Pareto Efficiency of the Pay-as-you-go Pension System in a Three-Period-OLG Model

by

Matthias Wrede

University of Bamberg and University of Essen, Germany*

December 1998

Abstract

The paper considers an unfunded linear pension system when workers make labor decisions more often than once in their life. To capture this feature, a three-period-overlapping-generations model is employed. On the one hand, the paper analyzes whether or not a Pay-as-you-go pension scheme is intergenerational Pareto efficient when labor is elastically supplied by the young and the middle-aged people. On the other hand, the focus is on the interregional efficiency of a Pay-as-you-go system when young and middle-aged workers are mobile.

Keywords: Pay-as-you-go pension system, overlapping-generations model, intergenerational fairness, labor mobility

JEL-Classification: H55, J26, J61

---

* Matthias Wrede, Department of Economics, University of Bamberg, D-96045 Bamberg, Germany, tel.: +49-951-8632549, fax: +49-951-8635549, e-mail: matthias.wrede@sowi.uni-bamberg.de

Pareto Efficiency of the Pay-as-you-go Pension System in a Three-Period-OLG Model

I. Introduction

The Pay-as-you-go (PAYG) pension system is under pressure from at least two sources. On the one hand, the aging population demands for higher contributions or lower benefits. On the other hand, increasing labor mobility, in particular in the European community, makes a race to the bottom in contributions possible.

Particularly in the US, a fully funded pension is proposed to avoid difficulties. A fully funded system will neither distort the leisure-labor decision nor the residence choice of mobile workers. On the other hand, to overcome inefficiencies in a common labor market, harmonization of PAYG pension systems has also been suggested.

There is a large strand of literature which discusses whether a fully funded system Pareto dominates a PAYG pension scheme. Although the rate of return of the fully funded system, i.e. the interest rate, exceeds the internal rate of return of a PAYG system, i.e. the growth rate of wages, this is not a trivial question since a PAYG system benefits the first generation which does not contribute but profits from the pension system. A Pareto-superior transition from an unfunded to a funded system has to take those gains into account. As, e.g., Sinn (1997) pointed out, any pension system is a zero-sum game for all the participating generations. Pension systems redistribute among generations. Therefore, a Pareto-improving transition from a PAYG system to a funded system is impossible when the pension system generates no static inefficiencies. Roughly ten years ago, this was presented rigorously by Breyer (1989) [see also Verbon (1989)].

However, contributions to the PAYG system are derived from wage earnings and distort therefore the leisure-labor decision. For a small open economy, Homburg (1990) argued that if some utility is attached to leisure, intergenerational Pareto efficiency is violated, and therefore a Pareto-improving transition from unfunded to funded pension schemes is possible. Ignoring the implementation of such a mechanism, Breyer and

Straub (1993) extended this result to a large closed economy. In any case, a static inefficiency gives way to a Pareto superior transition.

Fenge (1995) showed that the argument does not apply when the benefits of a retiree are proportional to the individual contributions. He called this feature partial equivalence and claimed that it ensures the intergenerational efficiency of a PAYG system. This result attracts attention, particularly in Germany, where the PAYG system is endowed with individual accounts. E.g. Sinn (1997) stated that „Fenge's result is important" because it doubts the merits of a transition from a funded to an unfunded system. „Appropriate isolated reforms of the tax system" would do much better than a fundamental change in the pension system.

The results, discussed so far, were derived in a standard two-period-overlapping-generations framework. To restrict pension schemes to partial equivalent systems which ensure that benefits are proportional to contributions, has a unique meaning. If, in contrast, people's lives are divided in more than just two periods, even partial equivalence gives scope for reforms of the pension system. Therefore, this paper employs a three-period-overlapping-generations model to discuss intergenerational (in-)efficiency of a PAYG system when labor supply is endogenous.

Two main results will be derived: On the one hand, a PAYG system is intergenerational PARETO efficient, if reforms are restricted to guarantee that *present values of benefits are proportional to present values of contributions*. This property will be called partial equivalence of present values. In that case, the argument of Fenge (1995) fully applies. Lacking in lump sum taxes and further instruments, the government is unable to reduce static inefficiencies. This result cannot be applied to any linear pension system where the benefits of a retiree are proportional to weighted individual contributions. Thus the German PAYG scheme is not perfectly intergenerational efficient, which has been pointed out before by the advisory board of the German ministry of economics [see Wissenschaftlicher Beirat beim Bundesministerium für Wirtschaft (1998), part 52].

Therefore, in general, designing the pension system becomes a sort of second-best-optimum taxation problem. If the actual PAYG scheme is not for every generation a second-best-optimum, a Pareto-improving transition from the PAYG system to a fully funded system becomes feasible. Hence, this papers shifts the attention from first-best policies to second-best policies. While some simulation studies considered second-best taxation to manage the transition from a PAYG system to a fully funded system [see, e.g., Hirte and Weber (1997) and Fehr (1998)] their is lack of theoretical analysis in a second-best framework.

The mobility issue has also been dealt with by some authors. Homburg and Richter (1993) made clear how important harmonization of premium payments is in order to ensure interregional efficiency, i.e. identical marginal products of labor, if labor is perfectly mobile. First and not surprisingly, they showed that a fully funded system does not distort the residence of mobile workers. Second, they claimed that harmonized premium payments will guarantee interregional efficiency only in the stationary state when workers do not expect future shifts in the distribution of the mobile workers. However, Breyer and Kolmar (1994) argued that a constant population distribution is the only plausible belief when the total population stays constant. Therefore, harmonization would ensure interregional efficiency just from the beginning.

Regarding mobility, the aim of the paper is to apply the three-period-overlapping generations model. For simplicity, the second part of the paper focuses on interregional efficiency by assuming inelastically supplied labor. While the analysis will show that some harmonization is necessary to avoid inefficiency, complete harmonization of the pension schemes is not necessary. Only in special cases, a justification of perfect harmonization can be found.

The remaining part of the paper is organized as follows. While section two discusses intergenerational Pareto efficiency, section three focuses on interregional efficiency. The final section concludes.

II. Intergenerational Pareto efficiency

Consider a small open economy in which labor and capital are used for production according to a linearly homogeneous production function which takes place in a competitive firm sector. If labor is internationally immobile but capital is perfectly mobile, at time t, the interest factor $R_t = 1 + r_t > 1$, therefore the capital intensity and, thus, the wage rate w_t are exogenous. Furthermore, we assume for simplicity that the interest rate stays constant and hence $R_t = R$, for all t. In the absence of technological change, this would also imply $w_t = w$.

The framework employed here is a simple overlapping-generations model where the lifespan of every individual consists of three periods. People consume in every period but supply labor only in the first two periods. For simplicity, the paper assumes that all individuals have identical utility functions and time endowments (and abilities) and that the population stays constant. It is also assumed that the ability does not change during an individual's working period. In any period, the wage rate is the same for young and middle-aged people. A young member of generation t faces the problem

$$\text{(1)} \qquad \max_{\substack{c_{t,t}, c_{t,t+1}, c_{t,t+2}, \\ l_{t,t}, l_{t,t+1}}} u\left(c_{t,t}, c_{t,t+1}, c_{t,t+2}, 1 - l_{t,t}, 1 - l_{t,t+1}\right)$$

$$\begin{aligned} \text{s.t.} \qquad c_{t,t} + s_{t,t} &= w_t l_{t,t}(1-\tau), \\ c_{t,t+1} + s_{t,t+1} &= w_{t+1} l_{t,t+1}(1-\tau) + s_{t,t} R, \\ c_{t,t+2} &= s_{t,t+1} R + p_{t+2}, \end{aligned}$$

where c denotes consumption, l labor, s savings, τ the contribution rate, i.e. the wage tax rate, and p the pension received by the individual. Total time in each period is normalized to one. If there is only one subscript (as with w and p), it indicates the period under consideration. If there are two subscripts (as with c, l, and s), the first one denotes the period of birth and the second one indicates the period under consideration. E.g., $c_{t,t+1}$ is consumption in period t+1 of an individual born in period t. The utility function u is well-behaved.

Combining the constraints, the intertemporal budget constraint can be described as

$$\text{(2)} \qquad c_{t,t} + \frac{c_{t,t+1}}{R} + \frac{c_{t,t+2}}{R^2} = w_t l_{t,t}(1-\tau) + \frac{w_{t+1} l_{t,t+1}(1-\tau)}{R} + \frac{p_{t+2}}{R^2}.$$

The focus in the remaining part of the paper is on *linear pension systems*, i.e. the pension paid in the retirement period is a linear combination of the contributions made by the individual in earlier times:

(3) $$p_{t+2} = \Omega_1 \tau w_t l_{t,t} + \Omega_2 \tau w_{t+1} l_{t,t+1},$$

where Ω_1 and Ω_2 are nonnegative numbers. It is assumed that $0 \leq \Omega_2 < R$ and $0 \leq \Omega_1 < R^2$. Under a linear pension system, the intertemporal budget constraint becomes

(4) $$c_{t,t} + \frac{c_{t,t+1}}{R} + \frac{c_{t,t+2}}{R^2} = w_t l_{t,t}(1-\tau_1) + \frac{w_{t+1} l_{t,t+1}(1-\tau_2)}{R},$$

where

(5) $$\tau_1 = \tau\left(1 - \frac{\Omega_1}{R^2}\right) \quad \text{and} \quad \tau_2 = \tau\left(1 - \frac{\Omega_2}{R}\right)$$

are the benefit-adjusted tax rates. If $\Omega_2 = R$ and $\Omega_1 = R^2$ were fulfilled, which is excluded by assumption, the effective tax rates would be equal to zero and the pension system would not distort the allocation of resources.[1]

Solving the utility maximization problem given above, yields the labor supply functions, the consumer goods demand functions and the indirect utility function

(6) $$v = v\left(w_t(1-\tau_1), w_{t+1}(1-\tau_2)/R, R, w_t(1-\tau_1) + w_{t+1}(1-\tau_2)/R\right)$$

where the last argument is the potential labor income.

Recalling that the population is assumed to be constant, the budget constraint of the pension system per capita reads as

(7) $$\tau\left(\Omega_1 w_t l_{t,t} + \Omega_2 w_{t+1} l_{t,t+1}\right) = \tau w_{t+2}\left(l_{t+1,t+2} + l_{t+2,t+2}\right).$$

---

[1] However, under a actuarial fair unfunded system, a stationary state does not exist, as discussed below [see also Breyer and Straub (1993), p. 83, fn 7].

At a stationary state, where $w_t = w$, $l_{t,t} = l_1$, and $l_{t,t+1} = l_2$, for all t, the budget constraint can be written as

$$(8) \qquad (\Omega_1 - 1)l_1 + (\Omega_2 - 1)l_2 = 0.$$

Thus, either both contribution rates are equal to one, i.e. the growth factor, or one is larger and the other one is smaller than one. Particularly, $\Omega_2 = R$ and $\Omega_1 = R^2$ is impossible.

Following Homburg (1990) and others, intergenerational Pareto efficiency will be analyzed by considering a one shot derivation from an established PAYG system with the help of public indebtedness. The government's optimization problem is to choose the tax rate, the benefit rates and the amount of public debt such as to maximize the utility of one generation without changing the contributions and benefits of other generations under a given PAYG system. It is worthwhile to stress that lump-sum taxes and transfers are not permissible.

The existing PAYG system is characterized by $\left(\bar{\tau}, \overline{\Omega}_1, \overline{\Omega}_2\right)$ and, therefore, a sequence of pensions payments $\bar{p}_t, \bar{p}_{t+1}, \ldots$. Hence, the constraints of the government's optimization problem are

$$(9) \qquad \begin{aligned} \bar{p}_t &= \tau w_t l_{t,t} + \bar{\tau} w_t \bar{l}_{t-1,t} + D_t, \\ \bar{p}_{t+1} &= \tau w_{t+1} l_{t,t+1} + \bar{\tau} w_{t+1} \bar{l}_{t+1,t+1} + D_{t+1} - D_t R, \\ \bar{p}_{t+2} &= \Omega_1 \tau w_t l_{t,t} + \Omega_2 \tau w_{t+1} l_{t,t+1} + D_{t+1} R. \end{aligned}$$

Recall that wage rates and interest factors are exogenous. While the first and the second constraint ensure that neither the old in period t nor the old in period t+1 suffer losses, the third constraint guarantees that generation t completely repays the debt and, therefore, the subsequent generations maintain the utility level they achieved under the preexisting PAYG system. For our purpose, these constraints can be combined such that public debt disappears from the constraints. The combined condition requires that the present value of the net payment of a member of generation t stays constant.

From the set of government instruments (the contribution rate τ and the two benefit rates Ω_1 and Ω_2), one instrument is redundant. Therefore, the paper takes the benefit-adjusted tax rates directly as government's control variables. The government faces the optimization problem

$$(10)\qquad \max_{\tau_1,\tau_2} v\left(w_t(1-\tau_1), w_{t+1}(1-\tau_2)/R, R, w_t(1-\tau_1)+w_{t+1}(1-\tau_2)/R\right)$$

$$\text{s.t. } \tau_1 w_t l_{t,t} + \tau_2 w_{t+1} l_{t,t+1}/R = \text{konstant} \geq 0\,.$$

If necessary, the optimum benefit-adjusted tax rates will be indicated by an asterisk.

If the required net payment were equal to zero, the optimum benefit-adjusted tax rates were also equal to zero and the first-best optimum would be achieved.

Otherwise, the government faces a standard second-best-optimum-taxation problem, of which a solution can be described by the Ramsey rule [see, e.g., Sandmo (1987)]:[2]

$$(11)\qquad \sum_{j=1}^{2} \frac{\tau_j * \tilde{w}_j}{l_{t,i}} \frac{\partial \hat{l}_{t,j}}{\partial\left(\tilde{w}_i(1-\tau_i *)\right)} \text{ independent of i, } i=1,2\,.$$

E.g., this formula simplifies to the (compensated) inverse elasticity rule, when the cross price elasticities of the Hicksian labor supply functions are zero.

Figure 1 shows the net-payment requirement and an indifference curve, assuming some regularities. In particular, it makes clear that both benefit-adjusted tax rates are determined by the constraint if the ratio of the benefit-adjusted tax rates is fixed.

Figure 1: Second-best-optimum taxation

To get additional insight into the structure of the solution, it is useful to combine the equations (5) in which the benefit-adjusted tax rates are defined by eliminating the contribution rate τ. This defines a linear relationship between Ω_1 and Ω_2:

$$(12)\qquad \Omega_1 = \left(1-\frac{\tau_1}{\tau_2}\right)R^2 + \frac{\tau_1}{\tau_2} R\Omega_2\,.$$

2 A hat indicates the Hicksian supply function, and a tilde indicates the present value of the variable.

Given the benefit-adjusted tax rates, neither labor supply and utility nor the net payment measured at time t changes along this line. To ensure that the benefit-adjusted tax rates are unaltered,

$$(13) \qquad \tau = \tau_2 R / (R - \Omega_2)$$

must *also* hold. However, as figure 1 has made clear, the benefit-adjusted tax rates τ_1 and τ_2 are determined by the net-payment requirement if τ_1/τ_2 is fixed. Hence, as long as the contribution rate τ is adjusted according to the net-payment requirement, (12) defines indifference curves. In figure 2 (b), three different indifference curves determined by (12) are depicted. Figure 2 (a) shows the corresponding contribution rate according to (13).

Figure 2: Benefit rates and indifference curves

In the feasible range,

$$(14) \qquad \tau_1 \underset{<}{\overset{>}{=}} \tau_2 \Leftrightarrow R\Omega_2 \underset{<}{\overset{>}{=}} \Omega_1$$

hold. The benefit-adjusted tax rate on labor income in the youth is higher than (equal to, lower than) the benefit-adjusted tax rate on labor income in the second period of life, iff the weight of the discounted contributions in the second period of life in the pension formula is higher than (equal to, lower than) the weight of the contributions in the first period of life.

This discussion leads immediately to the following proposition.

Proposition 1*: If the existing PAYG system fulfills and if every alternative has to fulfill* $\Omega_1 = R\Omega_2$, *no improvement of every single generation is possible without harming the predecessors and the successors.*

In order to fulfill the net-payment requirement, any decrease in the contribution rate of a generation t has to be accompanied by an adequate decrease in the benefit rates such that the uniform benefit-adjusted tax rate is unaltered. Even with zero-benefit rates

Ω_1 and Ω_2 (but still a positive contribution rate), utility is not higher than under the pre-existing PAYG scheme. By requiring $\Omega_1 = R\Omega_2$, reforms are restricted to one and the same indifference curve.

This proposition generalizes the result obtained by Fenge (1995) in a standard two-period OLG model.[3] He claims that partial equivalence (i.e. benefits are proportional to contributions) ensures the intergenerational efficiency of a PAYG system. However, Fenge's result could not be obtained in the three-period model if Ω_1 is fixed to some $\alpha R\Omega_2$ when $\alpha \neq 1$. This can be easily seen in figure 2, where every ray defined by $\Omega_1 = \alpha R\Omega_2$ crosses the indifference curves when $\alpha \neq 1$. Hence, it turns out that in a multi-period framework benefits have to *be proportional to the present value of contributions* to ensure intergenerational efficiency of a PAYG system.

Some further statements on the solution of the maximization problem of the government given above are possible.

Proposition 2*: Suppose, a unique solution to (10) exist with* $\tau_1^*, \tau_2^* > 0$ *and* $0 \le \Omega_2 < R$ *and* $0 \le \Omega_1 < R^2$*. Then, the following statements hold true:*

(a) The contribution rate t *is strictly positive.*

(b) The smallest possible contribution rate t *requires* $\Omega_1\Omega_2 = 0$.

(c) When $\Omega_2 \to R$*, then* $\Omega_1 \to R^2$ *and* $\tau \to \infty$.

(d) When $\alpha = \Omega_1/(R\Omega_2)$ *is exogenously fixed and* $0 < \alpha < \infty$*, the optimum can be achieved if either* $(\tau_1^* - \tau_2^*)(1-\alpha) > 0$ *or* $\alpha = \tau_1^*/\tau_2^* = 1$*. While in the former case* Ω_2 *is uniquely determined and* $\Omega_1, \Omega_2 > 0$ *has to hold, in the latter case* Ω_2 *is undetermined and* $\Omega_1 = \Omega_2 = 0$ *is possible.*

(e) If $\tau_1^* \neq \bar{\tau}_1$ *or* $\tau_2^* \neq \bar{\tau}_2$*, the existing PAYG system is not a second-best optimum.*

Dependent on the utility function, partial equivalence in present values, i.e. $\Omega_1 = R\Omega_2$, is or is not compatible with intergenerational efficiency.

---

3 Alternatively, the method used by Fenge (1995) can also be applied to prove the proposition in the three-period OLG model.

Proposition 3*: Consider a stationary state under a PAYG scheme characterized by* $\left(\bar{\tau}, \overline{\Omega}_1, \overline{\Omega}_2\right)$, *where* $w_t = w$, $\bar{l}_{t,t} = \bar{l}_1$, *and* $\bar{l}_{t,t+1} = \bar{l}_2$, *for all* $t \geq 0$. *Suppose the utility function is* $u = \phi\left(c_{t,t}\right) + \delta\phi\left(c_{t,t+1}\right) + \delta^2\phi\left(c_{t,t+2}\right) + \psi\left(1 - l_{t,t}\right) + \delta\psi\left(1 - l_{t,t+1}\right)$, *where* $\phi' > 0$, $\phi'' < 0$, $\psi' > 0$, $\psi'' < 0$. *Suppose further* $\delta R = 1$. *Then, the second-best-optimal tax system is uniform, and a PAYG scheme which satisfies* $\Omega_1 = R\Omega_2$ *is intergenerational efficient.*

Proof: Uniform taxation is optimal with this type of additive separable utility function, given that $\delta R = 1$ holds [see, e.g., Buchholz and Wiegard (1998)]. Since the specified PAYG system ensures uniform taxation at the second-best optimal level, a one-shot derivation is impossible and the pension system, therefore, intergenerational efficient. Q.E.D.

Although other utility functions might also require uniform taxation, it becomes clear that a PAYG system which establishes partial equivalence in present values and, therefore, uniform taxation is intergenerational efficient only under very specific circumstances.

Using the result (e) of proposition 2, the transition from a non-optimal PAYG system to a Pareto superior funded pension system can be considered in a stationary state.

Proposition 4*: Consider a stationary state under a PAYG scheme characterized by* $\left(\bar{\tau}, \overline{\Omega}_1, \overline{\Omega}_2\right)$ *where* $w_t = w$, $\bar{l}_{t,t} = \bar{l}_1$, *and* $\bar{l}_{t,t+1} = \bar{l}_2$, *for all* $t \geq 0$. *Suppose further that* $\tau_1{*} \neq \bar{\tau}\left(1 - \overline{\Omega}_1 / R^2\right)$ *or* $\tau_2{*} \neq \bar{\tau}\left(1 - \overline{\Omega}_2 / R\right)$, *for some* $t \geq 0$ *holds. Then, there exists a T, such that a complete transition beginning in period 0 from the PAYG pension system to a fully funded pension system becoming effective in period* $T+1$ *is possible without harming the generations -2 till T.*

Proof: If for some generation t, $\tau_1{*} \neq \bar{\tau}_1$ or $\tau_2{*} \neq \bar{\tau}_2$ holds, it holds for all $t \geq 0$ and the net payment of a member of every generation t in the transition period [0,T] can be increased by a change in the benefit-adjusted tax rates such that the utility level is equal to the level under the preexisting PAYG system without hurting generations t-2, t-1, t+1 and t+2. The increment which is denoted by Y will be paid in period t+2 and will be saved by a pension funds. In period T+1 and T+2 the funds wealth will be used to pay the pensions payable under the so modified PAYG system which is clearly possible if the (per capita) pensions under the preexisting PAYG system are lower than the (per capita) funds wealth:

$$\text{(15)} \qquad \bar{p}_{T+1} + \frac{\bar{p}_{T+2}}{R} \le Y \sum_{j=2}^{T} R^{(T+1-j)}$$

Since the LHS is bounded from above by $\bar{\tau} w(1+R)\left(\bar{\Omega}_1 + \bar{\Omega}_2\right)/R$ and the RHS is positive and unboundedly increasing in T, there exists a T such that the inequality is fulfilled.

Q.E.D.

If the PAYG misses the static second-best optimum, a Pareto superior transition from the PAYG system to a fully funded system becomes feasible. Note that the transition process can also be managed with government indebtedness.

III. Interregional efficiency

Next, consider a federation which consists of I jurisdictions. The production function in each region is linearly homogeneous. Internationally perfect mobile capital and two kinds of labor are used for production. One type is perfectly mobile, the other one is completely immobile. Total population of each type is constant. Individuals of one and the same type do not differ from each other. To make the distinction easier, variables for the immobile workers are denoted by uppercase letters, but lowercase letters are used for the mobile workers.

At time t, the federation faces a common interest factor $R_t = 1 + r_t > 1$ and a common wage rate for mobile labor w_t. Both are not exogenous for the entire federation and, therefore, not necessarily constant.

Apart from worker mobility, the framework employed here is essentially the same as in the previous section with one important difference: Each individual supplies inelastically one unit of labor.

A young immobile worker of generation t in region i faces the problem

$$\max_{C^i_{t,t},C^i_{t,t+1},C^i_{t,t+2}} U\left(C^i_{t,t}, C^i_{t,t+1}, C^i_{t,t+2}\right) \tag{16}$$

$$\begin{aligned} \text{s.t.} \qquad C^i_{t,t} + S^i_{t,t} &= W^i_t\left(1-\tau^i\right), \\ C^i_{t,t+1} + S^i_{t,t+1} &= W^i_{t+1}\left(1-\tau^i\right) + S^i_{t,t}R_{t+1}, \\ C^i_{t,t+2} &= S^i_{t,t+1}R_{t+2} + P^{i,i}_{t+2}, \end{aligned}$$

where C denotes consumption, S savings, τ the contribution rate, i.e. the wage tax rate, and p the pension received by the individual. The subscripts denote, if necessary, the time of birth and the period under consideration. The superscript i indicates the region. $P^{i,i}_{t+2}$ indicates the pension an (immobile) individual of generation t receives in period t+2.

Mobile workers decide upon consumption in each period and choose the location of residence when they are young and again when they are in middle age. A young mobile worker of generation t solves

$$\max_{\substack{c_{t,t},c_{t,t+1},c_{t,t+2},\\ i,j}} U\left(c_{t,t}, c_{t,t+1}, c_{t,t+2}\right) \tag{17}$$

$$\begin{aligned} \text{s.t.} \qquad c_{t,t} + s_{t,t} &= w^i_t\left(1-\tau^i\right), \\ c_{t,t+1} + s_{t,t+1} &= w^j_{t+1}\left(1-\tau^j\right) + s_{t,t}R_{t+1}, \\ c_{t,t+2} &= s_{t,t+1}R_{t+2} + p^{i,j}_{t+2}, \end{aligned}$$

where the variables are analogously defined. $p^{i,j}_{t+2}$ indicates the pension a mobile individual of generation t receives in period t+2 depending on the residence in the first and second period of his life (indicated by the superscripts).

Combining the second and the third constraint, yields

$$c_{t,t+1} + \frac{c_{t,t+2}}{R_{t+2}} = w^j_{t+1}\left(1-\tau^j\right) + s_{t,t}R_{t+1} + \frac{p^{i,j}_{t+2}}{R_{t+2}}. \tag{18}$$

If an individual, which stayed in some region i during his youth, is perfectly mobile and indifferent between this region and any other region,

$$w^j_{t+1}\left(1-\tau^j\right) + \frac{p^{i,j}_{t+2}}{R_{t+2}} \text{ is independent of j,} \tag{19}$$

must hold, i.e. the sum of net labor income in the second period of life and the present value of the pension is independent of the residence choice.

Similarly, combining all constraints leads to

(20) $$c_{t,t} + \frac{c_{t,t+1}}{R_{t+1}} + \frac{c_{t,t+2}}{R_{t+1}R_{t+2}} = w_t^i\left(1-\tau^i\right) + \frac{w_{t+1}^j\left(1-\tau^j\right)}{R_{t+1}} + \frac{p_{t+2}^{i,j}}{R_{t+1}R_{t+2}}.$$

A perfectly mobile young member of generation t is indifferent between the various regions if

(21) $$w_t^i\left(1-\tau^i\right) + \frac{w_{t+1}^j\left(1-\tau^j\right)}{R_{t+1}} + \frac{p_{t+2}^{i,j}}{R_{t+1}R_{t+2}} \text{ is independent of i.}$$

Thus, lifetime income is independent of the residence choice in the birth period. If, however, perfect mobility in the second period of life ensures (19) for each region i, condition (21) reduces to

(22) $$w_t^i\left(1-\tau^i\right) + \frac{p_{t+2}^{i,\sim}}{R_{t+1}R_{t+2}} \text{ is independent of i,}$$

because the residence of middle-aged people is independent of the residence choice in the youth (the tilde stands for any region).

Let us focus on interior solutions, i.e. we assume that (19) and (22) hold. The starting point is again a *linear pension system*. It is assumed that every region has established a pure linear pension scheme, which does not discriminate against migration. Thus, the pension of a mobile worker of generation t is given by[4]

(23) $$p_{t+2}^{i,j} = \Omega_1^i \tau^i w_t^i + \Omega_2^j \tau^j w_{t+1}^j .$$

The allocation is *interregional efficient* if the capital-immobile-labor ratio and the mobile-labor-immobile-labor ratio are the same in all regions, respectively. This implies that the marginal products of all inputs are independent of the location. From profit

[4] The benefit rates are non-negative numbers which are smaller than the interest factors.

maximization follows that the regions face a common interest factor, a common wage of mobile labor and a common wage of immobile labor in every period.

Hence, in the presence of perfect mobility of one type of labor, characterized by (19) and (22), and under linear pension systems

$$(24) \qquad \tau_t^i = \tau_t^j \quad \text{and} \quad \tau_{t+1}^i = \tau_{t+1}^j, \quad \text{for all i, j,}$$

is necessary for interregional efficiency, where

$$(25) \qquad \tau_t^i = \tau^i\left(1 - \frac{\Omega_1^i}{R_{t+1}R_{t+2}}\right) \quad \text{and} \quad \tau_{t+1}^i = \tau^i\left(1 - \frac{\Omega_2^i}{R_{t+2}}\right)$$

are the benefit-adjusted tax rates for the first and the second period of life of generation t. Putting the conditions together,

$$(26) \qquad \frac{\tau^j}{\tau^i} = \frac{R_{t+2} - \Omega_2^i}{R_{t+2} - \Omega_2^j} = \frac{R_{t+1}R_{t+2} - \Omega_1^i}{R_{t+1}R_{t+2} - \Omega_1^j}, \quad \text{for all i, j,}$$

is obtained. This allows the following statement:

Proposition 5*: Denote* $\Omega_1^i = \alpha^i R_{t+1}\Omega_2^i$*, for all i.*

(a) Suppose, the allocation is interregional efficient and the contribution rates are strictly positive. Then $\tau^i = \tau^j$ *,* $\Omega_2^i = \Omega_2^j$*, and* $\Omega_1^i = \Omega_1^j$ *are equivalent. These conditions also imply* $\alpha^i = \alpha^j$*, and they are implied by* $\alpha^i = \alpha^j \neq 1$*.*

(b) Suppose, the allocation is interregional efficient and the contribution and benefit rates are strictly positive. If $\alpha^i = 1$ *for some i,* $\alpha^j = 1$ *must hold for all j.*

(c) Suppose, the allocation is interregional efficient and the contribution and benefit rates are strictly positive. If $\alpha^i < 1$ *for some i,* $\alpha^j < 1$ *must hold for all j.*

(d) If $\tau^i = 0$ *holds for all i, the allocation is interregional efficient.*

Proof: Rearranging the interregional efficiency condition (26) for two regions i and j, yields

$$(27) \qquad \Omega_2^i\Omega_2^j\left(\alpha^j - \alpha^i\right) + R_{t+2}\left[\Omega_2^j\left(1-\alpha^j\right) - \Omega_2^i\left(1-\alpha^i\right)\right] = 0.$$

(a) While the equivalence and the first implication follow immediately from (26), the second implication can be shown by using (27). Recalling $\Omega_2^i = R_{t+2}$ is excluded by assumption, (b) is implied by (27). Recalling $\Omega_1^i < R_{t+1}R_{t+2}$ is also assumed, (c) can be proved:

$$\alpha^i < 1 \Leftrightarrow R_{t+2} - \alpha^i\Omega_2^i > R_{t+2} - \Omega_2^i \Leftrightarrow \frac{R_{t+2} - \alpha^i\Omega_2^i}{R_{t+2} - \Omega_2^j} > \frac{R_{t+2} - \Omega_2^i}{R_{t+2} - \Omega_2^j}.$$

Using (26), this implies

$$\frac{R_{t+2} - \alpha^i\Omega_2^i}{R_{t+2} - \Omega_2^j} > \frac{R_{t+2} - \alpha^i\Omega_2^i}{R_{t+2} - \alpha^j\Omega_2^j} \Leftrightarrow R_{t+2} - \alpha^j\Omega_2^j > R_{t+2} - \Omega_2^j \Leftrightarrow \alpha^j < 1.$$

(d) is obvious. Q.E.D.

On the one hand, harmonizing contribution rates is not necessary. On the other hand, harmonizing contribution rates requires also harmonization of benefit rates.

Furthermore, in particular, if benefits are proportional to present values of contributions in *all* regions, i.e. if $\alpha^i = 1$ for all i, neither contribution rates nor benefit rates have to be harmonized.

It is worthwhile to consider the budget constraint of the (linear) PAYG system. In region i in period t+2 it can be written as

$$\begin{aligned}(28)\qquad &\tau^i\left[\Omega_1^i\left(w_t^i n_{t,t}^i + W_t^i N_t^i\right) + \Omega_2^i\left(w_{t+1}^i n_{t,t+1}^i + W_{t+1}^i N_t^i\right)\right]\\ &= \tau^i\left[w_{t+2}^i\left(n_{t+1,t+2}^i + n_{t+2,t+2}^i\right) + W_{t+2}^i\left(N_{t+1}^i + N_{t+2}^i\right)\right],\end{aligned}$$

where n indicates a number of mobile worker and N indicates a number of immobile workers. The superscripts denote the region, and the subscripts the periods. E.g., $n_{t,t+1}^i$ denotes the number of the mobile workers born in period t which live in region i in period t+1, and, e.g., N_t^i indicates the number of immobile workers in region i which where born in period t. The LHS counts the benefits of the old, the RHS the contributions of the young and the middle-aged.

At a stationary state, where $N_t^i = N^i$, $n_{t,t}^i = n_1^i$, $n_{t,t+1}^i = n_2^i$, $R_t = R$, $w_t^i = w^i$, and $W_t^i = W^i$, the budget constraint becomes

(29) $$\left(\Omega_1^i - 1\right)\left(n_1^i w^i + N^i W^i\right) + \left(\Omega_2^i - 1\right)\left(n_2^i w^i + N^i W^i\right) = 0 .$$

If $\Omega_1^i = \Omega_2^i = 1$ holds, i.e. if the benefit rates are equal to the growth factors, the constraint is automatically fulfilled. Otherwise, the budget constraint requires $\left(\Omega_1^i - 1\right)\left(\Omega_2^i - 1\right) < 0$.

Consider now an interregional efficient stationary state in a federation which consists of two regions. Thus, the budget constraint (29) is fulfilled for $i = 1,2$, the wage rates fulfill $w^i = w$ and $W^i = W$, and the equalization of labor intensities requires

(30) $$\left(n_1^1 + n_2^1\right) / \left(2N^1\right) = \left(n_1^2 + n_2^2\right) / \left(2N^2\right).$$

In an equilibrium, where the young people are equally distributed, the allocation can be further described provided that the contribution rates are strictly positive:[5]

Proposition 6*: At an interregional efficient stationary state in a federation with two regions where the young people are equally distributed and where, therefore,* $N_t^i = N$, $n_{t,t}^i = n_1$, $n_{t,t+1}^i = n_2^i$, $R_t = R$, $w_t^i = w$, *and* ,$W_t^i = W$, *contribution rates and benefit rates have to be harmonized, i.e.* $\tau^1 = \tau^2$, $\Omega_2^1 = \Omega_2^2$, *and* $\Omega_1^1 = \Omega_1^2$.

Proof: Equal labor intensities ask for $n_2^1 = n_2^2$. Since the population does not grow, $n_1^1 = n_2^1 = n_1^2 = n_2^2$ follows immediately. Hence, the budget constraint of the pension system demands $\Omega_1^i = 2 - \Omega_2^i$, for $i = 1,2$. Thus, the efficiency condition (26) can be written as

(31) $$\frac{\tau^2}{\tau^1} = \frac{R - \Omega_2^1}{R - \Omega_2^2} = \frac{R^2 + \Omega_2^1 - 2}{R^2 + \Omega_2^2 - 2}.$$

This leads to $\Omega_2^1 = \Omega_2^2$. Now, part (a) of the previous proposition can be applied. Q.E.D.

---

[5] Note that the proposition also holds for more than two regions.

In general, and even if just the immobile workers were equally distributed, harmonization of contribution and benefit rates is not necessary.

IV. Concluding remarks

The paper has considered separately intergenerational Pareto efficiency and interregional efficiency of a PAYG system.

With respect to the first aspect, two results were derived. First, a PAYG system is intergenerational PARETO efficient, if reforms are restricted to guarantee that *present values of benefits are proportional to present values of contributions*. This result cannot be applied to any linear pension system where benefits are proportional to weighted contributions.

In general, partial equivalence of present values, which means uniform taxation of labor income, does not ensure intergenerational efficiency.

Second, if the actual PAYG scheme is not for every generation a second-best-optimum, a Pareto-improving transition from the PAYG system to a fully funded system becomes feasible. Hence, the paper shifts the attention from first-best policies to second-best policies.

With respect to the second problem, it was shown that some harmonization of pension schemes is necessary to achieve interregional efficiency. Complete harmonization, however, is not necessary. A justification for perfect harmonization can be found only in special, particular symmetric, cases.

There is, of course, scope for further research. First, the discussion of intergenerational efficiency could be extended to a large closed economy. Second, the debate concerning the relationship between labor mobility and pension schemes lacks in the strategic dimension of the problem. Hence, premium payment competition among jurisdictions should be taken into account.

References

Breyer, F. (1989). On the Intergenerational Pareto Efficiency of Pay-as-you-go Financed Pension Systems. Journal of Institutional and Theoretical Economics 145. 643 - 658.

Breyer, F. and M. Kolmar (1994). Does the Common Labor Market Imply the Need for a European Public Pension System. Working Paper 244. University of Konstanz.

Breyer, F. and M. Straub (1993). Welfare Effects of Unfunded Pension Systems When Labor Supply is Endogenous. Journal of Public Economics 50. 77 - 91.

Buchholz, W. and W. Wiegard (1998). Zeit(in)konsistente Steuerpolitik. In: Alois Oberhauser (Ed.). Probleme der Besteuerung. Duncker & Humblot. Berlin. 9 - 55.

Fehr, H. (1998). Privatization of Public Pensions in Germany: Who Gains and How Much? Diskussionsbeiträge 148. Universität Tübingen.

Fenge, R. (1995). Pareto-efficiency of the Pay-as-you-go Pension System with Intergenerational Fairness. Finanzarchiv N. F. 52. 357 - 363.

Hirte, G. and R. Weber (1997). Pareto improving Transitions from a Pay-as-you-go to a Fully Funded System - is it Politically Feasible? Finanzarchiv N.F. 54. 303 - 330.

Homburg, S. (1990). The Efficiency of Unfunded Pension Schemes. Journal of Institutional and Theoretical Economics 146. 640 - 674.

Homburg, S. and W.F. Richter (1993). Harmonizing Public Debt and Public Pension Schemes in the European Community. Journal of Economics Supplement 7. 51 - 63.

Sandmo, A. (1987). A Reinterpretation of Elasticity Formulae in Optimum Tax Theory. Economica 54. 89 - 96.

Sinn, H.-W. (1997). The Value of Children and Immigrants in a Pay-as-you-go Pension System. CES Working Paper 141. University of Munich.

Verbon, H.A.A. (1989): Conversion Policies for Public Pension Plans in a Small Open Economy. In: B.A. Gustafsson and N.A. Klevmarken (Ed.). The Political Economy of Social Security. Elsevier Science Publishers B.V. North-Holland.

Wissenschaftlicher Beirat beim Bundesministerium für Wirtschaft (1998). Grundlegende Reform der gesetzlichen Rentenversicherung. Bonn