tier or, put differently, the rental to land owners becomes negative and, therefore, they will close down their
operation. Thus the best adjustment in a linear v-a-c is to shift all labor resources to the output tier
and to produce output at a level $Q'$ in point C. This creates income $Y'$ at point B which clearly is
larger than income $Y^*=Q^*$ in autarchy at point A.\textsuperscript{27}

This is proof of the basic insight that slicing a national value-added chain must be improving welfare
of the economy as a whole. If we assume there are land owners and workers, however, the former
might lose by the opening to trade and the latter might gain. As usual, the increase of wage earners,
however, would be larger than the loss of the land owners.

4. Convexity in v-a-c models

In this Section we investigate v-a-c trading under the assumption that production is smooth in at least
one production tier. We turn first to a model with mixed technologies.

4.1 Mixed technologies: The MT\textsubscript{0} model

We know from Figure 6 that there are two variants of the v-a-c model with mixed technologies. For
convenience we speak of the MT\textsubscript{0} model if the output tier has smooth production possibilities but
the input tier continues to produce with fixed coefficients. Figure 11 illustrates this version of the v-
a-c model. We discuss this intermediate case in between the linear and the following convex v-a-c
model because it shares some similarity and isomorphism with traditional trade models. Moreover,
within this framework we can address a phenomenon of input trade which was named hinterland
effect in Jones (2000, chapt. 3).

In Figure 11 point A portrays the autarchic situation. Let the world market offer a lower material
price, such that the country imports material while utilizing just the same amount of labor, $L_Q^*$, allocated in autarchy at the output tier.\textsuperscript{28} Then the economy would produce output $Q'$ at point P and
the gain of vertical trading appears in point C where the output isoquant $Y'$ is reached. In contrast to
the linear v-a-c model the factorial t-o-t effect now creates income and welfare even though we keep
the autarchic labor allocation.\textsuperscript{29} However, the economy could reach an even higher income isoquant
(not shown in Figure 11) if it was restructured, i.e., specialized by a reallocation of labor. If labor
was completely pulled out of the input tier and employed in the output tier, the economy would
produce the output at an isoquant (not shown) which is given by the intersection of the intensity ray
from the origin through P and the perpendicular line above E.

\textsuperscript{27} The reader may inspect Figure 9 to find a geometric illustration of the trade gains from importing material in a linear
v-a-c. Figure 10 covers the case of a material-exporting economy in a not further discussed way analogously to the case
of material import.

\textsuperscript{28} Thus we freeze the allocation of labor at point A.

\textsuperscript{29} This experiment is completely equivalent to the welfare (income) improving factorial t-o-t effect we found in the
endowment model of section 2.1.
If we draw a line with the slope \( \gamma \) through point E (not shown) we will reach the maximal possible income isoquant. In conclusion we observe that the total vertical trade gain can be split into a factorial t-o-t effect and a specialization effect (hinterland effect). This observation is similar to what is known from the standard Ricardian model although the economic context is very different.

4.2 The convex v-a-c model

A major drawback of our discussion in sections 3 and 4.1. was the assumption of the linear technology in the input tier because specialization is always complete under this assumption. Furthermore if land serves as a specific factor in the input tier it will be very convenient to introduce the standard neoclassical production function, \( Q_N = G(L_N; T) \), to describe production in the input tier, and this yields diminishing returns to labor.\(^{31}\) This property is captured in Figure 12 which illustrates the general convex v-a-c. The autarchic equilibrium is given at point A. If the relative factor price, \( (w/P_N) \), decreases because at a lower world-market price for material, \( (w/P_N)^f < (w/P_N)^a \), the slope of the isoquant curve becomes steeper, we observe production of final goods at point P. Since \( L \) is a non-traded factor of production, the fixed labor supply is just exhausted by the final goods production, \( Q^f \), at P and the material production \( Q^f_N \) at \( P_N \). Income (= absorption), however, is shown by the output isoquant \( Y^f \) at point C. Clearly the import of material, \( N^f - Q^f-N^f = Z > 0 \), makes this productive scheme viable and more important it is income (welfare) improving at the same time. We observe that the closed value-added chain has been sliced by vertical trade which implies input trade of material. On the other hand, the material import sets free some of the labor input in the input tier which is shifted to the output tier to produce \( Q^f \) at full employment.\(^{32}\) Obviously, the convex v-a-c exhibits incomplete specialization depending upon the exogenously given factorial t-o-t = \( 1/P_N \). The lower the real material price the better the t-o-t and the bigger are output and income under normal circumstances.\(^{33}\) The convex v-a-c is a minimal full-scale general equilibrium model worthwhile to be sketched in the following ten equations.\(^{34}\)

\(^{30}\)Jones (2000, chapt. 3.1 and 3.2) has coined the name hinterland effect for this restructuring (specialization) in connection with an influx of a mobile factor of production (footloose capital in Jones’s analysis). The name alludes to the tapping of the hinterland, i.e., the other region(s) in a country which employ nationally trapped factors of production after one sector (or an enclave) attracted mobile capital. We believe the new wording is useful to distinguish the restructuring (specialization) of an economy due to factor trade from the familiar specialization due to final-goods trade.

\(^{31}\)The mixed technology model MT\( _I \) in Figure 6 has this property while the output tier produces with fixed coefficients. To save space we do not explicitly discuss the MT\( _I \) model, where the allocation of labor is dominated by the input tier.

\(^{32}\)The full-employment assumption could be dropped easily along lines discussed in Schmid (2001) in connection with the macroeconomics of raw-material price shocks. As shown by Neary (1980) and Bruno-Sachs (1985) there is always a classical and a Keynesian modus operandi for the open economy. This holds for v-a-c- models, too, as shown Schmid (2001).

\(^{33}\)Figure 12 also illustrates the fundamental split of output into both payments to domestic and to foreign factors of production. This split is shown in labor units and it is clearly a consequence of material import in a shared production process.

\(^{34}\)If we are willing to assume CES production functions the algebra of the convex v-a-c covers the linear and mixed technology models, too.
**Output tier:**

- **factor-price frontier**
  \[ 0 = f(w, p_N) \]  
  \[ \text{(16)} \]

- **demand for labor**
  \[ L_Q = L_Q(w, N) \]  
  \[ \text{(17)} \]

- **demand for material**
  \[ N = N(p_N, L_Q) \]  
  \[ \text{(18)} \]

- **output supply**
  \[ Q = Q(w, N) \]  
  \[ \text{(19)} \]

**Input tier:**

- **factor-price frontier**
  \[ p_N = g(w, r_T) \]  
  \[ \text{(20)} \]

- **demand for labor**
  \[ L_N = L_N(w/p_N; T) \]  
  \[ \text{(21)} \]

- **output supply**
  \[ Q_N = Q_N(w/p_N; T) \]  
  \[ \text{(22)} \]

**Labor market:**

- **full employment**
  \[ L_Q(w, N) + L_N(w/p_N; T) = L \]  
  \[ \text{(23)} \]

**Material market:**

- **import market**
  \[ N(p_N, L_Q) - Q_N(w/p_N; T) = Z \]  
  \[ \text{(24)} \]

- **income** (value-added)
  \[ Y = Q(w, N) - p_N[N(\cdot) - Q_N(\cdot)] \]  
  \[ \text{(25)} \]
Eqs. (16) and (20) are the factor-price frontiers in the output and input tier. Eqs. (17), (18), and (21) state the factor-constrained factor-demand functions in the production of $Q$ and $Q_n$, respectively. However, $T$ is considered a specific factor in the output tier while $N$ is a variable determined within the system. Eqs. (19) and (22) follow from (17) and (21) as the corresponding output-supply functions in the output and input tier. Eq. (23) states the full-employment condition of labor which is vertically allocated across the two stages of production. While the labor market is closed the material market is open to free trade and represented in eq. (24). Eq. (24) serves to define the material import $Z$ as the difference between material used in the output tier and material produced in the input tier. Eq. (25) defines value-added in a broken v-a-c as the difference between output and the real value of imported material. In autarchy the v-a-c is closed and with $Z=0$ the system determines $p_n^*$ given $L$ and $T$. In an open economy with a broken v-a-c $p_n$ is exogenously determined by the world market and material flows in or out, respectively. The system determines the nine variables $w, r, L_n, L_Q, N, Q_n, Q, Z, Y$, with $p_n, L, T$ given exogenously.

The main variables can be determined from the following subsystem of four equations:

\[ L_0 \left( w, N \right) + L_n \left( w/p_n; T \right) = L \]  \hspace{1cm} (26)

\[ w = f \left( p_n \right) \]  \hspace{1cm} (27)

\[ L_Q = L_Q \left( w, N \right) \]  \hspace{1cm} (28)

\[ N \left( p_n, L_Q \right) - Q_n \left( w/p_n; T \right) = Z \]  \hspace{1cm} (29)

Given $p_n, L, T^{35}$, the endogenous variables $w, N, L_Q, Z$ can be determined. This system is recursive such that we can solve (26) and (27) for $w$ and $N$ as functions of $p_n$, i.e., $w = f(p_n)$ and $N = h(p_n)$.

Using these functions in (28) yields $L_Q = L_Q \left[ f \left( p_n \right), h \left( p_n \right) \right] = L \left( p_n \right)$. Using this reduced form in (29) together with (27) yields an expression by which $Z$ is related to $p_n$, i.e., $Z \left( p_n \right)$.

From the definition of value added we have

\[ Y = Q \left( p_n, h \left( p_n \right) \right) - p_n \left( Z \left( p_n \right) \right) \]  \hspace{1cm} (30)

and we can solve for $Y$ if we plug the reduced forms $h \left( p_n \right)$ and $Z \left( p_n \right)$ into (30).

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In what follows $L$ and $T$ are no longer mentioned as exogenous variables.

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\(^{35}\) In what follows $L$ and $T$ are no longer mentioned as exogenous variables.
Most of this algebraic solution process can be illustrated geometrically in Figure 13. The left-hand side of the upper panel illustrates the factor-price frontier of the output tier. The right-hand side portrays the vertical factor allocation of labor in the V-a-c using the standard diagram of the specific-factor model adapted to the special assumptions of our V-a-c model. In Figure 13 the lower panel shows the material market with an upward sloping supply curve for material and a downward sloping demand curve for material. In both panels the autarchic situation is illustrated by point A where the general equilibrium is satisfied if we interpret this situation as a four-tuple \((p_n^*, Q_n^* = N^*, L_1^*, L_2^*)\) where the parameters shown for the four market curves are in line with each other. Assume the world market offers material at a lower price, \(p_n^f < p_n^*,\) then the economy adjusts in two steps. First, we observe only the trade effect of an improved factoral t-o-t, and we are holding the factor allocation of labor constant. Second, we allow the hinterland effect, meaning the economy draws labor from the input tier in order to produce more output because at the improved t-o-t the export of final goods has become advantageous. Thus the hinterland effect magnifies the initial gain in value added (income) derived from the initial t-o-t improvement.

The splitting of the total adjustment into two parts is standard procedure in trade theory and can be illustrated in Figure 13. With a frozen factor allocation a decrease in \(p_n\) rises the real wage from \(w^*\) to \(w^f > w^*\) as can be seen from the factor-price frontier. The lower material price induces an increase in material demand at a given autarchic material supply. The higher material input lifts labor productivity in the output tier, shown at \(A'\) as a rightward shift of the labor-demand function of the output tier in the upper panel of Figure 13. The trade gain is represented by the hatched triangle in the lower panel.

In Figure 13 the specialization (hinterland effect) appears as a movement from point \(A'\) towards \(A''.\) The reallocation of labor is dominated by the input tier which lays off labor because of the rising real wage in material units, \(W / P_n\). In the upper panel this appears as a downward shift of the marginal product curve \(G_1^* (\cdot)\). At the given \(w^f\) level of the real wage the \(F_1^* (\cdot)\) curve of the output tier must shift further to the right until it passes through \(A''\). This secondary shift indicates the rise in labor productivity in the output tier by using more material which must be imported when the input tier reduces material production. The hinterland effect is illustrated, too, by the rightward and leftward shifts of the \(F_n^* (\cdot)\) and \(Q_n (\cdot)\) functions, respectively, in the lower panel of Figure 13. If \(p_n^f\) keeps falling starting with \(p_n^*\) it is possible to imagine a flatter demand-for-material curve and a flatter supply-of-material curve originating in \(A\). These lines (not shown) describe the hinterland effect comprehensively. Created by the new flatter demand-for-material curve, the world market price \(p_n^f\), and the perpendicular line below \(A\) the size of that triangle measures the rise in national net gain or income because labor employed in both tiers gains more than landlords lose.

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36 A similar procedure of "solving" a model with input trade can be found in Brecher-Findlay (1983). They investigate the standard specific-factors model with mobile capital.
5. **An almost Ricardian v-a-c model with international production**

As we intend to model the phenomenon of geographic separation of production activities in the manufacturing of a final good (service) across two countries, we have to combine our raw-material importing economy with the raw-material exporter discussed briefly in Figures 2b and 5a, b. This world of an internationally shared production process is represented in Figure 14 where the endowments of land are explicitly shown by T and T*. We will be discussing next the effects afflicting the two countries if the North (center) decides to outsource some or all activity in its input tier into the South (periphery) and the South accepts this business opportunity.

Again it is useful to start with a short digression by considering a one-final-good two-country endowment model in Figure 15.\(^{37}\) Point E represents the fixed endowment of the North (South) with respect to origin O (O\(^*\)). The North (South) has initially a lower (higher) endowment of material while labor is assumed equally distributed among countries. For simplicity the technology is considered to be identical between North and South. If the production is assumed neoclassical with constant returns to scale any point within the world-factor box is a possible representation of the world production of the final good.

However, only points of the allocation of world-factor supply on the diagonal OO\(^*\) are Pareto-efficient. The diagonal OO\(^*\) represents the contract curve along which the wage-material price is invariant due to identical technologies. Evidently endowment point E is neither a point of efficient world production nor of efficient world income. World production of final goods is clearly higher at P if we assume that labor is trapped behind national borders but material can flow from the periphery to the center. The maximized world production at P must equal world income but the distribution of world income between countries is shown at point C. The distribution of world income depends upon the countries’ initial endowment valued at the free-trade factor prices\(^{38}\) shown by \(tg_y = W/P_n\).

Figure 15 combines just the H.G. Johnson diagrams from Figures 3 and 5a. The trade gains from vertical trading are evident because point C always lies within the autarchic lens created by the endowment point E. Thus at C both countries are better off, i.e., their real incomes (value added) are higher than in autarchy. Note that after opening to vertical trading the income must rise in both countries but the output increases in the North and it decreases in the South.

Keeping the endowment model in mind we recall that we can easily reproduce endowment point E within an autarchic v-a-c model with mixed technologies where the input tiers operate with fixed coefficients in both countries.\(^{39}\)

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\(^{37}\) The two-country endowment model is extensively analysed in Schmid (1999). Figure 15 is adopted from Bhagwati-Srinivasan (1983) who discuss the choice between labor and capital mobility in the spirit of Ramaswamy.

\(^{38}\) With equal technologies vertical trading equalizes both factor prices, i.e., the material prices and the wage rates. These obviously were different at point E, thus providing an incentive for material to move.

\(^{39}\) In Section 4.1. we called this a MT\(_y\) model which is now assumed in both countries.
Figure 16 illustrates the vertical trade gains in a two-country MT$_2$ model. In strong contrast to the endowment model of Figure 15, however, in Figure 16 point E is not a fixed endowment any longer. It rather illustrates a frozen production pattern which could be altered if the North (South) specialized in processing (raw-material production). It is evident and well known that the world output of final goods could be increased by this productive restructuring.

The Pareto-efficient world trading system after the restructuring is shown in Figure 17. Both countries have completely specialized in point S such that the South utilizes all labor resources, $\vec{L}^*$, in producing material, $Q^*_S$, and the North puts its total labor force, $\vec{L}$, into final-goods processing, $Q^*$. This way, the world, i.e., the North, produces a maximum of final goods at point P. This Pareto-efficient situation is represented by the output isoquant $Q^* = Y^*$ running through P. For the world as a whole world output must equal world income, but the distribution of world income among the two countries depends upon their (labor) resources$^{41}$ they put into the world-production scheme. With equal technologies in both countries the world-income distribution is at efficient point C being located by the tangent line at point S which shows the same slope, $\text{tg}_Y = W/P_N$, as the tangent line at the maximal world-output isoquant at point P. Evidently, the tangential real-income isoquants for both countries at C indicate higher real incomes for both countries. This demonstrates that North and South have gained in welfare by slicing their value-added chain.$^{42}$ Note that by construction of point C the North must give up some of the produced final goods to pay the South for the supply of material. This is completely analogous to the standard income-distribution problem where the patron has to pay the hired workforce and may keep the balance if there is something left.

6. The vertical North-South model with convex v-a-cs

From our discussion in Figure 17 it should be obvious by now that the introduction of totally convex v-a-cs in both countries only leads to minor adjustments in our argument.$^{43}$ Particularly, the assumption of convex production in the input tiers of the world economy permits the construction of an efficient world input tier$^{44}$ shown in Figure 18. With equal technologies in the output tiers of the world economy we find the efficient world output of final goods by looking for the maximal possible output isoquant in Figure 18. This determines both the point O* and the slope of the tangent line at point O*. The latter indicates an efficient world-factor price, $\text{tg}_f = W/P_N$, and the former determines the size of the efficient world-factor box. This box has the diagonal OO* representing a straight-line contract curve due to our assumption of equal technologies in the output tiers. Following our line of

$^{40}$ This restructuring after the initial influx of material is again the hinterland effect by Jones (2000).

$^{41}$ In addition to land resources which we do not assume in Figure 17.

$^{42}$ The autarchic situation is given by point $A_S$ and $A_N$ in Figure 17.

$^{43}$ The two-country model with convex v-a-cs was analysed in Schmid (1985). The reader should also consult an ingenious paper by Dixit-Grossmann (1982) who assume a continuum of stages in the manufacturing of final goods.

$^{44}$ We do not assume equal technologies and equal land endowments in the input tiers of the world. We assume rather $T^* > T$ and/or a bigger labor productivity in the input tier of the South.
argument given in Section 5 it is easy to determine the efficient distribution of world output between North and South. The efficient distribution of world output is given by point \( P \) which must be located on a perpendicular line above \( P_n \), the efficient production point in the world input tier.\(^{45}\) Because of the shared production scheme we applied in our world economy, the distribution of income in the world must be different from the distribution of output. We find the world-income distribution at point \( C \) which is positioned at the tangent line running through point \( P_n \) and which must be parallel to the factor-price line through \( O^* \). Point \( C \) unambiguously demonstrates that income, i.e., welfare, has been increased by vertical trade and specialization. In contrast to Figure 17 in Section 5, specialization has become imperfect due to our convexity assumption in the input tiers.\(^{46}\) Apparently in the North the hinterland effect operates by pulling some but not all labor from the input tier to final goods processing and in the South it works vice versa by pulling labor from the output tier into the input tier. The main result, however, is evident. (1) Both countries gain by slicing their value-added chains. (2) The total trade gain can be split into an initial gain from vertical trade and into a magnification via the hinterland effect.

7. Conclusions and qualifications

Although economics very often deals with value-adding and although even way down at the level of single business production plants there are value-added chains within such production units, in trade theory and macroeconomic theory models of the economy very much ignore this typical feature of production in stages.\(^{47}\) In this paper we have used a slightly modified labor theory of value (with land as a specific factor) where the same final good can be produced in principle in both parts of the world economy. We have shown that North and South are able to gain if the North (South) specializes in processing (raw-material production). This has been derived under the assumption of equal technologies in the two output tiers of the world and a relatively higher labor productivity in the southern input tier. The result could be strengthened if we assumed additionally the wasteful use of material in the North and a higher labor productivity in northern processing. Although both countries gain from the opening to trade (vertical integration) there is no trickle down of the economy-wide gains into both households of labor and landlords. It has been shown that in the North labor gains whereas the landlords lose while in the land-rich South labor loses and the landlords (oligarchs) gain. It is important to realize that in the political economy of v-a-c trading this result follows (1) from the transition of closed to open economies, (2) under the assumption that there are just workers employed at all stages and landlords in the input tier only. Our result would be changed

\(^{45}\) The vertical distance between \( P \) and \( P_n \) indicates the material import (export) of the North (South).

\(^{46}\) Note that the diagram in Figure 18 completely determines the allocation of labor among the two stages of production in each country.

\(^{47}\) Macroeconomics today is very much dominated by the Mundell-Fleming structure where only two final goods are traded. This totally ignores the reality of many successful economies which make a living by turning imported material or intermediates into final goods. In Schmid (2001) an anti-Mundell-Fleming structure is interpreted as a v-a-c model where unemployment of the domestic factor and a Keynesian modus operandi are assumed. The new structure is very important for the theory of devaluation.
if we assumed more realistically that value-added was created by a combined effort of labor and capital and if the possession of land, as very often is the case, was combined with the possession of capital. Moreover, the fate of the North and the South and of the social groups living within their borders might look as well very different if the shock occurred in a world-trading system. These shocks could be variations of the primary resource base $L$ or $L^*$ or changes in the endowments of the specific factors $T$ and $T^*$. It is interesting to observe that not all of these shocks (assumed expansionary) turn out to be welfare (value-added) improving within the country where the shock originates. In particular, an expansionary $L$ shock in the North (an expansionary $T^*$ shock in the South) may turn out as an immiserizing factor shock by decreasing the t-o-t of the North (South).\footnote{This possibility was shown to me by fit. R. Jones within the two-country fixed-endowment model in Schmid (1999). There is a strong presumption that it survives in a fully-fledged v-a-c model which incorporates the hinterland effect.}

All these main results should be seen only as benchmarks in the political economy of North-South relations, especially in the sense that there exists a sound economic reason for a gainful cooperation between North and South.\footnote{The trade-union argument that Eastern Germany should not be allowed to become the extended worktable of the West (verlängerte Werkbank) was very damaging to a fast recovery of the East and contributed to the Mezzogiorno problem. Thus western companies outsourced their activities to Eastern Europe and continue to do so.}

Secondly, if shared production schemes, i.e., international production processes and delocation of production, are about to become the production pattern of the future the problem of income distribution, which is very often not properly addressed within the borders of a country, re-emerges at a much larger scale between countries. The oil crisis was a first hint at this sort of conflict which is lingering on and which has to be solved before the distribution of income between local factors of production can be taken care of.\footnote{Ironically, 25 years after the oil shock most recent US textbooks in macroeconomics written during the 1990s completely miss this international distribution (transfer) problem when they treat oil shocks just equivalent to a drought, i.e., an adverse productivity shock.}

Thirdly, it is very clear from the geometry and algebra of our v-a-c models that material trading and especially the factoral t-o-t become very important. If the factoral t-o-t deteriorate the incomes of the local factors of production are squeezed. Put differently, if the size of the vertical trading shrinks the gains from trade fall and the income (value added) of the country must decrease. In a full employment model this amounts to a falling value-added productivity of labor and generates a falling wage rate if we assume wages being determined by v-a-c productivity in the long run.\footnote{The concept of productivity compounds through several stages of production and, therefore, the concept of comparative advantage become a bit more modern and sophisticated.}

Thus input trading, i.e., material trading in particular affects the wage rate of an economy and it does so according to the change in the factoral t-o-t.\footnote{In Germany for many years the Kiel Institute for World Economics under the leadership of H. Giersch argued that the t-o-t effect should be one component in the dispute among trade unions and employers when they look for the wage rate guided by productivity. The German council of economic advisers also applied this concept.}

The class of v-a-c models we have proposed here is only a starting point from which many useful extensions are possible. If we enlarge the number of stages (see Dixit-Grossman for a continuum of
vertical trade could be confined to intermediates only and there could be multistage trading channels. Another important extension would be to acknowledge more than one final good and thus more than one v-a-c within an economy. This line of research can be followed in Caves-Frankel-Jones (1995, chapt. 8) or Jones (2000, chapt. 5). As a very interesting extension with large empirical importance it would be useful to combine vertical material trading with input trade in primary factors of production which, however, were assumed completely trapped behind national borders in this paper. Finally, we mentioned several times that the v-a-c structure of an economy can be modelled using a classical or a more Keynesian modus operandi.
References


Open Economy with Broken Value-added Chain
Circuitry of the 1*1*1 model (with specific land)

assumptions: one final good, one trapped factor, one mobile

Production

\[ Z = N - Q_N = Q_N' - N' > 0 \]

\[ X = Q - C > 0 \]
Broken Value-added Chain: The Case of Imported Material

**Input-Tier**

product account: primary production

\[ \text{product account: } \]

\[ \text{primary production} \]

\[ \begin{array}{c|c}
WL_N + R_T \cdot T &= \tilde{Y}_N \\
\hline
p_N & p_N Q_N
\end{array} \]

**Output-Tier**

product account: processing

\[ \begin{array}{c|c|c}
P_N(N-Q_N) & P_N N & P_Q \\
\hline
P_N Q_N & W & L_Q = \tilde{Y}_Q
\end{array} \]

\[ \begin{array}{c|c}
P_N(N-Q_N) & P_Q \\
\hline
\hat{Y} & \hat{Y}_N
\end{array} \]

**Consolidated Productive System**

[aggregated product account]

\[ \begin{array}{c|c|c|c}
P_N(N-Q_N) & \hat{Y} & \hat{Y}_N & P_Q \\
\hline
Y & Y_N + Y_Q & W(L_N + L_Q) & \tilde{Y}_Q
\end{array} \]

Solution of income (value added) in output units

\[ Y = Q - p_N [N - Q_N] \]

\[ = w[L_Q + L_N] = wL \]

with labor only

\[ = w[L_Q + L_N] + r_T \bar{T} \]

with labor and land

**Figure 2a**

\[ p_N = \frac{P_N}{P} \]

real material price

\[ w \]

real wage

\[ r_T \]

real land rental
Broken Value-added Chain: The Case of Exported Material
In output (final goods) units

\[ r_T T - wL_N = Y_N \]

\[ p_N Q_N \]

\[ p_N N \]

\[ wL_Q = Y_Q \]

\[ Q \]

\[ Y = \begin{array}{c} Y_N \\ Y_Q \end{array} \]

\[ p_N [Q_N - N] \]

Income in output units

\[ Y = Q + p_N \left[ Q_N - N \right] \]

\[ = w \left[ L_Q + L_N \right] + r_T T \]

Figure 2b
Vertical Trade: Endowment-Model (no VAC!)

Trade gains in the H. G. Johnson diagram

Production function:

\[ Q = F(N, L) \]

\[ \tan \gamma = \frac{W}{P_N} = \frac{w}{p_N} \]

\[ p_N = \frac{P_N}{P} = \frac{1}{t \cdot o \cdot t} \]

\( t \cdot o \cdot t = \) factorial \( t \cdot o \cdot t \)

\( w = \) real wage (output units)

national income

\[ Y = Q(N, L) - p_N [N(\cdot) - \bar{N}] \]

\[ = wL + p_N \bar{N} \]

Figure 3
Vertical Trade: Trade gains with imported material
Trade-triangle diagram

- trade-triangle DBC in input-output space
- balanced trade account $X = p_N [N - \bar{N}]$ i.e. $BC = p_N \overline{DB}$
- trade gain with shared production $DA$

\[ \frac{w}{P} \]

\[ w = \frac{W}{P} \]

\[ \tan \alpha = p_N = \frac{p_N}{P} = \frac{1}{t - o - t} \]

national income

\[ Y = Q(N; \bar{L}) - p_N [N(\cdot) - \bar{N}] \]

\[ = w\bar{L} + p_N \bar{N} \]

Figure 4a
Vertical Trade: Trade gains with imported material

- \( Z = [N - \bar{N}] > 0 \) imported material
- \( p_N^f \) real material price (world market)
- \( \Delta \text{ABC} = e \) trade gain = value-added gain
- \( \square \text{BDEC} = d \) import of material in output units
- \( \text{ACED} = e + d \) output gain
- \( X = p_N [N - \bar{N}] \) balanced trade account

Figure 4b
Endowment Model
Trade Gains: case of material export

$\text{Figure 5a}$
Vertical Trade: raw material export
Trade triangle and trade gains

DA trade gain
DBC trade triangle

ABDE export of material
ACDE output loss
ABC trade gain

Figure 5b
Alternative modelling of value-added chains

linear technology (Schmid (1985))

mixed technology (Schmid (1985))

flexible technology (Schmid (1985))

MT₀ model

MT₁ model

convex VAC

Pareto-Optimality in convex VAC

\[ \tan \alpha = \frac{1}{a_{LN}} \]

\[ \tan \beta = \frac{N}{L_Q} \frac{a_N}{a_L} \]

\[ \tan \gamma = \frac{W}{P_N} \frac{w}{p_N} \]

Figure 6
Linear value-added chain (labor theory of value)  
Schmid (1985)

value-added chain with fixed coefficients (linear technology)

price equation output-tier:
\[ l = a_L w + a_N p_N \]  \hspace{1cm} (1)

price equation input-tier:
\[ p_N = a_{LN} w + a_{TN} r_T \]  \hspace{1cm} (2)

price equations:
real wage and real material price in autarchy follow from (2) in (1)

\[ w^a = \frac{1}{a_N a_{LN} + a_L} \]
\[ p^a_N = \frac{a_{LN}}{a_N a_{LN} + a_L} \]

\[ \frac{W}{p_N} = \frac{w^a}{p^a_N} = \frac{1}{a_{LN}} \]

Figure 7
quantity equations:

full employment: \( L_Q + L_N = \bar{L} \)  \( (3) \)

import of material: \( Z = N - Q_N > 0 \)  \( (4) \)

from (3) and (4) follows:

\[
a_L Q + a_{LN} Q_N = \bar{L} \tag{5}
\]

\[
a_N Q - Q_N = Z \tag{6}
\]

Solutions for \( Q \) and \( Q_N \)

\[
Q = \frac{\bar{L} + a_{LN} Z}{\Delta} = w \bar{L} + p^a_N Z \text{ with } \Delta = a_L + a_N a_{LN} > 0 \tag{7}
\]

\[
Q_N = \frac{a_N \bar{L} - a_L Z}{\Delta} \tag{8}
\]

Solution for \( Y \)

\[
Y = Q - p_N Z \tag{9}
\]

using (7) in (9):

\[
Y = \frac{\bar{L} + a_{LN} Z}{\Delta} - p_N Z
\]

\[
Y = \frac{\bar{L}}{\Delta} + \left[ \frac{a_{LN}}{\Delta} - p^f_N \right] Z \quad \text{with} \quad \frac{a_{LN}}{\Delta} = p^a_N \tag{10}
\]

Figure 8
Vertical Trade: The microfoundation trade gains from import of material

\[ Y = Q - p_N [N - Q_N] \] income

\[ L_Q + L_N = \bar{L} \] full employment

\[ Q_N - N = Z \] export of material

\[ \frac{a_N}{a_L} = \frac{W}{P_N} = \frac{w}{p_N} \]

Figure 9
Vertical Trade: The microfoundation trade gains from export of material

(1) \( Y = Q + p_N [Q_N - N] \)

(2) \( L_Q + L_N = L \)

(3) \( Q_N - N = -Z \)  

\[ \tan \alpha = \frac{1}{a_{LN}} \]

\[ \tan \gamma = \frac{W}{p_N} = \frac{w}{p_N} \]  

Figure 10
VAC: mixed technologies: MT₀ model

assumptions:  - t-o-t effect only
             - no structural adjustment

\[ f_Y = f_Q \]

\[ N^a = Q^a = N^f \]

\[ Z > 0 \]

\[ \tan \gamma = \frac{W}{P_N} = \frac{w}{p_N} \]

Figure 11
Convex VAC: trade gains $\triangleq$ value-added gain

assumption: flexible coefficients in output- and input-tier

Figure 12
Factor-Markets: case of flexible technology

labour market

factor-price frontier (output-tier)

material market

equations of the model:

labor market (full employment) \[ L_Q(w;N) + L_N(w/p_N;T) = \bar{L} \] (1)

factor-price-frontier (output-tier) \[ w = f(p_N) \] (2)

labor demand (output-tier) \[ L_Q = L_Q(w;N) \] (3)

material market (import function) \[ N(p_N;L_Q) - Q_N(w/p_N;T) = Z \] (4)

income (value-added) \[ Y = Q(w;N) - p_N[N(\cdot) - Q_N(\cdot)] \] (5)

Figure 13
Two-country-model with labor value-theory

North [Center]

\[ X = Q - C > 0 \]

\[ Z = N - Q_N = Q_N^* - N^* > 0 \]

South [Periphery]

Figure 14
Endowment-Model: two countries (Schmid(1999)) adopted from Bhagwati-Srinivasan (1983)

**Assumptions:** equal technologies (output-tier)
fixed endowments, $\bar{N}, \bar{N}^*$ (input-tier), fixed endowments, $\bar{L}, \bar{L}^*$

**Factor-box**

**world economy**

**Equations of the Endowment model**

\[
Q = F(N, \bar{L}) \quad \quad Q^* = F^*(N^*, \bar{L}^*) \quad \quad \text{with } F(\cdot) = F^*(\cdot)
\]

\[
p_N = F'_N(N, \bar{L}) = F'_{N^*}(N^*, \bar{L}^*) \quad \quad w = F'_L(\bar{L}, N) = F'_{L^*}(\bar{L}^*, N^*)
\]

**Factor-price frontier:**

\[
w = f(p_N)
\]

**World market for material:**

\[
N(w, \bar{L}) - \bar{N} = \bar{N}^* - N^*(w, \bar{L}^*) > 0
\]

**Income (value added):**

\[
Y = Q(\cdot) - p_N[N(\cdot) - \bar{N}] = w\bar{L} + p_N\bar{N}
\]

\[
Y^* = Q^*(\cdot) - p_N[N^*(\cdot) - \bar{N}^*] = w^*\bar{L}^* + p_N\bar{N}^*
\]

**Figure 15**
Gains from vertical trade with fixed production
Ricardian Model of North-South-Trade
Gains from complete vertical specialization

\[ Q_N^* = N \]

\[ \beta = \frac{a_N}{L} = \beta^* = \frac{a_N^*}{a_L} \]

\[ t g \, \gamma = \frac{W}{p_N} = \frac{w}{p_N} \]

Figure 17
World Trading System: convex technologies in the output and input tier

Figure 18