Vertically and Horizontally Interacting Newsvendors

Abstract

In this working paper general conditions for a global optimum of the production quantity of several vertically respectively horizontally interacting newsvendors are derived.

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1 Vertically interacting newsvendors

We propose a supply chain structure of N value stages, where every production site is responsible for one value stage. Horizontal interaction is not considered. We further assume a typical newsvendor situation with a normal distributed demand D~*N*(μ,σ), where μ represents the mean and σ denotes the standard deviation. The short sales season allows only one order point, capacities are unrestricted, and decision-makers behave rational. We further consider one season and one product. In order to produce a certain production quantity on a value stage, the same quantity of the upstream value stage is required. Hence, defectives are not considered. As all relevant production processes have to be finished before the sales season, lead times can be neglected. We further assume that a production process started at site 1 have to be finished at site N. Each site's production costs are given by c_1 to c_N. The revenue r is paid by end customers for one unit of the product. The salvage value v_0 of a product is the value of a product after the season.

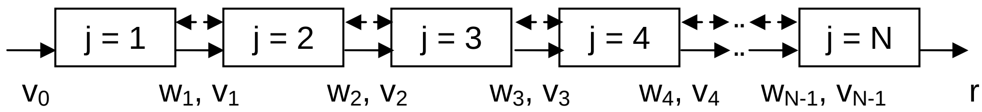

Figure 1: Decision situation for vertical interaction

The supply chain can be coordinated by the transfer prices w_1 to w_{N-1}, where w_1 represents the price, which site 1 achieves from site 2 for one unit. The buyback prices v_1 to v_{N-1} are paid by the supplying site for every unit, which has not been sold during sales season.

Theorem 1:

If $w_1,\ldots, w_{N-1}$ and $v_1,\ldots, v_{N-1}$ are determined in a way that $CR_j = CR_{j-1}$ holds for every j between 2 and N, then $CR_j = CR_g$ is true for every j between 1 and N, where

$$CR_1 = \frac{w_1 - c_1}{v_1 - v_0} \tag{1}$$

and

$$CR_j = \frac{w_j - w_{j-1} - c_j}{v_j - v_{j-1}} \quad \text{for } 2 \le j \le N-1 \tag{2}$$

and

$$CR_N = \frac{r - w_{N-1} - c_N}{r - v_{N-1}} \tag{3}$$

and

$$CR_g = \frac{r - \sum_{k=1}^{N} c_k}{r - v_0}\,. \tag{4}$$

Proof:

First it is shown by induction that the statement

$$\left(CR_m = CR_{m-1}\ \forall\ 2 \le m \le j\right) \Leftrightarrow v_j = \frac{v_1 - v_0}{w_1 - c_1}\left(w_j - w_1 - \sum_{k=2}^{j} c_k\right) + v_1 \tag{5}$$

is true for every j between 2 and N-1.

Basis: Showing that the statement (5) holds for j = 2 and j =3.

j = 2:

$$\begin{aligned} & CR_2 = CR_1 \\ \Leftrightarrow\ & \frac{w_2 - w_1 - c_2}{v_2 - v_1} = \frac{w_1 - c_1}{v_1 - v_0} \\ \Leftrightarrow\ & v_2 = \frac{v_1 - v_0}{w_1 - c_1}\left(w_2 - w_1 - c_2\right) + v_1 \end{aligned} \tag{6}$$

j = 3:

$$CR_3 = CR_2$$
$$\Leftrightarrow v_3 = \frac{v_2 - v_1}{w_2 - w_1 - c_2}\left(w_3 - w_2 - c_3\right) + v_2 \tag{7}$$

Replacing v_2 with the expression from (6) gives

$$\begin{aligned} v_3 &= \frac{\left(v_1 - v_0\right)\left(w_2 - w_1 - c_2\right)}{\left(w_1 - c_1\right)\left(w_2 - w_1 - c_2\right)}\left(w_3 - w_2 - c_3\right) + \frac{v_1 - v_0}{w_1 - c_1}\left(w_2 - w_1 - c_2\right) + v_1 = \\ &= \frac{v_1 - v_0}{w_1 - c_1}\left(w_3 - w_2 - c_3 + w_2 - w_1 - c_2\right) + v_1 = \\ &= \frac{v_1 - v_0}{w_1 - c_1}\left(w_3 - w_1 - c_3 - c_2\right) + v_1 \end{aligned} \tag{8}$$

Inductive step:

Induction hypothesis: Statement (5) holds for j-1 and for j.

Showing that Statement (5) then also holds for j+1:

$$CR_{j+1} = CR_j$$
$$\Leftrightarrow v_{j+1} = \frac{v_j - v_{j-1}}{w_j - w_{j-1} - c_j}\left(w_{j+1} - w_j - c_{j+1}\right) + v_j \tag{9}$$

Using the induction hypothesis v_{j+1} can be transformed to

$$\begin{aligned} v_{j+1} &= \frac{\left(v_1 - v_0\right)\left(w_j - w_{j-1} - c_j\right)}{\left(w_j - w_{j-1} - c_j\right)\left(w_1 - c_1\right)}\left(w_{j+1} - w_j - c_{j+1}\right) + \frac{v_1 - v_0}{w_1 - c_1}\left(w_j - w_1 - \sum_{k=2}^{j} c_k\right) + v_1 = \\ &= \frac{v_1 - v_0}{w_1 - c_1}\left(w_{j+1} - w_j - c_{j+1} + w_j - w_1 - \sum_{k=2}^{j} c_k\right) + v_1 = \\ &= \frac{v_1 - v_0}{w_1 - c_1}\left(w_{j+1} - w_1 - \sum_{k=2}^{j+1} c_k\right) + v_1 \end{aligned} \tag{10}$$

This way, it is proven that statement (5) holds for every j between 2 and N-1. We can now calculate an expression for v_1 from $CR_N = CR_{N-1}$:

$$\begin{aligned} & CR_N = CR_{N-1} \\ & \Leftrightarrow \frac{r - w_{N-1} - c_N}{r - v_{N-1}} = \frac{w_{N-1} - w_{N-2} - c_{N-1}}{v_{N-1} - v_{N-2}} \\ & \Leftrightarrow \left(r - w_{N-1} - c_N\right)\left(v_{N-1} - v_{N-2}\right) = \left(w_{N-1} - w_{N-2} - c_{N-1}\right)\left(r - v_{N-1}\right) \end{aligned} \tag{11}$$

Replacing v_{N-1} and v_{N-2} with the expression from (5):

$$\Leftrightarrow \left(r-w_{N-1}-c_N\right)\frac{v_1-v_0}{w_1-c_1}\left(w_{N-1}-w_{N-2}-c_{N-1}\right)=$$

$$=\left(w_{N-1}-w_{N-2}-c_{N-1}\right)\left[r-\frac{v_1-v_0}{w_1-c_1}\left(w_{N-1}-w_1-\sum_{k=2}^{N-1}c_k\right)-v_1\right]$$

$$\Leftrightarrow \left(r-w_{N-1}-c_N\right)\frac{v_1-v_0}{w_1-c_1}=r-\frac{v_1-v_0}{w_1-c_1}\left(w_{N-1}-w_1-\sum_{k=2}^{N-1}c_k\right)-v_1$$

$$\Leftrightarrow r\left(w_1-c_1\right)=$$

$$=\left(v_1-v_0\right)\left(w_{N-1}-w_1-\sum_{k=2}^{N-1}c_k\right)+v_1\left(w_1-c_1\right)+\left(r-w_{N-1}-c_N\right)\left(v_1-v_0\right)$$

$$\Leftrightarrow r\left(w_1-c_1\right)=v_1\left(w_{N-1}-w_1-\sum_{k=2}^{N-1}c_k+w_1-c_1+r-w_{N-1}-c_N\right)-$$

$$-v_0\left(w_{N-1}-w_1-\sum_{k=2}^{N-1}c_k+r-w_{N-1}-c_N\right)$$

$$\Leftrightarrow v_1=\frac{r\left(w_1-c_1\right)+v_0\left(r-w_1-\sum_{k=2}^{N}c_k\right)}{r-\sum_{k=1}^{N}c_k} \tag{12}$$

v_1 in CR_1 gives:

$$CR_1=\frac{\left(w_1-c_1\right)\left(r-\sum_{k=1}^{N}c_k\right)}{r\left(w_1-c_1\right)+v_0\left(r-w_1-\sum_{k=2}^{N}c_k\right)-v_0\left(r-\sum_{k=1}^{N}c_k\right)}=$$

$$=\frac{\left(w_1-c_1\right)\left(r-\sum_{k=1}^{N}c_k\right)}{r\left(w_1-c_1\right)-v_0\left(w_1-c_1\right)}=\frac{r-\sum_{k=1}^{N}c_k}{r-v_0}=CR_g \tag{13}$$

So $CR_j = CR_g$ is true for every j between 1 and N. □

2 Horizontally interacting newsvendors

In order to transfer the vertical model to horizontal supply chain interaction we assume an assembly chain without substitution. Each production competence is unique in the supply chain. The parameters $c_1,\ldots, c_N$ and r are defined as in section 1. Site N assembles components, which are supplied by the sites 1 to N-1. The transfer prices w_1 to w_{N-1} are paid by site N to the sites 1 to N-1 for one unit of a component to be assembled. The salvage value v_0 is paid directly to the assembly site. The prices v_1 to v_{N-1} represent transfer payments, which are introduced to share the overage risk. Vertical supply chain interaction between more than two value stages is not considered (cf. figure 2).

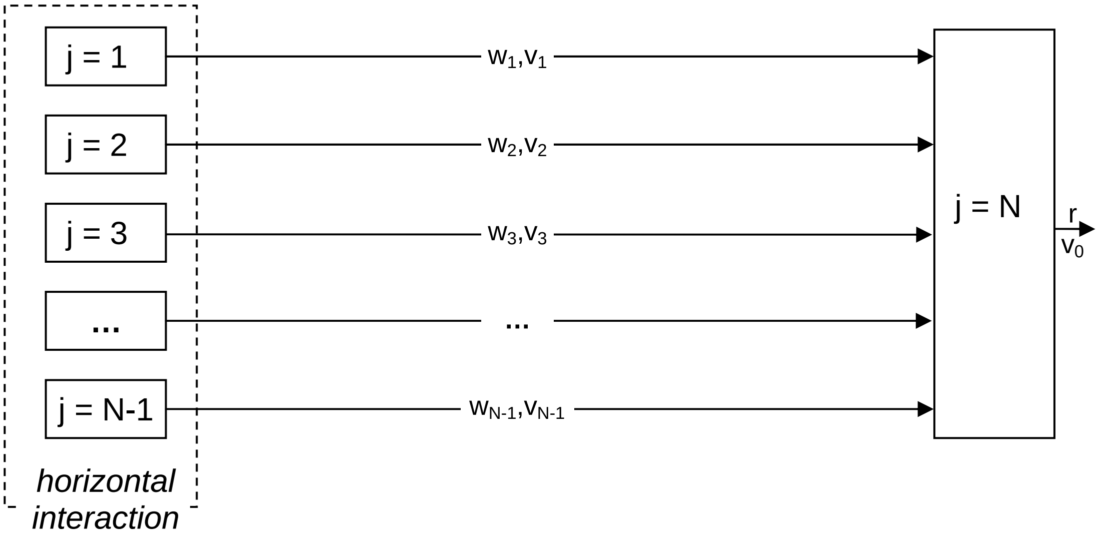

Figure 2: Decision situation for horizontal interaction

Theorem 2:

If $w_1,\ldots, w_{N-1}$ and $v_1,\ldots, v_{N-1}$ are determined in a way that $CR_j = CR_{j-1}$ holds for every j between 2 and N, then $CR_j = CR_g$ is true for every j between 1 and N, where

$$CR_j = \frac{w_j - c_j}{v_j} \quad \text{for } 1 \le j \le N-1 \tag{14}$$

and

$$CR_N = \frac{r - \sum_{k=1}^{N-1} w_k - c_N}{r - \sum_{k=0}^{N-1} v_k} \tag{15}$$

and

$$CR_g = \frac{r - \sum_{k=1}^{N} c_k}{r - v_0} \tag{16}$$

Proof:

First it is shown by induction that the statement

$$\left(CR_m = CR_{m-1} \ \ \forall\ 2 \le m \le j\right) \Leftrightarrow v_j = \frac{w_j - c_j}{w_1 - c_1} v_1 \tag{17}$$

is true for every j between 2 and N-1.

Basis: Showing that the statement (17) holds for j = 2:

$$CR_2 = CR_1 \Leftrightarrow \frac{w_2 - c_2}{v_2} = \frac{w_1 - c_1}{v_1} \Leftrightarrow v_2 = \frac{w_2 - c_2}{w_1 - c_1} v_1 \tag{18}$$

Inductive step:

Induction hypothesis: Statement (17) holds for j.

Showing that Statement (17) then also holds for j+1:

$$\begin{aligned} & CR_{j+1} = CR_j \\ & \Leftrightarrow \frac{w_{j+1} - c_{j+1}}{v_{j+1}} = \frac{w_j - c_j}{v_j} \Leftrightarrow v_{j+1} = \frac{w_{j+1} - c_{j+1}}{w_j - c_j} v_j \end{aligned} \tag{19}$$

Using the induction hypothesis v_{j+1} can be rewritten to

$$\begin{aligned} v_{j+1} &= \frac{w_{j+1} - c_{j+1}}{w_j - c_j} v_j = \frac{\left(w_{j+1} - c_{j+1}\right)\left(w_j - c_j\right)}{\left(w_j - c_j\right)\left(w_1 - c_1\right)} v_1 = \\ &= \frac{w_{j+1} - c_{j+1}}{w_1 - c_1} v_1 \end{aligned} \tag{20}$$

This way it is proven that statement (17) holds for every j between 2 and N-1. Now we can calculate an expression for v_1 from $CR_N = CR_{N-1}$:

$$
\begin{aligned}
& CR_N = CR_{N-1} \\
\Leftrightarrow & \frac{r - \sum_{k=1}^{N-1} w_k - c_N}{r - \sum_{k=0}^{N-1} v_k} = \frac{w_{N-1} - c_{N-1}}{v_{N-1}} \\
\Leftrightarrow & \left(r - \sum_{k=1}^{N-1} w_k - c_N \right) v_{N-1} = \left(w_{N-1} - c_{N-1} \right) \left(r - \sum_{k=0}^{N-1} v_k \right)
\end{aligned} \tag{21}
$$

Replacing all v_j with the expression from (17):

$$
\begin{aligned}
\Leftrightarrow & \left(r - \sum_{k=1}^{N-1} w_k - c_N \right) \frac{w_{N-1} - c_{N-1}}{w_1 - c_1} v_1 = \left(w_{N-1} - c_{N-1} \right) \left(r - v_0 - \sum_{k=1}^{N-1} \frac{w_k - c_k}{w_1 - c_1} v_1 \right) \\
\Leftrightarrow & \left(r - \sum_{k=1}^{N-1} w_k - c_N \right) v_1 = \left(r - v_0 \right) \left(w_1 - c_1 \right) - \sum_{k=1}^{N-1} \left(w_k - c_k \right) v_1 \\
\Leftrightarrow & v_1 \left[r - \sum_{k=1}^{N-1} w_k - c_N + \sum_{k=1}^{N-1} \left(w_k - c_k \right) \right] = \left(r - v_0 \right) \left(w_1 - c_1 \right) \\
\Leftrightarrow & v_1 \left(r - \sum_{k=1}^{N} c_k \right) = \left(r - v_0 \right) \left(w_1 - c_1 \right) \\
\Leftrightarrow & v_1 = \frac{\left(r - v_0 \right) \left(w_1 - c_1 \right)}{r - \sum_{k=1}^{N} c_k}
\end{aligned} \tag{22}
$$

v_1 in CR_1 gives:

$$
\begin{aligned}
CR_1 &= \frac{\left(w_1 - c_1 \right) \left(r - \sum_{k=1}^{N} c_k \right)}{\left(r - v_0 \right) \left(w_1 - c_1 \right)} = \\
&= \frac{r - \sum_{k=1}^{N} c_k}{r - v_0} = CR_g
\end{aligned} \tag{23}
$$

So $CR_j = CR_g$ is true for every j between 1 and N. □

3 Conclusion

In this paper we introduce the mathematical basis for a new and innovative solution method for the multi-location newsvendor problem. In particular, this method allows decentralized planning approaches in situations with incomplete information.