October 10, 2012

Advanced Microeconomics - Exercises Topic 3
Game Theory - Static Games (complete information)

Exercise 3.1 - Informational assumptions and outcome predictions

Consider the following game:

	A	B	C	D
a	0 , 3	3 , 5	6 , 2	4 , 1
b	3 , 1	2 , 2	1 , 0	6 , 4
c	4 , 4	1 , 1	5 , 2	3 , 3
d	2 , 6	4 , 2	2 , 3	5 , 5

(a) Assume the players only know that the other is *rational* (maximizes payoffs). Give the truncated game that results out of the thought process of the players.

(b) When the players possess an *iterated belief in rationality*, they will engage in a process of *iterated elimination of dominated strategies*. Again, give truncated game that results.

(c) Now assume there is *common knowledge of the game structure and the rationality* of the players. What do you predict as outcome(s) of the game?

Exercise 3.2 - Difference between *knowledge* and *common knowledge*: The blue-eyed islanders puzzle
(from Terence Tao's blog)

There is an island upon which a tribe resides. The tribe consists of 1000 people, with various eye colours. Yet, their religion forbids them to know their own eye color, or even to discuss the topic; thus, each resident can (and does) see the eye colors of all other residents, but has no way of discovering his or her own (there are no reflective surfaces). If a tribesperson does discover his or her own eye color, then their religion compels them to commit ritual suicide at noon the following day in the village square for all to witness.

All the tribespeople are highly logical and devout, and they all know that each other is also highly logical and devout (and they all know that they all know that each other is highly logical and devout, and so forth). [For the purposes of this logic puzzle, "highly logical" means that any conclusion that can logically deduced from the information and observations available to an islander, will automatically be known to that islander.]

Of the 1000 islanders, it turns out that 100 of them have blue eyes and 900 of them have brown eyes, although the islanders are not initially aware of these statistics (each of them can of course only see 999 of the 1000 tribespeople).

One day, a blue-eyed foreigner visits to the island and wins the complete trust of the tribe. One evening, he addresses the entire tribe to thank them for their hospitality. However, not knowing the customs, the foreigner makes the mistake of mentioning eye color in his address, remarking how unusual it is to see another blue-eyed person like myself in this region of the world.

What effect, if anything, does this faux pas have on the tribe?

Exercise 3.3 - (Weakly) dominant strategies: Second-price auction

Consider a setting in which there is a single unit of an indivisible good to be allocated to one of some identical agents. Each agent i has some private valuation $\theta_i > 0$, which follows an *iid* uniform distribution on $[0, 1]$, of the good. All agents can bid for the good (which can be represented for the strategic analysis by a bid function $b(\theta_i)$) with the highest bid winning the auction.
There are several auction formats which could potentially be used. In a *first-price* auction, the bidder has to pay her own bid to receive the good. Strategic analysis shows that in equilibrium *bid shading* ($b(\theta_i) < \theta_i$) occurs. This maximizes expected utility by an optimal tradeoff between the frequency of winning and the utility gain when winning.

In a *second-price auction* or *Vickrey auction* the winner bidder only has to pay the bid of the next highest bidder.

(a) Formulate such an auction as a strategic game.

(b) Show that $b_i(\theta_i) = \theta_i$ is a weakly dominant strategy for each agent i.

Exercise 3.4 - Nash equilibrium: Beauty contest

You are asked to participate in the following game:
Every student of this course chooses a real number between 0 and 100. You cannot observe what the others have chosen. The one who comes closest to $\frac{2}{3}$ of the *average* of all numbers wins a certain amount of money.

(a) Assuming you need the money, how would you play this game and why?

(b) Find the unique Nash equilibrium of the game.

(c) Show for N players that it is not profitable to unilaterally deviate by any $x > 0$ from the Nash equilibrium strategy for any player (i.e. the strategy combination of (b) is a *mutual best reply*).

Exercise 3.5 - Game types and mixed strategies

Find the best reply correspondences for the following games. Draw, or where appropriate, describe the reaction curves as in the lecture slides. How many Nash equilibria exist?

(a) With $V < C$, take $V = 2$ and $C = 4$:

	H	D
H	$\frac{V-C}{2}$, $\frac{V-C}{2}$	V , 0
D	0 , V	$\frac{V}{2}$, $\frac{V}{2}$

(b) With $x = y$ (zero-sum) or $x + y = c$ (constant-sum), for simplicity take $x = y = 1$:

	R	P	S
R	0 , 0	-x , y	x , -y
P	x , -y	0 , 0	-x , y
S	-x , y	x , -y	0 , 0

(c) With $a > b \geq d > c$, for simplicity take $a = 4$, $b = 3$, $c = 1$, $d = 3$:

	S	H
S	a , a	c , b
H	b , c	d , d