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An aggregate welfare optimizing interest rate rule under heterogeneous expectations

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An aggregate welfare optimizing interest rate rule under heterogeneous expectations

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Abstract

In this paper, I propose an optimal interest rate rule under heterogeneous expectations derived from a welfare criterion that is a second-order approximation of heterogeneous household utility following Di Bartolomeo et al. (2016). Additionally, I explore the agent level of the Branch and McGough (2009) framework in a more detailed fashion which is important as the central bank's welfare criterion depends on consumption inequality. I find that the consumption decision of "rational" agents in Di Bartolomeo et al. (2016) is inconsistent with the higher-order beliefs assumption of Branch and McGough (2009). Hence, consumption rules are derived that are consistent with the micro-foundations of Branch and McGough (2009) including a possible specification of agent's long-run beliefs. Further, the welfare analysis shows that the optimal interest rate rule yields welfare gains that range between 0.1 and 7.1 percent under the considered parameter values relative to a rule that is optimized under a conventional inflation-targeting objective as in Gasteiger (2014). Welfare gains are high when the underlying economy features a high degree of heterogeneity.

Keywords: Optimal monetary policy, policy implementation, heterogeneous expectations, inequality

JEL classifications: E52, D84.

1 Introduction

The New Keynesian model emphasizes the ability of monetary policy to stabilize the macroeconomy by taking into account agent's expectations. However, optimal monetary policy is usually studied within a framework that assumes agents to form their expectations rationally (Clarida et al., 1999). Yet, econometric studies based on inflation expectation surveys show that the data favors heterogeneous expectations with a certain degree of bounded rationality (Branch, 2004, 2007; Pfajfar and Santoro, 2010). Starting from this observation, several New Keynesian models including heterogeneous expectations were designed (Branch and McGough, 2009, 2010; De Grauwe, 2011; Massaro, 2013; Hommes et al., 2015).

A natural follow-up question is: How should central banks set interest rates optimally given its knowledge about the heterogeneity in expectations? To answer this question I derive an optimal interest rate rule under commitment based on the Branch and McGough (2009) framework and a model-consistent welfare criterion following Di Bartolomeo et al. (2016). To describe the micro level explicitly, I derive consumption Euler equations that adequately account for the assumptions of Branch and McGough (2009) about higher-order beliefs and long-run expectations. More specifically, the underlying model of this paper incorporates two types of agents. The more sophisticated agents, that I call "rational forecasters", are able to forecast *aggregate* variables rationally but are not smart enough to understand the micro level fully. In contrast, the boundedly rational forecasters use a systematically biased backward-looking heuristic instead for forecasting.

Although Di Bartolomeo et al. (2016) implicitly assume expectations-based reaction functions, they do not derive them. Thus, the literature has so far not provided a fully optimal interest rate rule under heterogeneous expectations based on the Branch and McGough (2009) model. Deriving an interest rate rule is important as well-grounded policy advice does require an actual rule for policymakers to apply. Further, such policy rules provide an intuitive way of identifying which variables are important in determining the interest rate and how their influence differs especially with the degree of heterogeneity. I will focus on the commitment case as it is typically superior to discretion.

As already indicated, I further add to the literature by exploring the agent level of the Branch and McGough (2009) framework in a more detailed fashion. The micro level has to be made explicit as the central bank's objective function introduced by Di Bartolomeo et al. (2016) depends on consumption inequality. I find that the *conventional* consumption Euler equation under *rational expectations* to describe the consumption decision of "rational forecasters" as done by Di Bartolomeo et al. (2016) is inconsistent with the higher-order beliefs assumption of Branch and McGough (2009). This assumption puts a particular (non-rational) structure on the beliefs of agents about the beliefs of other agents which implies that *no agent in this economy* can have rational expectations with respect to the distribution of individual consumption. Hence, even "rational forecasters" are not smart enough to sophisticatedly forecast individual consumption.

However, if the conventional Euler equation is applied, the first-order conditions of the central bank problem under commitment could only be reduced to a second-order difference equation in one of the Lagrange-multipliers to which the solution is fairly complicated and exponentially depends on time. Consequently, a meaningful interest rate rule under commitment in this case cannot be derived.

To fix this inconsistency, I derive consumption decisions that incorporate the higher-order beliefs assumption of Branch and McGough (2009) which show that agents rely on output gap expectations rather than expectations of individual consumption. Additionally, I provide a possible specification of Branch and McGough (2009)'s assumption that agents agree on expected differences in expected limiting consumption. To be precise, I assume that all agents believe that if their current bond holdings increase (decrease), they will expect to be able to consume more (less) in the long-run. This behavioral assumption results in a consumption equation which shows some similarities to Kurz et al. (2013). Consequently, explosive equilibria where agents would be able to roll over their debt until infinity are ruled out.

The final consumption rule solves the inconsistency mentioned above and also makes the derivation of a meaningful interest rate rule under commitment possible.

Additionally, I compare the optimal interest rate rule to a micro-founded version of the policy rule in Gasteiger (2014) and Gasteiger (2018) which is originally not done by Di Bartolomeo et al. (2016). However, a similar analysis is conducted by Beqiraj et al. (2017) in the Massaro (2013) framework. The policy rule in Gasteiger (2014) and Gasteiger (2018) recognizes the heterogeneity in expectations in the private sector equations but is optimized under a conventional inflation-targeting objective. The latter mimics the trade-off between inflation and output gap variations of real-world central banks. Thus, investigating the properties of an interest rate rule under the model-consistent welfare function potentially implies the recommendation for central banks to depart from their usual price-stability mandate.

The welfare analysis shows that the interest rate rule under the model-consistent loss function generates welfare gains that range from 0.1 to 7.1 percent under the considered parameter values. These results depend on the influence of heterogeneity on the equilibrium, i.e. the relative fraction of each agent type and whether bounded rational forecasters have mean-reverting, naive or trend-setting expectations. More specifically, the findings can be explained with the amplification mechanism that is due to the interaction of rational and bounded rational forecaster's expectations, the resulting complex nature of price dispersion and consumption inequality.

Also, welfare losses due to inequality are higher under the model-consistent policy which is due to the hawkishness of the central bank. However, inequality losses are not very large compared to overall welfare in this model.

Moreover, I find that welfare losses can substantially be underestimated when measured by the conventional inflation-targeting objective while the underlying economy features a certain fraction of backward-looking agents. Hence, this puts a warning sign on the use of the conventional inflation-targeting objective as a welfare measure to evaluate policy options in the presence of bounded rational forecasters.

Finally, I find that when the central bank takes into account the endogenous nature of rational expectations it gets a better grip on the amplification mechanism induced by the interaction of bounded rational and rational expectations compared to a situation where the central bank takes rational expectations as given. However, for small fractions of rational forecasters, the advantage of choosing rational expectations diminishes.

Hence, I extend the existing literature in *four* ways. Firstly, I explicitly consider the implementation of fully optimal monetary policy under heterogeneous expectations. Second, I derive consumption rules that account for the assumptions of Branch and McGough (2009) regarding higher-order beliefs and long-run expectations. Third, I compare the optimal interest rate rule to a rule that is optimized under a conventional-inflation targeting objective but the same private sector equations. Fourth, I find that the conventional inflation-targeting objective substantially underestimates welfare losses when bounded rational forecasters are present.

The remainder of the paper is organized as follows. In Section 2 the relevant literature is introduced. The underlying model, the critique to Di Bartolomeo et al. (2016) and the modified consumption rules are presented in Section 3. The optimal interest rate rule under heterogeneous expectations is derived in the subsequent section. Section 5 shows the impulse responses under optimal monetary policy with an emphasis on the micro-behavior followed by the welfare analysis in Section 6. Finally, the conclusion is given in Section 7.

2 Literature review

2.1 Bounded rationality, heterogeneous expectations and empirical evidence

The aggregate model in this paper is developed in Branch and McGough (2009). The authors slightly deviate from the canonical New Keynesian model by allowing for two types of different forecasting rules with fixed fractions. Forecasters have either rational expectations with respect to *aggregate* variables or employ a static backward-looking and hence biased rule. One advantage of this model is that it can intrinsically account for persistence of macroeconomic aggregates. For instance, Fuhrer (2017) identifies slowly moving expectations as a source of persistence by incorporating survey data on inflation expectations into a New Keynesian model. In Branch and McGough (2009) all agents are assumed to satisfy their individual Euler equation given their subjective forecasts but disregard their intertemporal budget constraint as an optimality condition. An advantage of this approach is that Branch and McGough (2009) facilitate aggregation so as to obtain the same functional forms of inflation and output as in the canonical model.

The work of Massaro (2013) is closely related as he also incorporates heterogeneous expectations into a New Keynesian model but follows Preston and Parker (2005) in that he assumes agents to forecast over an infinite horizon. This stems from the fact that agents do not disregard their intertemporal budget constraint as an optimality condition. In a model with fully rational expectations only the one-period-ahead forecast matters because agents already incorporate every available information in an optimal way in forecasting the next period. This implies an infinite horizon by the forward-looking structure of the canonical model. Consequently, agents that are boundedly rational have to forecast over an infinite horizon since their one-period-ahead forecast is not optimal. The advantage of this approach is that Massaro (2013) does not need to impose further assumptions on heterogeneous expectations to facilitate aggregation. However, this comes at the cost of not obtaining the same functional forms as in the canonical model which impairs comparability.

Branch and McGough (2010) extend their 2009-model by allowing agents to switch forecasting rules based on Brock and Hommes (1997).[1] In the same spirit Hommes et al. (2015) allow for expectations that are consistent with the central bank's inflation target and naive backward-looking expectations. By evaluating the relative forecasting performance agents can switch between these two heuristics which implies that the central bank's credibility is endogenous. Further mentionable work is also done by De Grauwe (2011) who introduces herding behavior by assuming heterogeneous and bounded rational agents with cross-correlations in beliefs.

The introduction of heterogeneous expectations is empirically relevant. For instance, Branch (2004) sets up a model in which agents choose rationally from a set of costly forecast instruments that vary in the degree of sophistication and thereby lead to bounded rationality. This model is tested in a statistical discrete choice setting which shows, first, that US inflation expectation data favors heterogeneous expec-

[1] The so-called misspecification equilibrium is first introduced into dynamic macroeconomics by Branch and Evans (2006) where agents are able to choose from a set of different forecasting rules based on the relative forecasting performance.

tations over rational expectations and, second, the presence of dynamic predictor selection. In his later work, he also confirms heterogeneous expectations in which he compares different models of expectations heterogeneity including sticky information (Branch, 2007). Further support is provided by Pfajfar and Santoro (2010). They identify three types of agents in the distribution of US inflation expectations that differ in their forecasting behavior: static backward-looking (as in Branch and McGough (2009)), near-rational and adaptive learning behavior.

Thus, incorporating heterogeneous expectations into macroeconomic models is of practical relevance and more than being merely a theorist's "what if" thought experiment.

2.2 (Optimal) Monetary policy under bounded rationality and heterogeneous expectations

Optimal monetary policy under homogeneous rational expectations is well known and extensively studied (see for instance Clarida et al. (1999); Woodford (1999); McCallum (1999)).

However, the strand of literature that focuses on optimal monetary policy under heterogeneous expectations is rather new. For instance, Di Bartolomeo et al. (2016) contribute to the literature originating from Branch and McGough (2009) by deriving a central bank's welfare criterion that is consistent with heterogeneous expectations. Recognizing heterogeneous expectations explicitly in the welfare function can yield substantial welfare gains when heterogeneity is high and is further important to avoid welfare measurement errors as I show in this paper. One very interesting implication of this welfare function is the introduction of consumption inequality as a third dimension of central bank policy. Additionally, they show that price dispersion is not only caused by inflation and price stickiness alone but also by different expectations regarding future marginal costs. Further, the authors investigate the implied policies under discretion and commitment. However, they assume rational forecasters to determine their consumption by the standard Euler equation under rational expectations. I will argue that assuming rational forecasters to have rational expectations with respect to individual consumption is inconsistent with the higher-order beliefs assumption of Branch and McGough (2009). Therefore, I depart from Di Bartolomeo et al. (2016) in that I derive consumption rules that are consistent with this higher-order beliefs assumption. Also, Di Bartolomeo et al. (2016) do not provide interest rate rules while I do. However, as commitment is typically superior to discretion, I focus on the commitment case in this paper.

Closely related is the work of Beqiraj et al. (2017) who also derive a model-consistent loss function for the Massaro (2013) framework. The discretionary policy implied by the model-consistent loss function is then compared to the same objective excluding consumption inequality and to a conventional inflation-targeting objective. First, consumption inequality is minimized by the policy under the model-consistent loss function and, second, yields the lowest welfare costs overall. The former result stands in contrast to the findings in this paper which is due to the different micro-foundations while the latter also holds here. Further, the inflation targeting objective in Beqiraj et al. (2017) performs worst since it neglects consumption inequality and the more complex nature of price dispersion which is in line with the findings in this paper. Also, as in Di Bartolomeo et al. (2016) the authors do not derive interest rate rules.

In contrast to Di Bartolomeo et al. (2016) and Beqiraj et al. (2017), Gasteiger (2014) and Gasteiger (2018) assume the conventional inflation-targeting objective to study optimal monetary policy in the Branch and McGough (2009) framework. Their main contribution is to show that expectations-based reaction functions perform exceptionally well, i.e. yield determinacy for a large part of the parameter space. However, the expectations-based property of the reaction function does not guarantee determinacy throughout the considered parameter space as long as the conventional inflation-targeting objective is used.

On the other hand, Bullard and Mitra (2002), Evans and Honkapohja (2003a,b, 2006) and Duffy and Xiao (2007) investigate (optimal) monetary policy under *homogeneous* adaptive learning of agents. Agents thereby make biased forecasts but learn over time, i.e. update their parameters via recursive least squares. The convergence of the learning process to a determinate and stable equilibrium critically depends on the design of monetary policy. Evans and Honkapohja (2003a,b) therefore emphasize the importance of expectations-based reaction functions and discard fundamentals-based reaction functions, i.e. interest rate rules incorporating only shocks and predetermined variables. The reason is that slight deviations of inflation expectations from the rational benchmark induce divergence from the rational expectations equilibrium that is not offset by the fundamentals-based reaction function. This finding is in line with Bullard and Mitra (2002) who recommend policy functions that bring about learnable rational expectations equilibria. However, Duffy and Xiao (2007) find that fundamentals-based reaction functions can yield stability and determinacy if the central bank considers also interest rate stabilization in their objective function. Further, Evans and Honkapohja (2006) show the importance of commitment in the case of learning agents and thereby present a further case where discretion is suboptimal compared to commitment.

3 Model

In this section, I use and further explore the Branch and McGough (2009) model. However, the notation of Di Bartolomeo et al. (2016) is maintained for comparability reasons.

It is assumed that the economy is populated by an exogenous fraction α of rational forecasters (R) which have rational expectations with respect to *aggregate* variables and a fraction $1-\alpha$ of boundedly rational forecasters (B) that employ a simple backward-looking heuristic. The general forecasting rule of bounded rational forecasters takes the form of $E_t^B x_{t+1} = \theta^2 x_{t-1}$ while rational forecasters simply use conditional statistical expectations, i.e. $E_t^R x_{t+1} = E_t x_{t+1}$, for *forecasting the output gap and inflation.* Backward-looking expectations for $\theta < 1$ are called steady-state-reverting, for $\theta = 1$ naive and for $\theta > 1$ trend-setting. Steady-state-reverting expectations constitute a stabilizing force while trend-setting expectations imply a further amplification of macroeconomic variables.

Assuming perfect consumption insurance *within* each of the two agent groups the model can be expressed in terms of two representative agents (RA). Both RA's maximize their individual expected discounted lifetime utility $E_t^\tau \sum_{t=0}^{\infty} \beta^t U_t$ given their subjective expectations E_t^τ with $\tau \in \{R, B\}$. However, as in Branch and McGough (2009) agents follow Euler equation learning, i.e. they disregard their intertemporal budget constraint as an optimality condition and solely base their consumption decision on the variational intuition of the Euler equation. The period utility function is of CES-form and is given by

$$U_t = \frac{(C_t^\tau)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{(Y_t^\tau)^{1+\eta}}{1+\eta} \tag{1}$$

with C_t^τ being consumption of type τ, Y_t^τ the output that each RA τ produces, $\frac{1}{\sigma}$ the coefficient of relative risk aversion and η the elasticity of marginal disutility of producing output. Agents must satisfy their real budget constraint

$$C_t^\tau + B_t^\tau = \frac{1+i_{t-1}}{\Pi_t} B_{t-1}^\tau + \Psi_t^\tau \tag{2}$$

with B_t^τ being real bond holdings, i_{t-1} the nominal interest rate in $t-1$, Π_t gross inflation and Ψ_t^τ real income of agent τ. For the log-linearized version and specification of real income Ψ_t^τ consider the Appendix A.

I will now turn to the derivation of agent's consumption decisions. As already indicated, the individual level is important for monetary policy as the central bank's objective function derived by Di Bartolomeo et al. (2016) depends on consumption inequality. Therefore, I will discuss the implications of Branch and McGough (2009)'s assumptions for the consumption decisions at length and show that the conventional

Euler-Equation under rational expectations as applied by Di Bartolomeo et al. (2016) is at odds with these assumptions. Incorporating these assumptions into the consumption decisions is crucial as a meaningful interest rate rule under commitment cannot be derived otherwise (see Appendix D) while it also matters for welfare (see Section 6). Further, the welfare criterion of the central bank can be re-written using market clearing in a way that only the consumption decision of rational forecasters is needed as an extra equation. Hence, the consumption Euler equation for $\tau = R$ reads

$$(C_t^R)^{-\frac{1}{\sigma}} = \beta E_t^R \left[(C_{t+1}^R)^{-\frac{1}{\sigma}} \frac{1+i_t}{\Pi_{t+1}} \right]. \tag{3}$$

Log-linearizing (3) gives

$$c_t^R = E_t^R c_{t+1}^R - \sigma(i_t - E_t^R \pi_{t+1}) \tag{4}$$

where lower case letters indicate log-deviations from steady state. Forward iteration yields

$$c_t^R = E_t^R c_\infty^R - \sigma \sum_{k=0}^{\infty} (i_{t+k} - E_t^R \pi_{t+k+1}). \tag{5}$$

It is assumed that rational forecasters know that market clearing $y_t = \alpha c_t^R + (1-\alpha) c_t^B$ holds and that bounded rational forecasters will also satisfy their consumption Euler equation. Writing market clearing one period forward and inserting equation (5) gives

$$\begin{aligned} E_t^R y_{t+1} = E_t^R \Big[& \alpha (E_t^R c_\infty^R - \sigma E_{t+1}^R \sum_{k=1}^{\infty} (i_{t+k} - \pi_{t+k+1})) \\ & + (1-\alpha)(E_t^B c_\infty^B - \sigma E_{t+1}^B \sum_{k=1}^{\infty} (i_{t+k} - \pi_{t+k+1})) \Big]. \end{aligned} \tag{6}$$

Note that (6) contains higher-order beliefs, in particular beliefs of rational forecasters E_t^R about the beliefs of bounded rational forecasters E_t^B and E_{t+1}^B. In order to arrive at the IS curve that has the same functional form as in the model under homogeneous rational expectations, Branch and McGough (2009) need to impose a specific (non-rational) structure on higher-order beliefs on consumption. This assumption states that agent's believe that other agents will forecast their consumption in the same way they do. Mathematically and in the context of rational forecasters: $E_t^R E_{t+k}^B c_{t+l} = E_t^R c_{t+l}$ with $l > k$. Hence, bounded rational expectations just drop out under this assumption. Further, note that making an alternative assumption, e.g. allowing rational forecasters to be fully rational, would result in a different system of aggregate equations.

Using the higher-order beliefs assumption and the law of iterated expectations at the individual level yields

$$E_t^R y_{t+1} = E_t^R y_\infty - \sigma \sum_{k=1}^{\infty} (i_{t+k} - E_t^R \pi_{t+k+1}). \tag{7}$$

It becomes obvious that (7) cannot hold under conventional rational expectations, i.e. when $E_t^R = E_t$ would hold, as bounded rational expectations, E_t^B and E_{t+1}^B, would *not drop out* and thus show up in (7). Mathematically written: $E_t E_t^B = E_t^B$ and $E_t E_{t+1}^B = E_{t+1}^B$, which contradicts the higher-order beliefs assumption of Branch and McGough (2009). Equation (7) would only hold under rational expectations when boundedly rational forecasters were absent, i.e. under *homogeneous rational expectations.* In this case (6) would collapse to (7) without any further assumption. It follows that rational forecasters in this model are not smart enough to have rational expectations with respect to the distribution of individual consumption *when there is heterogeneity.* Thus, using the conventional Euler equation under rational expectations to describe the individual consumption of rational forecasters as in Di Bartolomeo et al. (2016) is inconsistent with the underlying framework.

Using (7) to replace the infinite sum in (5), one obtains

$$c_t^R = E_t^R y_{t+1} + E_t^R (c_\infty^R - y_\infty) - \sigma(i_t - E_t^R \pi_{t+1}) \tag{8}$$

which is the true consumption decision of rational forecasters satisfying the higher-order beliefs assumption from above.

Equation (8) could have been derived for the general case of agent τ as the assumption on higher-order beliefs holds for both agent types. In the general case (8) reads

$$c_t^\tau = E_t^\tau y_{t+1} + E_t^\tau (c_\infty^\tau - y_\infty) - \sigma(i_t - E_t^\tau \pi_{t+1}). \tag{9}$$

Still, it has to be defined what $E_t^\tau(c_\infty^\tau - y_\infty)$ is. In Branch and McGough (2009) these terms drop out when aggregating (9) and when the assumption that agents agree on expected differences in expected limiting consumption is used (A7 in Branch and McGough (2009)). However, in this case, already an assumption at the individual level is needed.

For the rational agents in this model one might think of assuming that the long-run beliefs $E_t^R(c_\infty^R - y_\infty)$ adjust such that the consumption expectations are indeed rational, i.e. $E_t c_{t+1} = E_t^R y_{t+1} + E_t^R(c_\infty^\tau - y_\infty)$. This could then serve as a justification for Di Bartolomeo et al. (2016) to assume rational consumption expectations. However, as I showed above, individual consumption expectations cannot be rational under the micro-foundations of Branch and McGough (2009). Hence, this possibility can be ruled out.

Further, it could be tempting to assume that all agents will expect the steady state to realize in the limit and that they know that all agents are identical in the steady state. As a consequence $E_t^\tau(c_\infty^\tau - y_\infty)$ would drop out as $E_t^\tau c_\infty^\tau = E_t^\tau y_\infty = 0$. However, this assumption might induce agents to roll-over their debt/ accumulate bonds until infinity as there is no offsetting force in their consumption decision. This would be a particularly unrealistic feature of the model as households in reality wouldn't be able to get any more funds from a bank at some point when they are already heavily indebted. Thus, to rule out explosive equilibria, I introduce a behavioral rule in the form of $E_t^\tau(c_\infty^\tau - y_\infty) = \phi b_{t-1}^\tau$ where b_{t-1}^τ are the beginning-of-period real bonds. This rule states that when individual savings increase (decrease) agents will believe to be able to consume more (less) than the average in the long-run and thereby adjust their consumption today already. Hence, there will be an inherent stabilizing force to debt and savings which replaces a transversality condition or a borrowing constraint similar to Kurz et al. (2013).

As Branch and McGough (2009) are more interested in the aggregate behavior of their model, they just assume that agents agree on expected differences in expected limiting consumption which makes these long-run expectations irrelevant for the aggregate. The assumption I propose above provides a possible specification of what these long-run expectations actually can be.

Further, while I model long-run expectations in a boundedly rational manner here, households in Kurz et al. (2013) derive dis-utility from holding bonds. Making the same assumption in this context would complicate the analysis drastically as the model-consistent welfare criterion of Di Bartolomeo et al. (2016) (see next section) would change.

Hence, (9) becomes

$$c_t^\tau = E_t^\tau y_{t+1} + \phi b_{t-1}^\tau - \sigma(i_t - E_t^\tau \pi_{t+1}). \tag{10}$$

The parameter ϕ is chosen to be 0.015 for the baseline scenario as it is high enough to rule out explosive equilibria and low enough for the analysis to be still somehow comparable to Di Bartolomeo et al. (2016). The robustness check in Appendix E shows that welfare is quite robust with respect to variations in ϕ.

From (10) one can infer that agents only forecast aggregate variables when making consumption decisions. Note that, as rational forecasters have rational expectations with respect to aggregate variables, E_t^R can be replaced by E_t in the consumption decision of rational forecasters.

Now, using goods market clearing, bond market clearing and (10) the IS curve is given by

$$y_t = \alpha E_t y_{t+1} + (1-\alpha)\theta^2 y_{t-1} - \sigma(i_t - \alpha E_t \pi_{t+1} - (1-\alpha)\theta^2 \pi_{t-1}) \tag{11}$$

which is the same as in Branch and McGough (2009).

Further, all agents produce output under monopolistic competition. Calvo pricing is assumed where a fixed fraction ξ_p of agents cannot reset their prices in a given period (Calvo, 1983). Price dispersion arises because, first, optimal prices are different *between* expectation types since they depend on *expected* future marginal costs and, second, they differ *within* each type due to the fact that only a fraction of firms can reset prices. Following Di Bartolomeo et al. (2016) and Branch and McGough (2009) the Phillips-curve reads

$$\pi_t = \alpha\beta E_t \pi_{t+1} + (1-\alpha)\beta\theta^2\pi_{t-1} + \kappa y_t + e_t \tag{12}$$

with $\kappa = \frac{(1-\xi_p)(1-\beta\xi_p)(\eta+\sigma^{-1})}{\xi_p(1+\epsilon\eta)}$ where ϵ is the price elasticity of demand for a differentiated good.[2] As in Di Bartolomeo et al. (2016) the Phillips curve is augmented with a random cost-push shock e_t with expected value zero and finite variance.[3]

Note that inflation and output exhibit some degree of *persistence* due to the presence of backward-looking expectations. The degree of persistence depends on the fraction of bounded rational forecasters $1-\alpha$ and their forecasting coefficient θ. The higher the two parameters the higher the degree of persistence. Further, as rational forecasters use the aggregate equations (11) and (12) to forecast output and inflation, they are aware of this persistence. Hence, even if a *transitory* cost-push shock hits the economy, rational forecasters will expect inflation to be above the steady state in the next period. This non-zero inflation expectation then feeds back into current inflation via (12) and thus causes an *amplification* of inflation (and via the central bank in output). This amplification mechanism is strongest for intermediate values of α as already investigated by Gasteiger (2018). The reason is that for large values of α only a minority of agents is backward-looking and thus persistence becomes less pronounced while for low values of α there are not enough rational forecasters through which the amplification works. Hence, this model associates an amplification of macroeconomic variables with the presence of boundedly rational agents.

The model is calibrated as in Di Bartolomeo et al. (2016) for the US economy following Rotemberg and Woodford (1997) with the time unit being one quarter.

$\alpha = 0.7$	$\theta = 1$	$\beta = 0.99$	$\sigma = 6.25$	$\epsilon = 7.84$	$\eta = 0.47$	$\xi_p = 0.66$	$\phi = 0.015$

Table 1: Baseline calibration

[2]Part of the derivation is given in the Appendix A which is showing explicitly how the higher-order beliefs assumption matters for individual pricing. This is missing in both Branch and McGough (2009) and Di Bartolomeo et al. (2016).

[3]This shock can for instance be micro-founded by assuming an exogenous time-varying wage mark-up as in Gasteiger (2018).

4 An optimal interest rate rule

As in Gasteiger (2014), Gasteiger (2018) and Di Bartolomeo et al. (2016) a paternalistic central bank is assumed, i.e. the central bank's aim is to maximize social welfare. I follow the approach of Di Bartolomeo et al. (2016) where the central bank exploits its detailed knowledge about the heterogeneity in expectations and minimizes a social welfare loss that is a second-order approximation of household utility (1). Di Bartolomeo et al. (2016) obtain the following instantaneous utility-based loss function

$$L_t = \frac{1}{2}\left[\left(\eta + \frac{1}{\sigma}\right) y_t^2 + (\epsilon^2\eta) var_i(p_t(i)) + \frac{1}{\sigma} var_i(c_t(i))\right]. \tag{13}$$

with

$$var_i(p_t(i)) = \delta\pi_t^2 + \frac{\delta\xi_p(1-\alpha)}{\alpha}\left[\pi_t - \beta\theta^2\pi_{t-1} - \kappa\frac{c_t^B + \eta\sigma y_t}{1+\eta\sigma}\right]^2 \tag{14}$$

$$var_i(c_t(i)) = \alpha(1-\alpha)(c_t^R - c_t^B)^2. \tag{15}$$

where $\delta = \frac{\xi_p}{(1-\beta\xi_p)(1-\xi_p)}$ is a measure of price stickiness.

In the rational case where the economy is populated by rational forecasters only, i.e for $\alpha = 1$, price dispersion reduces to $var_i(p_t(i)) = \delta\pi_t^2$ and $var_i(c_t(i))$ to zero. Hence, (13) reduces to

$$L_t^{\alpha=1} = \frac{1}{2}\left[\left(\eta + \frac{1}{\sigma}\right) y_t^2 + \epsilon^2\eta\delta\pi_t^2\right]. \tag{16}$$

Equation (13) is called the model-consistent loss function and (16) the conventional (inflation-targeting) loss function. In the rational case (16), agents suffer from variations in output that is due to the concave nature of consumption utility (risk aversion) and price dispersion due to inflation variations in the presence of price stickiness (represented by δ). The welfare loss (16) is called conventional as it implicitly assumes rational expectations. Further, it represents the trade-off between inflation and output gap stabilization of real-world central banks. In the heterogeneous expectations case (13), price dispersion (14) additionally depends on lagged inflation due to the forecasting heuristic of bounded rational forecasters and terms that are due to differences in expected future marginal costs between types. An interesting novelty of (13) is the appearance of consumption inequality $var_i(c_t(i))$. Hence, agents suffer from consumption inequality as it represents a departure from the more efficient rational expectations benchmark where all choices are optimal and inequality is zero.[4]

[4] Note that agents do not explicitly dislike inequality as it is not part of their utility function but are affected by it as it renders the economy more inefficient.

The model-consistent loss function (16) can be rewritten using market clearing to eliminate c_t^B, (14) and (15) as

$$L_t = \frac{\sigma\eta + 1}{\sigma} y_t^2 + \frac{\alpha(y_t - c_t^R)^2}{(1-\alpha)\sigma} + \epsilon^2 \eta\delta \left\{ \pi_t^2 + \frac{\xi_p(1-\alpha)}{\alpha} \left[\pi_t - \beta\theta^2\pi_{t-1} - \kappa y_t - \frac{\alpha\kappa(y_t - c_t^R)}{(1+\eta\sigma)(1-\alpha)} \right]^2 \right\}. \tag{17}$$

Further, the central bank is assumed to set its interest rate so as to minimize (17) or, respectively, (13) subject to the private sector equations[5]

$$y_t = \alpha E_t y_{t+1} + (1-\alpha)\theta^2 y_{t-1} - \sigma[i_t - \alpha E_t \pi_{t+1} - (1-\alpha)\theta^2 \pi_{t-1}] \tag{18}$$

$$\pi_t = \alpha\beta E_t \pi_{t+1} + (1-\alpha)\beta\theta^2 \pi_{t-1} + \kappa y_t + e_t \tag{19}$$

$$c_t^R = E_t y_{t+1} + \phi b_{t-1}^R - \sigma(i_t - E_t \pi_{t+1}). \tag{20}$$

Note that the change in the consumption decisions relative to Di Bartolomeo et al. (2016) does not affect (17). Hence, (17) as derived by Di Bartolomeo et al. (2016) can be applied.

4.1 Taking rational expectations as given

I start by assuming that the central bank will take, for whatever reason, the rational expectations *as given* (in other words: as exogenous) although it is perfectly informed about the functioning of the economy. Taking rational expectations as given means that the central bank reacts to them when setting its interest rate but neglects that these expectations are endogenous with respect to its policy.[6] Therefore, the central bank reacts suboptimally to private sector rational expectations. I do this, first, to emphasize later on that incorporating the endogenous nature of rational expectations into monetary policy considerations is very important and, second, to present a less complicated intermediate step that already reveals some mechanisms that can be translated to the fully optimal case.

Also, one should not confuse "taking rational expectations as given" with discretionary policy as the central bank still minimizes over an infinite horizon which implies a commitment. The resulting interest rate rule can at best be interpreted as some kind of "limited" commitment.

The optimization procedure under timeless commitment[7] while taking rational expectations as given

[5] To minimize (16) only (19) is needed.

[6] Mathematically this means that the central bank does not take derivatives with respect to rational expectations when optimizing its objective function.

[7] The timeless commitment approach of Woodford (2003) assumes that the optimal commitment policy was implemented in the distant past so as to omit the first period's optimality condition. The problem of the latter is that it renders the

yields

$$
\begin{aligned}
i_t = & \Omega_1 y_{t-1} + \Omega_2 E_t y_{t+1} + \Omega_3 E_t y_{t+2} + \Omega_4 \pi_{t-1} + \Omega_5 E_t \pi_{t+1} + \Omega_6 E_t \pi_{t+2} \\
& + \Omega_7 E_t c^R_{t+1} + \Omega_8 E_t c^R_{t+2} + \Omega_9 e_t
\end{aligned} \tag{21}
$$

where the reaction coefficients Ω_x and derivations are given in the Appendix C.2.[8] The corresponding interest rate rule under the conventional inflation-targeting objective (16) is given by

$$
i_t = \gamma_1 y_{t-1} + \gamma_2 E_t y_{t+1} + \gamma_3 \pi_{t-1} + \gamma_4 E_t \pi_{t+1} + \gamma_5 e_t \tag{22}
$$

with

$$
\gamma_1 = (1-\alpha)\left[\frac{\theta^2}{\sigma}\right] \tag{23}
$$

$$
\gamma_2 = \alpha\frac{1}{\sigma} - \frac{(1-\alpha)}{\sigma}\left[\frac{\beta^2\theta^2(1+\eta\sigma)}{1+\eta\sigma+\delta\epsilon^2\eta\kappa^2\sigma}\right] \tag{24}
$$

$$
\gamma_3 = (1-\alpha)\left[\frac{\theta^2(1+\eta(\sigma+\delta\epsilon^2\kappa(\beta+\kappa\sigma)))}{1+\eta\sigma+\delta\epsilon^2\eta\kappa^2\sigma}\right] \tag{25}
$$

$$
\gamma_4 = \alpha\left[1+\frac{\beta\delta\epsilon^2\eta\kappa}{1+\eta\sigma+\delta\epsilon^2\eta\kappa^2\sigma}\right] \tag{26}
$$

$$
\gamma_5 = \frac{\delta\epsilon^2\eta\kappa}{1+\eta\sigma+\delta\epsilon^2\eta\kappa^2\sigma}. \tag{27}
$$

Equation (22) recognizes the heterogeneity in expectations by introducing lagged terms of output and inflation which drop out for $\alpha = 1$. However, the appearance of lagged terms is due to the private sector equations (18) and (19) only since the conventional objective (16) implicitly assumes rational expectations. I will refer to (21) as the model-consistent interest rate rule and to (22) as the policy/interest rate rule under the conventional objective from now on (and equivalently under full commitment as below).

The model-consistent interest rate rule (21) introduces four additional terms that are due to the different expectations regarding future marginal costs and consumption inequality. As the heterogeneity in expectations causes both phenomena, the central bank ultimately tries to stabilize expectations. This still holds true under the hypothetical situation here where the central bank takes rational expectations as given. The central bank is still able to align both expectation types by exerting influence on the

policy time-inconsistent. Hence, the drop of the initial period's optimality condition solves this problem.

[8] These coefficients are large composite parameters and therefore not useful for interpretation. However, calculating them numerically gives a good intuition on how the different terms affect the interest rate.

backward-looking expectations and therefore *indirectly* the rational ones. To understand the appearance of the $t+2$ terms consider figure (1) which displays the reaction of the inflation expectations of both agent types following a one standard deviation i.i.d. cost-push shock under the policy rule (21). Since all subsequent shock realizations are zero and rational forecasters know the true *aggregate* equations, they have *de facto* perfect foresight. Thus, rational forecasters' expectations in $t=1$ about inflation in $t+1$ will be correct, i.e. $E_t\pi_{t+1} = \pi_{t+1}$. However, the backward-looking expectations of bounded rational forecasters in t about inflation in $t+1$ will be zero, i.e. $E_t^B\pi_{t+1} = \pi_{t-1} = 0$ (where $\theta = 1$ for simplicity). In period $t+1$ bounded rational forecasters will expect inflation to *increase* drastically in $t+2$ as their expectations are based on the period where the shock hits, i.e. $E_{t+1}^B\pi_{t+2} = \pi_t$. On the contrary, rational agents correctly expect inflation to *decrease* further as the central bank causes the output gap to be still negative (not shown here). Thus, the different expectations in $t+1$ about $t+2$ diverge transitorily. Therefore, the central bank should set interest rates so as to align the two expectation types in order to minimize the effects of different expectations regarding future marginal costs and consumption inequality. These expectation mechanics apply to output gap expectations in the same way and can be translated to the fully optimal case.

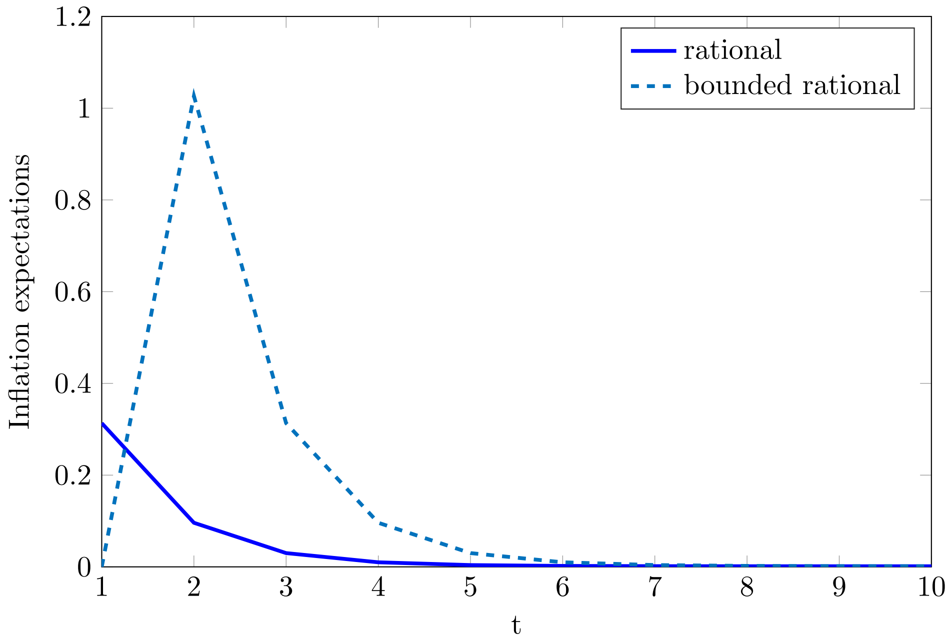

Figure 1: Individual inflation expectations in percentage deviation from steady state following a single, non-autocorrelated cost-push shock of one percent with monetary policy given by (21).

It is important to note that the Ω_x-coefficients for the four additional terms that are in (21) but not in (22) are comparatively small. To see that consider Table 2. All coefficients are calculated under baseline calibration introduced in Section 3 with variations in α (baseline $\alpha = 0.7$). Intuitively, all coefficients of terms that are due to heterogeneous expectations decrease in α and vanish for the rational expectations limit ($\alpha = 1$). In turn, the conventional terms, i.e. one period-ahead rational expectations with respect to output gap and inflation, gain strength in determining the interest rate as α increases. Also, note that when α decreases and thus the fraction of bounded rational forecasters increases, the backward-looking terms, especially for inflation, increase very strongly. However, the central bank finds it optimal to partly offset this effect by reacting to one-period-ahead rational expectations of inflation and output in the opposite way for low values of α.

The same exercise can also be done with variations in θ. For $\theta < 1$ there is a stabilizing force as bounded rational forecasters expect a gradual steady state reversion and for $\theta > 1$ there is an additional amplifying effect as a further departure from the steady state is expected. As a result, all coefficients that are due to heterogeneous expectations increase in θ for a reasonable parameter space to counteract the trend-setting expectations.

Ω_x	$\alpha = 0.1$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$	$\alpha = 1$ (RE)
y_{t-1}	0.144	0.112	0.080	0.048	0.016	0
$E_t y_{t+1}$	-0.078	-0.017	0.045	0.108	0.172	0.160
$E_t y_{t+2}$	-0.005	-0.008	-0.009	-0.007	-0.003	0
π_{t-1}	2.119	1.576	1.071	0.609	0.191	0
$E_t \pi_{t+1}$	-0.458	0.143	0.697	1.199	1.648	1.851
$E_t \pi_{t+2}$	0.103	0.064	0.033	0.012	0.001	0
$E_t c^R_{t+1}$	-0.004	-0.012	-0.021	-0.030	-0.040	0
$E_t c^R_{t+2}$	0.002	0.007	0.008	0.007	0.003	0
e_t	1.369	1.264	1.154	1.040	0.921	0.859

Table 2: Values of reaction coefficients Ω_x in the interest rate rule (21) for different values of the share of rational forecasters α

4.2 Incorporating the endogeneous nature of rational expectations

Now, I will relax the assumption that the central bank takes the rational expectations as given and consequently assume that it explicitly incorporates the effect it has on rational expectations when setting its policy rate. Hence, the central bank *chooses* rational expectations, i.e. it takes derivatives with respect to rational expectations when optimizing its objective function, which constitutes the fully optimal case.

The interest rate rule under timeless commitment and the model-consistent welfare loss (13) is given by

$$\begin{aligned} i_t =&\Omega_1^c E_t\pi_{t+1} + \Omega_2^c E_t\pi_{t+2} + \Omega_3^c\pi_{t-3} + \Omega_4^c\pi_{t-2} + \Omega_5^c\pi_{t-1} + \Omega_6^c E_t y_{t+1} + \Omega_7^c E_t y_{t+2} \\ &+\Omega_8^c y_{t-2} + \Omega_9^c y_{t-1} + \Omega_{10}^c E_t c_{t+1}^R + \Omega_{11}^c E_t c_{t+2}^R + \Omega_{12}^c c_{t-2}^R + \Omega_{13}^c c_{t-1}^R + \Omega_{14}^c e_t \end{aligned} \tag{28}$$

where the reaction coefficients Ω_x^c and derivations are given in the Appendix C.3. The corresponding interest rate rule under the conventional inflation-targeting objective (16) and commitment is given by

$$i_t = \gamma_1^c y_{t-1} + \gamma_2 y_{t+1} + \gamma_3\pi_{t-1} + \gamma_4\pi_{t+1} + \gamma_5 e_t \tag{29}$$

with

$$\gamma_1^c = \frac{(1-\alpha)\theta - \alpha}{\sigma} + \alpha\frac{\delta\epsilon^2\eta\kappa^2}{1+\eta\sigma+\delta\epsilon^2\eta\kappa^2\sigma} \tag{30}$$

and γ_2, γ_3, γ_4 and γ_5 as under "limited" commitment. Equation (29) is similar to the rule derived by Gasteiger (2018) and Gasteiger (2014).[9] Note that choosing rational expectations introduces additional persistence as can be seen from γ_1^c. While in the "limited commitment" case γ_1 is zero when $\alpha = 1$, γ_1^c becomes $\frac{\delta\epsilon^2\eta\kappa^2}{1+\eta\sigma+\delta\epsilon^2\eta\kappa^2\sigma} - \frac{1}{\sigma}$. Hence, full commitment introduces persistence even for $\alpha = 1$. It follows that any difference between the interest rate rule (28) and (21) must stem from the additional persistence caused by the decision of the central bank to explicitly choose rational expectations. Note that in the rational limit all coefficients associated with heterogeneous expectations vanish as well as the additional coefficients due to commitment except for y_{t-1} as expected which can be seen in table (5) in the appendix.

5 Impulse Responses

This section briefly describes the simulation outcomes under baseline calibration. Determinacy issues are not discussed as the model is determinate for all considered parameter constellations. There are two

[9] Gasteiger (2018) uses a non-micro-founded version of (16), i.e. $L_t = \frac{1}{2}(\pi_t^2 + \omega y_t^2)$. Setting $\omega = \frac{\sigma\eta+1}{\epsilon^2\eta\delta\sigma}$ and calculating through the optimization problem yields the interest rate rule (29).

reasons for this. First, an expectations-based interest rate rule is derived, i.e. it properly accounts for private sector expectations which are known to perform exceptionally well as opposed to fundamentals-based reaction functions (Evans and Honkapohja, 2006). Second, the interest rate rule is derived from the fully model-consistent loss function. Hence, the proposed interest rate rule is a good proxy for the fully optimal (non-linear) policy.

It should be noted that Di Bartolomeo et al. (2016) show impulse responses as well. They do this by using the solution algorithm of Söderlind (1999) that takes the model-consistent welfare criterion and private sector equations, i.e. the conventional Euler equation under rational expectations, the Phillips and IS curve, as inputs. An expectations-based reaction function under commitment is implicitly assumed. Aggregate variables in the present paper follow a very similar pattern.

The impulse responses of a one percent i.i.d cost-push shock with monetary policy given by (28) are depicted in figure 2.

The aggregate behavior of the model is straight-forward: taking into account subjective expectations, the real interest rates of both agent types, $r_t^{\tau} = i_t - E_t^{\tau}\pi_{t+1}$, increase due to an increase of the nominal rate by the central bank. Hence, both agent types cut their individual consumption which leads to a quite severe recession which counteracts the cost-push shock to some extent. Consequently, inflation increases by less than one percent. Thus, the central bank that finds it optimal to be extraordinarily hawkish with respect to inflation which comes at the cost of a significant recession.

On the individual level, the disparity between the consumption adjustment paths of both agent types becomes obvious. While bounded rational forecasters cut their consumption by approximately three percent on impact, rational forecasters decrease consumption by almost ten percent. This is, first, because of substantially negative rational output gap expectations and, second, due to a slightly higher subjective real interest rate. On the other hand, as bounded rational forecasters are backward-looking, their output gap expectations are zero on impact and hence cut their consumption because of the increase in the subjective real interest rate only. This results in a consumption cut that is far smaller compared to rational forecasters and thereby in significant inequality in individual consumption on impact.[10]

Thus, bounded rational forecaster seem to be better off than rational ones at first. However, bounded rational forecasters make less smart decisions than rational forecasters by definition. This becomes clear when looking at the following periods where bounded rational forecasters have to pay for their initially higher consumption by giving up a lot of future consumption. On average, bounded rational forecasters are worse off. Specifically, one can observe that bounded rational output gap expectations

[10]I define consumption inequality here as $ci_t = \alpha c_t^R - (1-\alpha)c_t^B$ which should not be confused with the contribution of consumption inequality to welfare in (13), i.e. the cross-sectional variance of consumption.

in the second period $(t+1)$ *drop* drastically to $E^B_{t+1}y_{t+2} = y_t$ which is approximately minus 8 percent, resulting in a further cut of consumption. This is the case even though the subjective real interest rate of bounded rational forecasters becomes negative which is due to the jump of their inflation expectations to $E^B_{t+1}\pi_{t+2} = \pi_t$. At the same time, output gap expectations of rational forecasters *increase* as the output gap recovers. From this period onwards rational forecasters are able to consume more than bounded rational ones for a prolonged time.

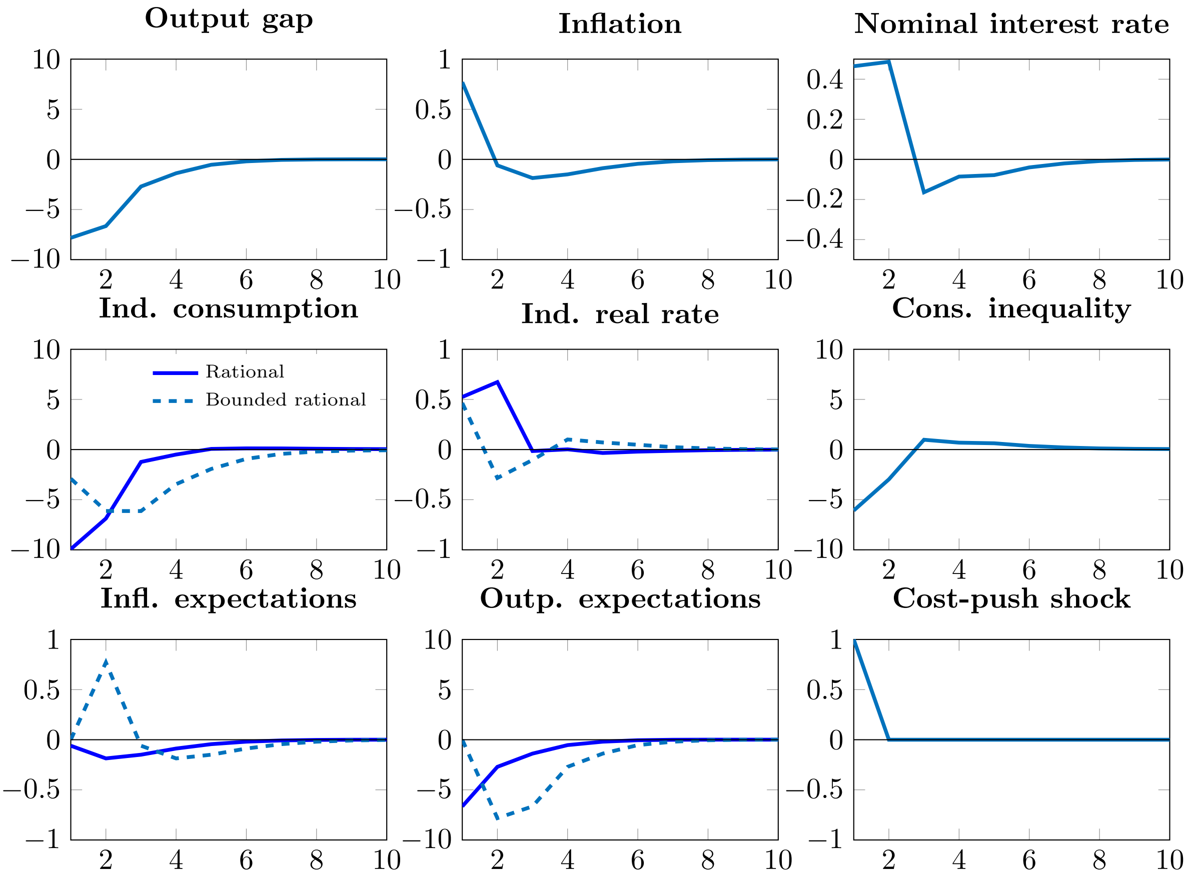

Figure 2: Impulse responses in %-deviations from steady state following a single, non-autocorrelated cost-push shock.

6 Welfare evaluation

This section answers the following questions: first, does the adjustment to the consumption decisions in Section 3 matter for welfare? Second, is the model-consistent interest rate rule in fact superior to the policy under the conventional objective? And what about the effect of inequality on welfare? Third, to which extent does the conventional objective underestimate the welfare losses when the underlying economy features a certain degree of bounded rational forecasters?

6.1 Consumption decisions and welfare

Adjusting the consumption decisions as shown in Section 3 relative to Di Bartolomeo et al. (2016) matters for welfare. While there is no economically significant difference under mean-reverting expectations, they become non-negligible for naive and trend-setting expectations when heterogeneity is high. For instance, Di Bartolomeo et al. (2016) report a welfare loss of 409.3 for $\theta = 1$ and $\alpha = 0.5$ which is 394.7 in this paper. This disparity becomes more pronounced for $\theta = 1.1$ and $\alpha = 0.5$: Di Bartolomeo et al. (2016) calculate a welfare loss of 492.9 whereas the corresponding loss here is 455.55. Thus, deriving consumption rules that incorporate the micro-foundations of Branch and McGough (2009) explicitly is not only important for being able to derive an interest rate rule under commitment in the first place but also as it affects welfare.

6.2 Optimal policy vs. policy under the inflation targeting objective

To answer the second question from above the percentage welfare *gain* of the model-consistent policy over the policy under the conventional objective is calculated which can be found in table 3. This welfare gain is thereby the difference between the *true* welfare losses under these policies following a one percent i.i.d. cost-push shock in percent.

α	$\theta = 0.8$	$\theta = 1$	$\theta = 1.2$
0.9	0.1	0.2	0.6
0.7	1.3	1.3	3.8
0.5	3.2	2.0	7.1
0.3	4.2	1.8	4.9
0.1	2.3	1.0	1.9

Table 3: Welfare gains of fully optimal interest rate rule (28) over the policy under the conventional objective (29) in percent for different fractions of rational forecasters α and different forecasting coefficients of bounded rational forecasters θ.

Recall that for intermediate values of α, i.e. high heterogeneity, and higher values of θ the amplification mechanism becomes more influential in determining the equilibrium. Also, recall that the model-consistent loss function collapses to the conventional inflation-targeting objective when $\alpha = 1$. Hence, both policies must be identical in the case of full rationality.

First, note that all values are positive as the policy under the model-consistent objective must be superior to the policy under the conventional inflation-targeting objective in terms of welfare as long as $\alpha < 1$. Second, with up to 7.1 percent welfare gains are highest when heterogeneity is high, i.e. around $\alpha = 0.5$, and when accompanied by trend-setting expectations ($\theta = 1.2$) of bounded rational forecasters. In this case, the amplification mechanism that is due to the interaction of rational and bounded rational expectations is strongest which implies high price dispersion and consumption inequality. While the commitment of the central bank alone is already effective in mitigating this mechanism by choosing rational expectations explicitly (Gasteiger, 2018), it becomes even more effective under the model-consistent loss function that recognizes both agent types and, therefore, the amplification mechanism.

Under the baseline calibration, $\alpha = 0.7$ and $\theta = 1$, one can observe also a non-negligible welfare gain of 1.3 percent. However, when α approaches one, welfare gains are intuitively quite small, especially under mean-reverting expectations ($\theta = 0.8$), as there is only a minor deviation from the rational benchmark where both policies are identical. Further, there are still some recognizable welfare gains for low values of α. The reason is that the model-consistent objective acknowledges the presence of bounded rational forecasters whereas the conventional inflation-targeting objective doesn't.

6.3 What about inequality?

In Section 5 it was shown how consumption inequality is caused and how it evolves following a one-percent cost-push shock. In this section, I analyze the effect of consumption inequality on welfare according to the model-consistent loss function (13). To that end, the consumption inequality component of (13) is calculated based on theoretical (co-)variances as depicted in figure (3).[11]

The most striking observation is that the inequality losses are *higher* under the model-consistent policy compared to the policy under the conventional objective. In order to understand this one has to point out that the central bank minimizes over several terms jointly. As was already shown in Section 5, the central bank's preferences under the model-consistent loss function are such that it considers it to be very important to control inflation at the cost of a quite severe recession. This effect can also be seen by looking at the theoretical variances as shown in Table (4). Using the model-consistent policy the central bank can achieve a slight decrease in the inflation variance relative to the policy under the conventional objective. However, this comes at the cost of increasing the variance of the output gap significantly, especially for lower values of α. These output gap variations directly feed back into consumption as the consumption equations depend on the output gap. Hence, a higher output gap variance causes a higher consumption variance (and the other way around). This ultimately leads to a higher cross-sectional

[11] Theoretical moments were calculated using Dynare.

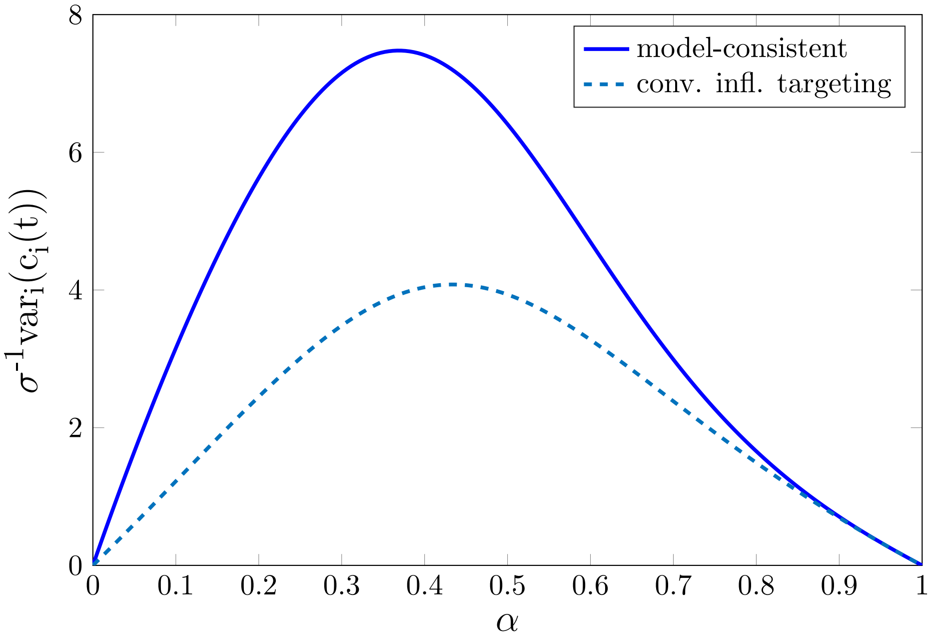

Figure 3: Welfare losses due to inequality under the model-consistent policy (28) and under the conventional inflation targeting objective (29) for different fractions of rational forecaster α.

variance of consumption (inequality loss) compared to the policy under the conventional objective. The latter effect can thus be explained by the specific micro-foundations of Branch and McGough (2009) which imply that agents forecast the output gap rather than individual consumption.

However, the magnitude of inequality losses relative to overall losses are quantitatively very small but increase in absolute value if bounded rational expectations gain influence on determining the equilibrium.[12]

	$Var(y)$				$Var(\pi)$			
policy	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$
L_t	342.40	231.45	115.03	50.65	0.65	0.67	0.67	0.60
$L_t^{\alpha=1}$	272.89	179.89	88.98	44.80	0.86	0.83	0.75	0.62

Table 4: Theoretical variances of inflation and output gap under the model-consistent policy (28) and under the conventional inflation-targeting objective (29) for different values of rational forecaster α.

[12]For instance, the absolute model-consistent loss under baseline calibration and the optimal policy is 243.6.

6.4 The value of choosing rational expectations

The value of commitment lies in the ability of the central bank to explicitly choose and manipulate rational expectations rather than taking them as given. Figure (4) displays an quite intuitive result. The

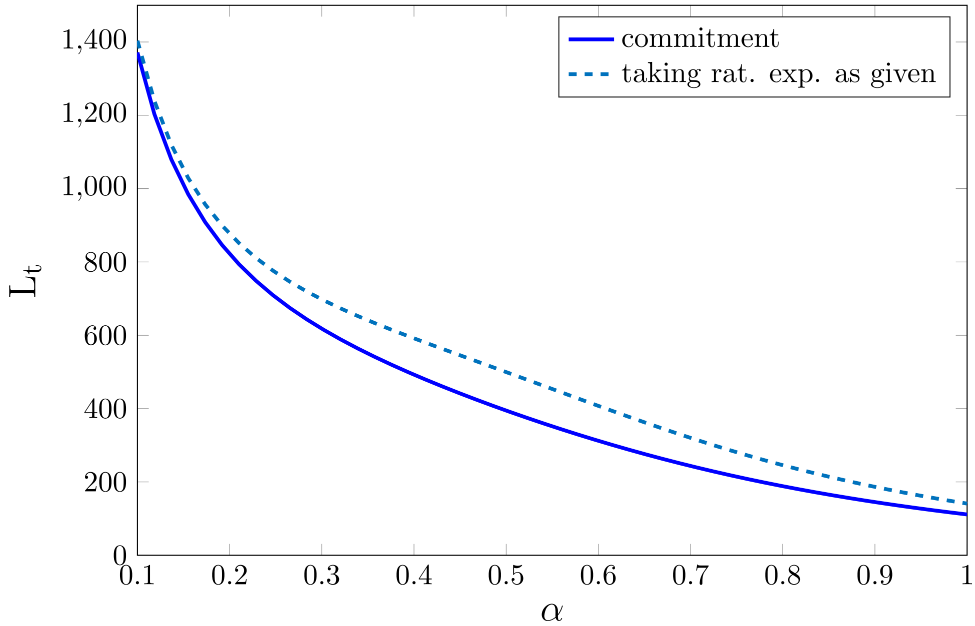

Figure 4: Welfare losses under the fully optimal model-consistent policy (28) vs. the model-consistent policy (21) where the central bank takes rational expectations as given for different fractions of rational forecasters.

central bank generates quite substantial welfare gains by choosing rational expectations, i.e. when it explicitly incorporates the endogenous nature of rational expectations, for the largest interval of α. For instance, there are enormous welfare gains under baseline calibration ($\alpha = 0.7$) of 31.4 percent. Note also that the absolute difference between commitment and discretion is highest around $\alpha = 0.5$ which is where the amplification mechanism is strongest. Hence, by manipulating rational expectations explicitly the channel through which the amplification works is controlled by the central bank which is already emphasized by Gasteiger (2018). However, "choosing rational expectations" loses its grip for low values of α as there are only a few rational forecasters left whose expectations can be manipulated.

6.5 Measuring welfare

Welfare losses can be substantially underestimated when they are measured by the conventional inflation-targeting objective that implicitly assumes rational expectations while the underlying economy features heterogeneous expectations as can be seen in figure (5). The heuristics of bounded rational forecasters are systematically biased which leads them to be more negatively affected by an unexpected cost-push shock than rational forecasters. Thus, true welfare losses, i.e. a weighted average of heterogeneous utility losses, are substantially higher than measured by the conventional objective. For instance, under baseline calibration ($\alpha = 0.7$) the conventional objective underestimates true welfare losses by 35.3. Further, the measurement error is monotonically increasing with a decreasing α. Hence, the conventional inflation-targeting objective performs particularly bad in measuring welfare when the fraction of bounded rational forecasters is high (for $\alpha = 0.3$ the difference is already 92.7 percent).

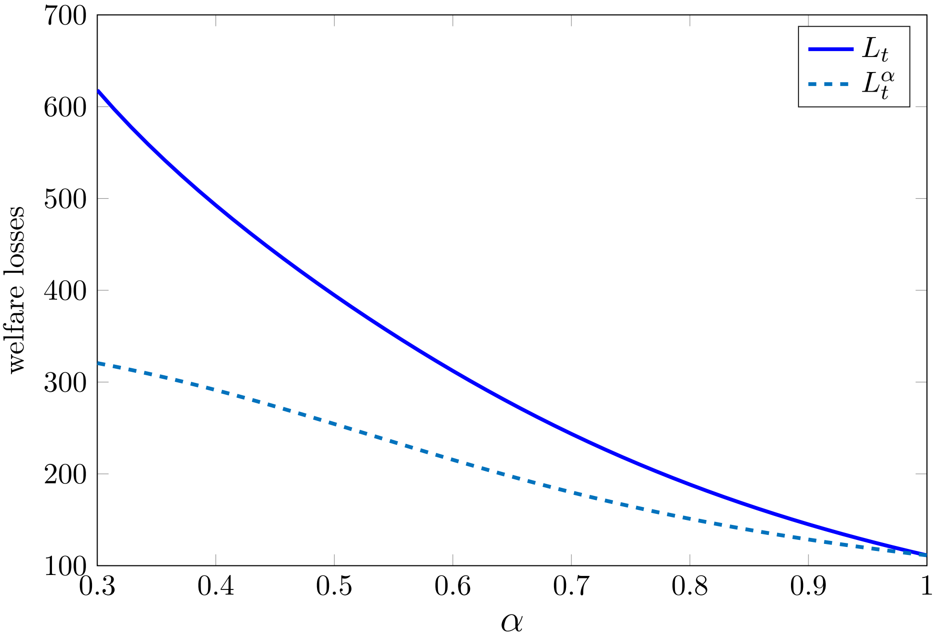

Figure 5: Welfare losses are *measured* either by L_t or L_t^α given the model-consistent policy (28) for different fractions of rational forecaster α.

7 Conclusion

In this paper, I propose a fully optimal interest rate rule under heterogeneous expectations where the central bank commits to its policy from a timeless perspective. This rule incorporates the more complex nature of price dispersion and consumption inequality under heterogeneous expectations as identified by Di Bartolomeo et al. (2016). Further, this rules performs considerably better than a micro-founded version of the interest rate rule as in Gasteiger (2014) and Gasteiger (2018) when heterogeneity is high and boundedly rational forecasters have trend-setting expectations. There are also non-negligible welfare gains for the baseline calibration (30 percent of bounded rational forecasters with naive expectations). However, when either the fraction of bounded rational agents is very low or very high and when their expectations are steady-state-reverting, I only find small benefits from the model-consistent interest rate rule. These findings can be explained with the amplification mechanism that is due to the interaction of rational and bounded rational forecaster's expectations, the resulting complex nature of price dispersion and consumption inequality. These effects are strongest for intermediate fractions of rational forecasters, i.e. high heterogeneity.

I additionally explore the agent level of the Branch and McGough (2009) framework and find that the consumption Euler equation under rational expectations as in Di Bartolomeo et al. (2016) is inconsistent with the higher-order beliefs assumption of Branch and McGough (2009). This assumption puts a specific (non-rational) structure on higher-order beliefs which implies that not even "rational forecasters" understand the micro level fully. Therefore, I derive consumption decisions that account for this particular assumption which makes it possible to derive an optimal interest rate rule in the first place. These consumption equations also require to specify agent's long-run beliefs. To rule out explosive equilibria, agents are assumed to believe that if their current wealth holdings increase (decrease), they will be able to consume more (less) in the long-run which feeds back into current consumption.

Further, welfare losses due to consumption inequality are higher under the model-consistent policy which is due to the hawkishness of the central bank with respect to inflation. This hawkishness causes high output gap variations which feed back into the consumption decisions and ultimately results in higher inequality losses. However, the inequality losses are small relative to overall welfare in this model.

Also, there is a huge benefit from choosing rational expectations opposed to taking them as given. By manipulating rational expectations directly, the central bank is able to dampen the effect of the amplification mechanism caused by the interaction of rational and bounded rational expectations. However, for small fractions of rational forecasters, this benefit diminishes as the channel through which the amplification mechanism works tightens.

Finally, policymakers should be aware of possibly high measurement errors when evaluating policy by the conventional inflation-targeting objective while the underlying economy features a certain fraction of backward-looking agents.

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A Individual pricing und rational budget

Following Di Bartolomeo et al. (2016) and Branch and McGough (2009) one can obtain the following log-linear pricing equation of a yeoman-farmer j

$$p_t(j)^* = E_t^j \sum_{s=0}^{\infty} (\xi_p\beta)^s \big[(1-\xi_p\beta)\Big(\frac{\sigma^{-1}}{1+\eta\epsilon}c_{t+s}^\tau + \frac{\eta}{1+\eta\epsilon}y_{t+s}\Big) + \xi_p\beta\pi_{t+s+1}\big] \tag{31}$$

where $p_t(j)^* = \log\big(\frac{P_t(j)^*}{P_t}\big)$ is the log-deviation of the price that firm j sets when it is chosen by the calvo lottery. Writing it one period ahead from the point of view of an agent of type τ gives

$$\xi_p\beta E_t^\tau p_{t+1}(j)^* = E_t^\tau E_{t+1}^j \sum_{s=1}^{\infty} (\xi_p\beta)^s \big[(1-\xi_p\beta)\Big(\frac{\sigma^{-1}}{1+\eta\epsilon}c_{t+s}^\tau + \frac{\eta}{1+\eta\epsilon}y_{t+s}\Big) + \xi_p\beta\pi_{t+s+1}\big]. \tag{32}$$

Integrating over all j and using the higher-order beliefs assumption of Branch and McGough (2009) delivers

$$\xi_p\beta E_t^\tau p_{t+1}^* = E_t^\tau \sum_{s=1}^{\infty} (\xi_p\beta)^s \big[(1-\xi_p\beta)\Big(\frac{\sigma^{-1}}{1+\eta\epsilon}c_{t+s}^\tau + \frac{\eta}{1+\eta\epsilon}y_{t+s}\Big) + \xi_p\beta\pi_{t+s+1}\big]. \tag{33}$$

Taking (31), setting $j=\tau$ and substracting (33) gives

$$p_t^{\tau *} = (1-\xi_p\beta)\Big(\frac{\sigma^{-1}}{1+\eta\epsilon}c_t^\tau + \frac{\eta}{1+\eta\epsilon}y_t\Big) + \xi_p\beta E_t^\tau(\pi_{t+1} + p_{t+1}^*). \tag{34}$$

which shows that agents forecast optimal aggregate prices which is needed for the aggregation result, i.e. the Phillips curve, in both Branch and McGough (2009) and Di Bartolomeo et al. (2016) to hold. Combining (34) with $\pi_t = \frac{1-\xi_p}{\xi_p}(\alpha p_t^{R*} + (1-\alpha)p_t^{B*})$ gives (12).

In the simulation I also use the log-linearized budget constraint of rational forecasters which is given by

$$c_t^R + b_t^R = \psi_t^R + \beta^{-1} b_{t-1}^R. \tag{35}$$

Given perfect insurance within groups, real income of type $\tau = R$ is defined as

$$\Psi_t^R = \frac{1}{\alpha P_t} \int_0^\alpha P_t(j) Y_t(j) dj \tag{36}$$

with $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$ being the demand for a particular good j. Hence,

$$\Psi_t^R = \frac{1}{\alpha P_t} \int_0^\alpha P_t(j) \Big(\frac{P_t(j)}{P_t}\Big)^{-\epsilon} Y_t dj. \tag{37}$$

which can be rewritten as

$$\Psi_t^R = \frac{1}{\alpha} \int_0^\alpha \Big(\frac{P_t(j)}{P_t}\Big)^{1-\epsilon} Y_t dj. \tag{38}$$

Defining $\frac{1}{\alpha} \int_0^\alpha \left(\frac{P_t(j)}{P_t}\right)^{1-\epsilon} = \tilde{P}_t^R$, (38) becomes

$$\Psi_t^R = \tilde{P}_t^R Y_t \tag{39}$$

and log-linearized

$$\psi_t^R = \tilde{p}_t^R + y_t. \tag{40}$$

The evolution of prices within the group of rational forecasters is given by

$$\int_0^\alpha (P_t(j))^{1-\epsilon} = \xi_p \int_0^\alpha (P_{t-1}(j))^{1-\epsilon} + (1-\xi_p)\alpha (P_t^{R*})^{1-\epsilon}. \tag{41}$$

Dividing both sides by $(P_t)^{1-\epsilon}$ yields

$$\int_0^\alpha \Big(\frac{P_t(j)}{P_t}\Big)^{1-\epsilon} = \xi_p \int_0^\alpha \Big(\frac{P_{t-1}(j)}{P_{t-1}}\Big)^{1-\epsilon} (\Pi_t)^{\epsilon-1} + (1-\xi_p)\alpha \Big(\frac{P_t^{R*}}{P_t}\Big)^{1-\epsilon}. \tag{42}$$

With the definition from above, (42) becomes

$$\tilde{P}_t^R = \xi_p \tilde{P}_{t-1}^R (\Pi_t)^{\epsilon-1} + (1-\xi_p) \Big(\frac{P_t^{R*}}{P_t}\Big)^{1-\epsilon}. \tag{43}$$

and log-linearized

$$\tilde{p}_t^R = \xi_p \tilde{p}_{t-1}^R - (1-\epsilon)\xi_p \pi_t + (1-\xi_p)(1-\epsilon) p_t^{R*} \tag{44}$$

B Implementation under the conventional inflation-targeting objective

B.1 Taking rational expectations as given

Defining $g_{t+s} = E_t \pi_{t+s+1}$, the policy problem where the central bank takes rational expectations as given and the conventional inflation-targeting objective takes the following form:

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \frac{1}{2} \left[\left(\eta + \frac{1}{\sigma} \right) y_{t+s}^2 + \epsilon^2 \eta \delta \pi_{t+s}^2 \right] \tag{45}$$

$$+ \lambda_{t+s} [\pi_{t+s} - \alpha \beta g_{t+s} - (1-\alpha)\beta\theta^2 \pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}]$$

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+s}} : \; E_t \left\{ \epsilon^2 \eta \delta \pi_{t+s} + \frac{\lambda_{t+s}}{2} - (1-\alpha)\beta^2\theta^2 \frac{\lambda_{t+s+1}}{2} \right\} \stackrel{!}{=} 0 \tag{46}$$

$$\frac{\partial \mathcal{L}}{\partial y_{t+s}} : \; E_t \left\{ \beta^s \left[\left(\eta + \frac{1}{\sigma} \right) y_{t+s} - \frac{\kappa}{2} \lambda_{t+s} \right] \right\} \stackrel{!}{=} 0. \tag{47}$$

Combining and solving for inflation gives

$$\pi_t = -\frac{1+\eta\sigma}{\sigma\epsilon^2\eta\delta\kappa} [y_t - (1-\alpha)\beta^2\theta^2 E_t y_{t+1}] \tag{48}$$

where the index s was dropped since the central bank commits to its policy from a timeless perspective. Combining with the Phillips and IS curve and solving for i_t yields (22).

B.2 Commitment

The policy problem under discretion and the conventional inflation-targeting objective is given by

$$\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \frac{1}{2} \left[\left(\eta + \frac{1}{\sigma} \right) y_{t+s}^2 + \epsilon^2 \eta \delta \pi_{t+s}^2 \right] \tag{49}$$

$$+ \lambda_{t+s} [\pi_{t+s} - \alpha \beta E_t \pi_{t+s+1} - (1-\alpha)\beta\theta^2 \pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}]$$

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+s}} : \; E_t \left\{ \epsilon^2 \eta \delta \pi_{t+s} + \frac{\lambda_{t+s}}{2} - (1-\alpha)\beta^2\theta^2 \frac{\lambda_{t+s+1}}{2} - \alpha \frac{\lambda_{t+s-1}}{2} \right\} \stackrel{!}{=} 0 \tag{50}$$

$$\frac{\partial \mathcal{L}}{\partial y_{t+s}} : \; E_t \left\{ \beta^s \left[\left(\eta + \frac{1}{\sigma} \right) y_{t+s} - \frac{\kappa}{2} \lambda_{t+s} \right] \right\} \stackrel{!}{=} 0. \tag{51}$$

Combining and solving for inflation gives

$$\pi_t = -\frac{1+\eta\sigma}{\sigma\epsilon^2\eta\delta\kappa}[y_t - \alpha y_{t-1} - (1-\alpha)\beta^2\theta^2 E_t y_{t+1}] \tag{52}$$

where the index s was dropped as the central bank employs timeless commitment. Combining with the Phillips and IS curve yields (29).

C Optimal monetary policy

C.1 Rewriting the model-consistent loss function

The period loss function L_t is given by

$$\begin{aligned} L_t = & \frac{\sigma\eta+1}{\sigma} y_t^2 + \frac{\alpha(y_t - c_t^R)^2}{(1-\alpha)\sigma} \\ & + \epsilon^2\eta\delta \left\{ \pi_t^2 + \frac{\xi_p(1-\alpha)}{\alpha}\left[\pi_t - \beta\theta^2\pi_{t-1} - \kappa y_t - \frac{\alpha\kappa(y_t - c_t^R)}{(1+\eta\sigma)(1-\alpha)}\right]^2 \right\}. \end{aligned} \tag{53}$$

By multiplying out, L_t can be re-written as

$$\begin{aligned} L_t = & \Gamma_1 y_t^2 + \Gamma_2 \pi_t^2 + \Gamma_3 \pi_{t-1}^2 + \Gamma_4 (c_t^R)^2 \\ & + \Gamma_5 y_t c_t^R + \Gamma_6 \pi_t c_t^R + \Gamma_7 \pi_{t-1} c_t^R + \Gamma_8 \pi_t \pi_{t-1} + \Gamma_9 \pi_t y_t + \Gamma_{10} \pi_{t-1} y_t \end{aligned} \tag{54}$$

with

$$\Gamma_1 = \frac{((\alpha-1)\eta\sigma-1)(\alpha(\eta^2\sigma^2(\delta\epsilon^2\kappa^2\xi_p-1)-1-2\eta\sigma)-\delta\epsilon^2\eta\kappa^2\xi_p\sigma(1+\eta\sigma))}{(1-\alpha)\alpha\sigma(1+\eta\sigma)^2} \tag{55}$$

$$\Gamma_2 = \frac{\delta\epsilon^2\eta(\alpha+\xi_p-\alpha\xi_p)}{\alpha} \tag{56}$$

$$\Gamma_3 = \frac{(1-\alpha)\beta^2\delta\epsilon^2\eta\theta^4\xi_p}{\alpha} \tag{57}$$

$$\Gamma_4 = \frac{\alpha(1+\eta\sigma(2+\delta\epsilon^2\kappa^2\xi_p)+\eta^2\sigma^2)}{(1-\alpha)\sigma(1+\eta\sigma)^2} \tag{58}$$

$$\Gamma_5 = \frac{2(\alpha+2\alpha\eta\sigma+\alpha\eta^2\sigma^2(1-\delta\epsilon^2\kappa^2\xi_p)+\delta\epsilon^2\eta\kappa^2\xi_p\sigma(1+\eta\sigma))}{(\alpha-1)\sigma(1+\eta\sigma)^2} \tag{59}$$

$$\Gamma_6 = \frac{2\delta\epsilon^2\eta\kappa\xi_p}{1+\eta\sigma} \tag{60}$$

$$\Gamma_7 = -\frac{2\beta\delta\epsilon^2\eta\theta^2\kappa\xi_p}{1+\eta\sigma} \tag{61}$$

$$\Gamma_8 = \frac{2(\alpha-1)\beta\delta\epsilon^2\eta\theta^2\xi_p}{\alpha} \tag{62}$$

$$\Gamma_9 = \frac{2\delta\epsilon^2\eta\kappa\xi_p((\alpha-1)\eta\sigma-1)}{\alpha+\alpha\eta\sigma} \tag{63}$$

$$\Gamma_{10} = \frac{2\beta\delta\epsilon^2\eta\theta^2\kappa\xi_p(1+\eta\sigma(1-\alpha))}{\alpha+\alpha\eta\sigma}. \tag{64}$$

C.2 Taking rational expectations as given

Defining $f_{t+s} = E_t y_{t+s+1}$, $g_{t+s} = E_t \pi_{t+s+1}$ and $h_{t+s} = E_t c^R_{t+s+1}$, the policy problem is

$$\begin{aligned}
\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Bigg[& \Gamma_1 y_{t+s}^2 + \Gamma_2 \pi_{t+s}^2 + \Gamma_3 \pi_{t+s-1}^2 + \Gamma_4 (c^R_{t+s})^2 \\
& + \Gamma_5 y_{t+s} c^R_{t+s} + \Gamma_6 \pi_{t+s} c^R_{t+s} + \Gamma_7 \pi_{t+s-1} c^R_{t+s} + \Gamma_8 \pi_{t+s}\pi_{t+s-1} + \Gamma_9 \pi_{t+s} y_{t+s} + \Gamma_{10} \pi_{t+s-1} y_{t+s} \\
& + \lambda_{1,t+s}[y_{t+s} - \alpha f_{t+s} - (1-\alpha)\theta^2 y_{t+s-1} + \sigma[i_{t+s} - \alpha g_{t+s} - (1-\alpha)\theta^2 \pi_{t+s-1}]] \\
& + \lambda_{2,t+s}[\pi_{t+s} - \alpha\beta g_{t+s} - (1-\alpha)\beta\theta^2 \pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}] \\
& + \lambda_{3,t+s}[c^R_{t+s} - h_{t+s} - \phi b^R_{t+s-1} + \sigma(i_{t+s} - g_{t+s})] \Bigg].
\end{aligned} \tag{65}$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial y_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_1 y_{t+s} + \Gamma_5 c^R_{t+s} + \Gamma_9 \pi_{t+s} + \Gamma_{10} \pi_{t+s-1} + \lambda_{1,t+s} - \kappa \lambda_{2,t+s}] \\ - \beta^{s+1}(1-\alpha)\theta^2 \lambda_{1,t+s+1} \Big\} \stackrel{!}{=} 0 \tag{66}$$

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_2 \pi_{t+s} + \Gamma_6 c^R_{t+s} + \Gamma_8 \pi_{t+s-1} + \Gamma_9 y_{t+s} + \lambda_{2,t+s}] \\ + \beta^{s+1} [2\Gamma_3 \pi_{t+s} + \Gamma_7 c_{t+s+1} + \Gamma_8 \pi_{t+s+1} + \Gamma_{10} y_{t+s+1} - (1-\alpha)\theta^2 \sigma \lambda_{1,t+s+1} \\ - (1-\alpha)\beta\theta^2 \lambda_{2,t+s+1}] \Big\} \stackrel{!}{=} 0 \tag{67}$$

$$\frac{\partial \mathcal{L}}{\partial c^R_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_4 c^R_{t+s} + \Gamma_5 y_{t+s} + \Gamma_6 \pi_{t+s} + \Gamma_7 \pi_{t+s-1} + \lambda_{3,t+s}] \Big\} \stackrel{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial i_{t+s}} : E_t \Big\{ \beta^s \sigma \lambda_{1,t+s} + \beta^s \sigma \lambda_{3,t+s} \Big\} \stackrel{!}{=} 0. \tag{68}$$

Since the central bank acts under timeless commitment, the index s can be dropped. Using $\lambda_{3,t} = -\lambda_{1,t}$ the FOCs can equivalently be written as

$$2\Gamma_1 y_t + \Gamma_5 c^R_t + \Gamma_9 \pi_t + \Gamma_{10} \pi_{t-1} + \lambda_{1,t} - \kappa \lambda_{2,t} - (1-\alpha)\beta\theta^2 E_t \lambda_{1,t+1} \stackrel{!}{=} 0 \tag{69}$$

$$2\Gamma_2 \pi_t + \Gamma_6 c^R_t + \Gamma_8 \pi_{t-1} + \Gamma_9 y_t + \lambda_{2,t} + 2\Gamma_3 \beta \pi_t + \Gamma_7 \beta E_t c^R_{t+1} + \Gamma_8 \beta E_t \pi_{t+1} \\ + \Gamma_{10} \beta E_t y_{t+1} - (1-\alpha)\beta\theta^2 \sigma E_t \lambda_{1,t+1} - (1-\alpha)\beta^2\theta^2 E_t \lambda_{2,t+1} \stackrel{!}{=} 0 \tag{70}$$

$$2\Gamma_4 c^R_t + \Gamma_5 y_t + \Gamma_6 \pi_t + \Gamma_7 \pi_{t-1} - \lambda_{1,t} \stackrel{!}{=} 0. \tag{71}$$

Eliminating the Lagrange multipliers yields the reduced-form FOC

$$\Delta_1 \pi_t + \Delta_2 E_t \pi_{t+1} + \Delta_3 E_t \pi_{t+2} + \Delta_4 \pi_{t-1} + \Delta_5 y_t + \Delta_6 E_t y_{t+1} + \Delta_7 E_t y_{t+2} \\ + \Delta_8 c^R_t + \Delta_9 E_t c^R_{t+1} + \Delta_{10} E_t c^R_{t+2} \stackrel{!}{=} 0 \tag{72}$$

with

$$\Delta_1 = -\frac{\Gamma_6 + \Gamma_9 + 2\Gamma_2\kappa + 2\beta\Gamma_3\kappa - (1-\alpha)\beta\theta^2(\Gamma_7 + \beta(\Gamma_{10} + \Gamma_7) + \Gamma_7\kappa\sigma)}{\kappa} \tag{73}$$

$$\Delta_2 = \frac{(1-\alpha)\beta\theta^2(\beta(\Gamma_9 - (1-\alpha)\beta\Gamma_7\theta^2) + \Gamma_6(1+\beta+\kappa\sigma))}{\kappa} - \beta\Gamma_8 \tag{74}$$

$$\Delta_3 = -\frac{(\alpha-1)^2\beta^3\theta^4\Gamma_6}{\kappa} \tag{75}$$

$$\Delta_4 = -\frac{\Gamma_{10} + \Gamma_7 + \Gamma_8\kappa}{\kappa} \tag{76}$$

$$\Delta_5 = -\frac{2\Gamma_1 + \Gamma_5 + \Gamma_9\kappa}{\kappa} \tag{77}$$

$$\Delta_6 = -\frac{\beta((\alpha-1)\beta(2\Gamma_1 + \Gamma_5)\theta^2 + \Gamma_{10}\kappa + (\alpha-1)\Gamma_5\theta^2(1+\kappa\sigma))}{\kappa} \tag{78}$$

$$\Delta_7 = -\frac{(\alpha-1)^2\beta^3\theta^4\Gamma_5}{\kappa} \tag{79}$$

$$\Delta_8 = -\frac{2\Gamma_4 + \Gamma_5 + \Gamma_6\kappa}{\kappa} \tag{80}$$

$$\Delta_9 = -\frac{\beta((\alpha-1)\beta\Gamma_5\theta^2 + \Gamma_7\kappa + 2(\alpha-1)\Gamma_4\theta^2(1+\beta+\kappa\sigma))}{\kappa} \tag{81}$$

$$\Delta_{10} = -\frac{2(\alpha-1)^2\beta^3\theta^4\Gamma_4}{\kappa}. \tag{82}$$

Solving (72) for π_t and setting it equal to the NK Phillips curve yields

$$\begin{aligned} y_t = & -\frac{1}{\Delta_5 + \Delta_1\kappa}(\Delta_6 y_{t+1} + \Delta_7 y_{t+2} + (\Delta_2 + \alpha\beta\Delta_1)\pi_{t+1} + \Delta_3\pi_{t+2} + (\Delta_4 + (1-\alpha)\beta\theta^2\Delta_1)\pi_{t-1} \\ & + \Delta_8 c_t^R + \Delta_9 c_{t+1}^R + \Delta_{10} c_{t+2}^R + \Delta_1 e_t). \end{aligned} \tag{83}$$

Setting (83) equal to the New IS curve, substituting c_t^R for consumption demand and solving for i_t gives the reaction function under the assumption that the central bank takes rational expectations as given

$$\begin{aligned} i_t = & \Omega_1 y_{t-1} + \Omega_2 E_t y_{t+1} + \Omega_3 E_t y_{t+2} + \Omega_4 \pi_{t-1} + \Omega_5 E_t \pi_{t+1} + \Omega_6 E_t \pi_{t+2} \\ & + \Omega_7 E_t c_{t+1}^R + \Omega_8 E_t c_{t+2}^R + \Omega_9 e_t \end{aligned} \tag{84}$$

with

$$\Omega_1 = \frac{(1-\alpha)\theta^2(\Delta_5 + \Delta_1\kappa)}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{85}$$

$$\Omega_2 = \frac{\Delta_6 + \Delta_8 + \alpha(\Delta_5 + \Delta_1\kappa)}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{86}$$

$$\Omega_3 = \frac{\Delta_7}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{87}$$

$$\Omega_4 = \frac{\Delta_4 + (1-\alpha)\theta^2(\beta\Delta_1 + \sigma(\Delta_5 + \Delta_1\kappa))}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{88}$$

$$\Omega_5 = \frac{\alpha\beta\Delta_1 + \Delta_2 + \sigma\Delta_8 + \alpha\sigma(\Delta_5 + \Delta_1\kappa)}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{89}$$

$$\Omega_6 = \frac{\Delta_3}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{90}$$

$$\Omega_7 = \frac{\Delta_9}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{91}$$

$$\Omega_8 = \frac{\Delta_{10}}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)} \tag{92}$$

$$\Omega_9 = \frac{\Delta_1}{\sigma(\Delta_5 + \Delta_8 + \Delta_1\kappa)}. \tag{93}$$

The Ω-coefficients are expressed in terms of the targeting rule coefficients for simplicity. Writing them in terms of the deep model parameters would yield in part far to big expression.

C.3 Incorporating the endogeneous nature of rational expectations

The policy problem under full commitment takes the following form:

$$\begin{aligned}
\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Bigg[& \Gamma_1 y_{t+s}^2 + \Gamma_2 \pi_{t+s}^2 + \Gamma_3 \pi_{t+s-1}^2 + \Gamma_4 (c_{t+s}^R)^2 \\
& + \Gamma_5 y_{t+s} c_{t+s}^R + \Gamma_6 \pi_{t+s} c_{t+s}^R + \Gamma_7 \pi_{t+s-1} c_{t+s}^R + \Gamma_8 \pi_{t+s}\pi_{t+s-1} + \Gamma_9 \pi_{t+s} y_{t+s} + \Gamma_{10} \pi_{t+s-1} y_{t+s} \\
& + \lambda_{1,t+s}[y_{t+s} - \alpha E_t y_{t+s+1} - (1-\alpha)\theta^2 y_{t+s-1} + \sigma[i_{t+s} - \alpha E_t \pi_{t+s+1} - (1-\alpha)\theta^2 \pi_{t+s-1}]] \\
& + \lambda_{2,t+s}[\pi_{t+s} - \alpha\beta E_t \pi_{t+s+1} - (1-\alpha)\beta\theta^2 \pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}] \\
& + \lambda_{3,t+s}[c_{t+s}^R - E_t y_{t+s+1} - \phi b_{t+s-1}^R + \sigma(i_{t+s} - E_t \pi_{t+s+1})] \Bigg].
\end{aligned} \tag{94}$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial y_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_1 y_{t+s} + \Gamma_5 c^R_{t+s} + \Gamma_9 \pi_{t+s} + \Gamma_{10} \pi_{t+s-1} + \lambda_{1,t+s} - \kappa \lambda_{2,t+s}] \tag{95}$$

$$- \beta^{s+1}(1-\alpha)\theta^2 \lambda_{1,t+s+1} - \beta^{s-1}[\alpha \lambda_{1,t+s-1} + \lambda_{3,t+s-1}] \Big\} \overset{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_2 \pi_{t+s} + \Gamma_6 c^R_{t+s} + \Gamma_8 \pi_{t+s-1} + \Gamma_9 y_{t+s} + \lambda_{2,t+s}]$$

$$+ \beta^{s+1}[2\Gamma_3 \pi_{t+s} + \Gamma_7 c_{t+s+1} + \Gamma_8 \pi_{t+s+1} + \Gamma_{10} y_{t+s+1} - (1-\alpha)\theta^2 \sigma \lambda_{1,t+s+1} \tag{96}$$

$$- (1-\alpha)\beta\theta^2 \lambda_{2,t+s+1}] - \beta^{s-1}[\alpha\sigma\lambda_{1,t+s-1} + \alpha\beta\lambda_{2,t+s-1} + \sigma\ \lambda_{3,t+s-1}] \Big\} \overset{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial c^R_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_4 c^R_{t+s} + \Gamma_5 y_{t+s} + \Gamma_6 \pi_{t+s} + \Gamma_7 \pi_{t+s-1} \tag{97}$$

$$+ \lambda_{3,t+s}] \Big\} \overset{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial i_{t+s}} : E_t \Big\{ \beta^s \sigma \lambda_{1,t+s} + \beta^s \sigma \lambda_{3,t+s} \Big\} \overset{!}{=} 0. \tag{98}$$

Again, the index s can be dropped assuming commitment from a timeless perspective. Using $\lambda_{3,t} = -\lambda_{1,t}$ the FOCs can equivalently be written as

$$2\Gamma_1 y_t + \Gamma_5 c^R_t + \Gamma_9 \pi_t + \Gamma_{10} \pi_{t-1} + \lambda_{1,t} - \kappa \lambda_{2,t} - (1-\alpha)\beta\theta^2 \lambda_{1,t+1} -$$

$$\beta^{-1}(\alpha - 1)\lambda_{1,t-1} \overset{!}{=} 0 \tag{99}$$

$$2\Gamma_2 \pi_t + \Gamma_6 c^R_t + \Gamma_8 \pi_{t-1} + \Gamma_9 y_t + \lambda_{2,t}$$

$$+ 2\Gamma_3 \beta \pi_t + \Gamma_7 \beta c_{t+1} + \Gamma_8 \beta \pi_{t+1} + \Gamma_{10} \beta y_{t+1} - (1-\alpha)\beta\theta^2 \sigma \lambda_{1,t+1} \tag{100}$$

$$- (1-\alpha)\beta^2\theta^2 \lambda_{2,t+1} + \beta^{-1}(1-\alpha)\sigma\lambda_{1,t-1} - \alpha\lambda_{2,t-1} \overset{!}{=} 0$$

$$2\Gamma_4 c^R_t + \Gamma_5 y_t + \Gamma_6 \pi_t + \Gamma_7 \pi_{t-1} - \lambda_{1,t} \overset{!}{=} 0. \tag{101}$$

Eliminating the Lagrange multipliers yields the reduced-form FOC

$$\Delta^c_1 \pi_t + \Delta^c_2 \pi_{t+1} + \Delta^c_3 \pi_{t+2} + \Delta^c_4 \pi_{t-3} + \Delta^c_5 \pi_{t-2} + \Delta^c_6 \pi_{t-1} + \Delta_7 y_t + \Delta^c_8 y_{t+1} \tag{102}$$

$$+\Delta^c_9 y_{t+2} + \Delta^c_{10} y_{t-2} + \Delta^c_{11} y_{t-1} + \Delta_{12} c^R_t + \Delta^c_{13} c^R_{t+1} + \Delta^c_{14} c^R_{t+2} + \Delta^c_{15} c^R_{t-2} + \Delta^c_{16} c^R_{t-1} \overset{!}{=} 0.$$

with

$$\Delta_1 = \Gamma_6 + \Gamma_9 + (1-\alpha)(-1+2\alpha)\beta\Gamma_6\theta^2 + 2\Gamma_2\kappa + 2\beta\Gamma_3\kappa \tag{103}$$

$$+ (\alpha-1)\beta\theta^2(\Gamma_7 + \beta(\Gamma_{10}+\Gamma_7) + \Gamma_7\kappa\sigma) \tag{104}$$

$$\Delta_2 = \beta(\Gamma_8\kappa + (\alpha-1)\theta^2(\beta(\Gamma_9 + (\alpha-1)\beta\Gamma_7\theta^2) + \Gamma_6(1+\beta+\kappa\sigma))) \tag{105}$$

$$\Delta_3 = (\alpha-1)^2\beta^3\Gamma_6\theta^4 \tag{106}$$

$$\Delta_4 = \frac{(\alpha-1)\alpha\Gamma_7}{\beta} \tag{107}$$

$$\Delta_5 = \frac{\alpha^2\Gamma_6 + \Gamma_7 + \Gamma_7\kappa\sigma - \alpha(\Gamma_6 + \Gamma_7 + \beta(\Gamma_{10}+\Gamma_7) + \Gamma_7\kappa\sigma)}{\beta} \tag{108}$$

$$\Delta_6 = \Gamma_{10} + \Gamma_7 - \alpha(\Gamma_6+\Gamma_9) + (-1+(3-2\alpha)\alpha)\beta\Gamma_7\theta^2 + \Gamma_8\kappa \tag{109}$$

$$- \frac{(\alpha-1)\Gamma_6(1+\kappa\sigma)}{\beta} \tag{110}$$

$$\Delta_7 = 2\Gamma_1 + \Gamma_5 + (1-\alpha)(2\alpha-1)\beta\Gamma_5\theta^2 + \Gamma_9\kappa \tag{111}$$

$$\Delta_8 = \beta(\Gamma_{10}\kappa + (\alpha-1)\theta^2(\Gamma_5 + \beta(2\Gamma_1+\Gamma_5) + \Gamma_5\kappa\sigma) \tag{112}$$

$$\Delta_9 = (\alpha-1)^2\beta^3\Gamma_5\theta^4 \tag{113}$$

$$\Delta_{10} = \frac{(\alpha-1)\alpha\Gamma_5}{\beta} \tag{114}$$

$$\Delta_{11} = \frac{\Gamma_5 + \Gamma_5\kappa\sigma - \alpha(\Gamma_5 + \beta(2\Gamma_1+\Gamma_5) + \Gamma_5\kappa\sigma)}{\beta} \tag{115}$$

$$\Delta_{12} = 2\Gamma_4 + \Gamma_5 - 2(\alpha-1)(2\alpha-1)\beta\Gamma_4\theta^2 + \Gamma_6\kappa) \tag{116}$$

$$\Delta_{13} = \beta(\Gamma_7\kappa + (\alpha-1)\theta^2(\beta\Gamma_5 + 2\Gamma_4(1+\beta+\kappa\sigma)) \tag{117}$$

$$\Delta_{14} = 2(\alpha-1)^2\beta^3\Gamma_4\theta^4 \tag{118}$$

$$\Delta_{15} = \frac{2(\alpha-1)\alpha\Gamma_4}{\beta} \tag{119}$$

$$\Delta_{16} = -\alpha\Gamma_5 - \frac{2\Gamma_4(-1+\alpha+\alpha\beta+(\alpha-1)\kappa\sigma)}{\beta}. \tag{120}$$

Solving (102) for π_t and setting it equal to the NK Phillips curve yields

$$\begin{aligned} y_t = -\frac{1}{\Delta_7+\Delta_1\kappa}((\alpha\beta\Delta_1 + \Delta_2)\pi_{t+1} + \Delta_3\pi_{t+2} + \Delta_4\pi_{t-3} + \Delta_5\pi_{t-2} \\ + (\Delta_6 + (1-\alpha)\beta\theta^2\Delta_1)\pi_{t-1} + \Delta_8 y_{t+1} + \Delta_9 y_{t+2} + \Delta_{10}y_{t-2} + \Delta_{11}y_{t-1} \\ + \Delta_{13}c^R_{t+1} + \Delta_{14}c^R_{t+2} + \Delta_{15}c^R_{t-2} + \Delta_{16}c^R_{t-1}) \end{aligned} \tag{121}$$

Setting (121) equal to the New IS curve, substituting c_t^R for consumption demand and solving for i_t gives the central bank's reaction function under commitment (28):

$$\begin{aligned} i_t =& \Omega_1^c E_t \pi_{t+1} + \Omega_2^c E_t \pi_{t+2} + \Omega_3^c \pi_{t-3} + \Omega_4^c \pi_{t-2} + \Omega_5^c \pi_{t-1} + \Omega_6^c E_t y_{t+1} + \Omega_7^c E_t y_{t+2} \\ &+ \Omega_8^c y_{t-2} + \Omega_9^c y_{t-1} + \Omega_{10}^c E_t c_{t+1}^R + \Omega_{11}^c E_t c_{t+2}^R + \Omega_{12}^c c_{t-2}^R + \Omega_{13}^c c_{t-1}^R + \Omega_{14}^c e_t \end{aligned} \tag{122}$$

with

$$\Omega_1^c = \frac{\alpha\beta\Delta_1 + \Delta_2 + \sigma\Delta_{12} + \alpha\sigma(\Delta_7 + \Delta_1\kappa)}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{123}$$

$$\Omega_2^c = \frac{\Delta_3}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{124}$$

$$\Omega_3^c = \frac{\Delta_4}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{125}$$

$$\Omega_4^c = \frac{\Delta_5}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{126}$$

$$\Omega_5^c = \frac{\Delta_6 + (1-\alpha)\theta^2(\beta\Delta_1 + \sigma(\Delta_7 + \Delta_1\kappa))}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{127}$$

$$\Omega_6^c = \frac{\Delta_{12} + \alpha\Delta_7 + \Delta_8 + \alpha\kappa\Delta_1}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{128}$$

$$\Omega_7^c = \frac{\Delta_9}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{129}$$

$$\Omega_8^c = \frac{\Delta_{10}}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{130}$$

$$\Omega_9^c = \frac{\Delta_{11} + (1-\alpha)\theta^2(\Delta_7 + \Delta_1\kappa)}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{131}$$

$$\Omega_{10}^c = \frac{\Delta_{13}}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{132}$$

$$\Omega_{11}^c = \frac{\Delta_{14}}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{133}$$

$$\Omega_{12}^c = \frac{\Delta_{15}}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{134}$$

$$\Omega_{13}^c = \frac{\Delta_{16}}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)} \tag{135}$$

$$\Omega_{14}^c = \frac{\Delta_1}{\sigma(\Delta_7 + \Delta_{12} + \Delta_1\kappa)}. \tag{136}$$

C.4 Tables

Ω_x^C	$\alpha = 0.1$	$\alpha = 0.3$	$\alpha = 0.5$	$\alpha = 0.7$	$\alpha = 0.9$	$\alpha \to 1$ (RE)
y_{t-2}	0.001	0.004	0.01	0.018	0.029	0
y_{t-1}	0.115	0.052	-0.016	-0.085	-0.153	-0.139
$E_t y_{t+1}$	-0.08	-0.02	0.045	0.111	0.175	0.160
$E_t y_{t+2}$	-0.005	-0.008	-0.009	-0.007	-0.003	0
π_{t-3}	0.012	0.028	0.034	0.029	0.013	0
π_{t-2}	-0.609	-0.502	-0.38	-0.241	-0.085	0
π_{t-1}	2.747	2.082	1.420	0.792	0.238	0
$E_t \pi_{t+1}$	-0.482	0.115	0.697	1.229	1.639	1.851
$E_t \pi_{t+2}$	0.103	0.064	0.033	0.012	0.001	0
$E_t c^R_{t-2}$	0	-0.003	-0.009	-0.017	-0.029	0
$E_t c^R_{t-1}$	0.004	0.013	0.022	0.031	0.041	0
$E_t c^R_{t+1}$	-0.004	-0.012	-0.021	-0.031	-0.040	0
$E_t c^R_{t+2}$	0.003	0.007	0.008	0.007	0.003	0
e_t	1.275	1.227	1.154	1.056	0.932	0.859

Table 5: Values of reaction coefficients Ω_x^C in the interest rate rule (28) for different values of the share of rational forecasters α

D Implementation under the conventional Euler equation

The policy problem under commitment and the conventional Euler equation takes the following form:

$$
\begin{aligned}
\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Bigg[& \Gamma_1 y_{t+s}^2 + \Gamma_2 \pi_{t+s}^2 + \Gamma_3 \pi_{t+s-1}^2 + \Gamma_4 (c^R_{t+s})^2 \\
& + \Gamma_5 y_{t+s} c^R_{t+s} + \Gamma_6 \pi_{t+s} c^R_{t+s} + \Gamma_7 \pi_{t+s-1} c^R_{t+s} + \Gamma_8 \pi_{t+s} \pi_{t+s-1} + \Gamma_9 \pi_{t+s} y_{t+s} + \Gamma_{10} \pi_{t+s-1} y_{t+s} \\
& + \lambda_{1,t+s}[y_{t+s} - \alpha E_t y_{t+s+1} - (1-\alpha)\theta^2 y_{t+s-1} + \sigma[i_{t+s} - \alpha E_t \pi_{t+s+1} - (1-\alpha)\theta^2 \pi_{t+s-1}]] \\
& + \lambda_{2,t+s}[\pi_{t+s} - \alpha\beta E_t \pi_{t+s+1} - (1-\alpha)\beta\theta^2 \pi_{t+s-1} - \kappa y_{t+s} - e_{t+s}] \\
& + \lambda_{3,t+s}[c^R_{t+s} - E_t c^R_{t+s+1} + \sigma(i_{t+s} - E_t \pi_{t+s+1})] \Bigg]. \qquad (137)
\end{aligned}
$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial y_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_1 y_{t+s} + \Gamma_5 c^R_{t+s} + \Gamma_9 \pi_{t+s} + \Gamma_{10} \pi_{t+s-1} + \lambda_{1,t+s} - \kappa \lambda_{2,t+s}] \tag{138}$$
$$- \beta^{s+1}(1-\alpha)\theta^2 \lambda_{1,t+s+1} - \beta^{s-1} \alpha \lambda_{1,t+s-1} \Big\} \stackrel{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial \pi_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_2 \pi_{t+s} + \Gamma_6 c^R_{t+s} + \Gamma_8 \pi_{t+s-1} + \Gamma_9 y_{t+s} + \lambda_{2,t+s}]$$
$$+ \beta^{s+1} [2\Gamma_3 \pi_{t+s} + \Gamma_7 c_{t+s+1} + \Gamma_8 \pi_{t+s+1} + \Gamma_{10} y_{t+s+1} - (1-\alpha)\theta^2 \sigma \lambda_{1,t+s+1} \tag{139}$$
$$- (1-\alpha)\beta\theta^2 \lambda_{2,t+s+1}] - \beta^{s-1}[\alpha\sigma\lambda_{1,t+s-1} + \alpha\beta\lambda_{2,t+s-1} + \sigma\ \lambda_{3,t+s-1}] \Big\} \stackrel{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial c^R_{t+s}} : E_t \Big\{ \beta^s [2\Gamma_4 c^R_{t+s} + \Gamma_5 y_{t+s} + \Gamma_6 \pi_{t+s} + \Gamma_7 \pi_{t+s-1} \tag{140}$$
$$+ \lambda_{3,t+s}] - \beta^{s-1} \lambda_{3,t+s-1} \Big\} \stackrel{!}{=} 0$$

$$\frac{\partial \mathcal{L}}{\partial i_{t+s}} : E_t \Big\{ \beta^s \sigma \lambda_{1,t+s} + \beta^s \sigma \lambda_{3,t+s} \Big\} \stackrel{!}{=} 0. \tag{141}$$

Again, the index s can be dropped assuming commitment from a timeless perspective. Using $\lambda_{3,t} = -\lambda_{1,t}$ the FOCs can equivalently be written as

$$2\Gamma_1 y_t + \Gamma_5 c^R_t + \Gamma_9 \pi_t + \Gamma_{10} \pi_{t-1} + \lambda_{1,t} - \kappa \lambda_{2,t} - (1-\alpha)\beta\theta^2 \lambda_{1,t+1} -$$
$$\beta^{-1} \alpha \lambda_{1,t-1} \stackrel{!}{=} 0 \tag{142}$$

$$2\Gamma_2 \pi_t + \Gamma_6 c^R_t + \Gamma_8 \pi_{t-1} + \Gamma_9 y_t + \lambda_{2,t}$$
$$+ 2\Gamma_3 \beta \pi_t + \Gamma_7 \beta c_{t+1} + \Gamma_8 \beta \pi_{t+1} + \Gamma_{10} \beta y_{t+1} - (1-\alpha)\beta\theta^2 \sigma \lambda_{1,t+1} \tag{143}$$
$$- (1-\alpha)\beta^2\theta^2 \lambda_{2,t+1} + \beta^{-1}(1-\alpha)\sigma\lambda_{1,t-1} - \alpha\lambda_{2,t-1} \stackrel{!}{=} 0$$

$$2\Gamma_4 c^R_t + \Gamma_5 y_t + \Gamma_6 \pi_t + \Gamma_7 \pi_{t-1} - \lambda_{1,t} + \beta^{-1} \lambda_{1,t-1} \stackrel{!}{=} 0.. \tag{144}$$

(144) can be used to replace $\lambda_{1,t-1}$ and $\lambda_{1,t+1}$ with $\lambda_{1,t}$ in (142). Then, solving (142) for $\lambda_{1,t}$ and inserting in (143) yields a second-order difference equation in $\lambda_{2,t}$. A solution to this equation can in principle be substituted back into the difference equation, which would give a targeting rule. However, this solution is fairly complicated in which some parameter terms exponentially depend on time. The

solution is available upon request. The resulting targeting rule and, hence, a reaction function would also be of such a complicated form where parameters exponentially depend on time. Consequently, there is no meaningful interest rate rule under commitment and the conventional Euler equation under rational expectations.

E Robustness with respect to ϕ

Finally, it needs to be checked whether the welfare results in this paper are sensitive towards different values of ϕ. Therefore, I re-did the welfare analysis for $\phi = \{0.015, 0.1, 0.5, 1\}$.

First, as can be seen in table 6 the welfare gains of the optimal policy relative to the policy under the conventional inflation-targeting objective analyzed in Section 6 do not vary much with respect to ϕ (where θ is fixed to one). The same picture emerges for $\theta = 0.8$ and $\theta = 1.2$. Additionally, absolute welfare does not vary in an economically significant way with ϕ as well, even for different values of θ and α. Also, while inequality losses do change a little bit they are still negligible with respect to overall welfare.

α	$\phi = 0.015$	$\phi = 0.1$	$\phi = 0.5$	$\phi = 1$
0.9	0.2	0.2	0.2	0.2
0.7	1.3	1.3	1.3	1.2
0.5	2.0	2.0	1.8	1.5
0.3	1.8	1.8	1.6	1.4
0.1	1.0	1.0	1.0	0.9

Table 6: Welfare gains of the model consistent policy (28) over the policy under the conventional objective (29) in percent for different fractions of rational forecasters α and different values of the bond-adjustment parameter ϕ in the consumption decisions of agents. The forecasting coefficient of bounded rational forecaster θ is fixed to one.

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