Sentiment-Driven Investment, Non-Linear Corporate Debt Dynamics and Co-Existing Business Cycle Regimes

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Abstract

Recent evidence on the development of corporate debt suggests that firms' leverage ratios increased enormously during the past few decades. Taking into account firms financing concerns, the present work provides a dynamic disequilibrium model that is able to generate cyclical patterns of various key economic variables. One of the main features of the model is that a dynamic law governing the evolution of investor sentiment determines firms' investment through their sales expectations according to recurrent and endogenously determined waves of optimism and pessimism. The model further incorporates commercial banks providing loans to firms with the respective lending rate exhibiting a mark-up that changes endogenously with the evolution of the firms' indebtedness. It is shown that the model generates sentiment-driven business cycle fluctuations for two economic environments that exist contemporaneously: a "normal-" and "high-indebted" regime.

Keywords: Minsky Cycles, De-Leveraging, Paradox of Debt, Financial Accelerator, Business Sentiment, Kaleckian Model, Disequilibrium Model

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1 Introduction

During the last few decades, corporate leveraging has been increased extensively. After the dramatic rise of private as well as business sector debt in the US, episodes of de-leveraging were followed and associated with a rapid economic downturn. Some authors even contribute the rapid reduction in private sectors' debt as one of the key factors responsible for the recent economic recession and slow recovery in the US, see for instance Eggertsson and Krugman (2012). Focusing on the business sector, the effects of increasing leverages on the state of the business cycle are one of the main concerns of Minsky's "Financial Instability Hypothesis". But the interdependency of corporate debt and the economic system's (in-)stability is not without ambiguity. According to Minsky (2008), increasing leverage ratios go hand in hand with economic expansions but in contrary, a branch of literature, e.g. Lavoie (1995) and Hein (2006, 2007) among others suggests that the "paradox of debt", a phenomenon which states that de-leveraging leads to increases in real economic activity takes place instead. However, Minsky also made the point that firms external financing structure is closely linked to the current state of the business cycle and the investor sentiment. He explicitly stressed this fact when he wrote: "Acceptable financing techniques are not technological constrained; they depend upon the subjective preferences and views of bankers and businessmen about prospects. [...] However, success breeds a disregard of the possibility of failure" (Minsky, 2008, p. 237).

There are various works that elaborate the interdependency of firms financing decisions and the state of confidence formally, e.g. Taylor and O'Connell (1985), Flaschel et al. (1997, ch. 12) and Ryoo (2013*a*,*b*), just to name a few. Partly motivated by the aforementioned works, this paper proposes a dynamic macroeconomic framework that incorporates various important features which have not been studied enough by the existing literature. The model also considers heterogeneous firms that determine their investment demand by taking into account their subjective beliefs about future sales, i.e. according to recurrent and endogenously determined waves of optimism and pessimism. The evolution of the respective firm shares are then composed into a sentiment index that represents a further non-linear dynamic law which is elaborated formally along the lines of Franke (2012).

In the proposed theoretical framework, firms finance their investment by means of retained earnings or by taking out loans from commercial banks. Corporate indebtedness or leverage is expressed in terms of the debt-asset ratio and is derived residually from their financing constraint.

Commercial banks charge a loan rate on outstanding debt which consists of a mark-up that varies endogenously depending on deviations of firms' leverage ratio from a target benchmark value. In particular, the loan rate exhibits a financial accelerator term that amplifies destabilizing effects. We show that this feature implies some important consequences for the dynamics of the model framework. The interaction of firm's leverage ratio and the investor sentiment gives rise to a two-dimensional system of differential equations which is able to produce periodic business cycle fluctuations. It is shown that the resulting dynamics strictly depend on the responsiveness of the endogenous loan rate. In some cases of sufficiently large disturbances the model generates co-existing periodic orbits, that give rise to a "normal-" and "high-indebted" regime.

The properties of the dynamics of the debt-asset ratio are then discussed for different scenarios of how aggressive commercial banks adjust the mark-up component. Similar to the theoretical explorations of Ryoo (2013*b*), it turns out that the differential equation determining the dynamics of the debt-asset ratio is non-linear. However, in contrast to Ryoo (2013*b*), where the differential equation of the debt-asset ratio is given by a parabola shaped function, the endogenously determined loan rate within the current framework produces a cubic shaped leverage function and leads thus to a diversity of dynamic scenarios and more interestingly, to the co-existence of different business regimes.

The remainder of the paper is as follows. Section 2 introduces the basic model framework where an investment driven IS-relationship expressed in terms of effective demand is derived. A law of motion governing the dynamics of corporate debt by means of firm's debt-asset ratio is then introduced. Section 3 analyzes the properties of the model for different parameter settings of the loan rate's responsiveness to changes in the debt ratio. Section 4 extends the basic model framework by the formulation of the sentiment dynamics, where subsection 4.2 and 4.3 analyze the resulting dynamics for different scenarios of commercial banks' behavior. Section 5 concludes.

2 The Structure of the Real Sector

The following model considers a closed economy without government activity and Harrod-neutral technological progress. Under these conditions it is assumed that firms merely produce one type of commodity that can be used for investment as well as consumption. Firms can finance their investment either by internal or by external sources, whereas the only external financing structure is the granting of loans from commercial banks. Firms do not issue equities or bonds, i.e. portfolio decisions should be neglected. Internal finance refers to the firms retained earnings.

2.1 Entrepreneurial Sector

Firms investment demand is assumed to be in accordance to the principle of effective demand. Along the lines of Ryoo (2013*b*), real investment per unit of capital stock (in real terms) is

$$\frac{I}{K} = \gamma_o + \gamma_u u - \gamma_\lambda \lambda = g \tag{1}$$

where γ_o is a positive constant representing the trend rate of growth or even the expected growth rate of the capital stock (to be endogenized in section 4). The variable $u = Y/K$ is the output-capital ratio or the rate of (capacity) utilization of capital (assuming that the capacity is represented by the capital stock) and $\lambda = \Lambda/pK$ represents the debt-asset ratio, i.e. leverage. The coefficients γ_u and γ_λ are both constant and positive. For the sake of simplicity, corporate equities should be neglected in the following.

Consider further a banking system which in turn extends loans to firms Λ and collects deposits from households Λ_d. For simplicity, let us assume that loans to firms are the only asset of banks. Therefore, the lending rate is equal to the deposit rate, the public does not hold any cash and commercial banks do not hold reserves. Therefore, the issuance of bank loans generates a corresponding increase in liabilities, i.e. deposits, since $\Lambda = \Lambda_d$.

2.2 Household's Sector

Turning to the households, their consumption is determined along the lines of Flaschel et al. (1997, ch. 12). Thus, it reads

$$pC = wL + (1 - s_c)[(1 - s_f)(\Pi - \delta - j\Lambda) + j\Lambda] \tag{2}$$

where C is real consumption; δ is capital depreciation, w and L are the nominal wage rate and the number of the employed; p describes the aggregated price level which is assumed to be constant for simplicity (see Franke (2007, 2012, 2015*a*) for a related model with a varying price level expressed in terms of a general inflation climate); s_c is the capitalists propensity to save; s_f is the firms retention rate; $(1 - s_f)(\Pi - j\Lambda)$ are firms net profits or dividend income (Π are the gross profits) where $j\Lambda$ are firm's interest payments to the commercial banks. Within the present paper, the lending rate j plays a crucial role. Instead of assuming that it is being fixed, as in Hein (2007), Sasaki and Fujita (2012) and Ryoo (2013*b*) among others, we configure it to develop endogenously according to a simple rule. For the sake of simplicity, we assume that it consists of a component measuring a constant long-term average lending rate ι_j plus a spread banks charge to cover possible credit (default) losses. It is assumed to depend on the deviations of firms leverage λ from a desired or targeted ratio λ^d. It is hence given by

$$j = j(\lambda) = \iota_j + \iota_\lambda(\lambda - \lambda^d). \tag{3}$$

The parameter ι_λ measures the sensitivity with which banks update their mark-up w.r.t. changes in firms' indebtedness. When the firms' leverage ratio exceeds the benchmark value λ^d, banks interpret it as a deterioration of firm's solvency and charge thus a higher mark-up. Consequently, if banks perceive a declining credit risk by a shrinking spread, they in response will reduce their loan rate. Although this specification is not widely used in the literature, we employ it as a plausible configuration of a financial accelerator, not just to spice up the dynamics of the model, but to emphasize that bank's lending behavior has important effects on the economy's (in-)stability. As it turns out, the arising non-linear features of the model are worth studying.

Real consumption (2) can be expressed in terms of effective demand by normalizing with the capital stock

$$\frac{C}{K} = (1 - \sigma\pi)u - (1 - s_f)(1 - s_c)\delta + s_f(1 - s_c)j(\lambda)\lambda = c(\lambda^2) \tag{4}$$

where $\omega = (wL)/(pY) = 1 - \pi$ is the wage share, $\pi = \Pi/(pY)$ the profit share and $\sigma = s_c + (1 - s_c)s_f$ is a composite parameter. Correspondingly, the profit rate can be decomposed into the profit share, determined by unit costs, times the utilization rate, reflecting the development of effective demand. The consumption equation (4) gives rise to be a quadratic function $c(\lambda^2)$ of the debt-asset ratio.

2.3 Market Clearing

The goods market equilibrium is brought about by

$$u = \frac{C}{K} + \frac{I}{K} + \delta. \tag{5}$$

In the Kaleckian literature it is common to use capacity utilization as a measure of economic activity. For simplicity, we use the output-capital ratio u instead. Accordingly, the utilization rate adjusts to establish goods market equilibrium. Replacing C/K by (4) and I/K by (1) and solving for u gives

$$u = \frac{\gamma_o + \sigma\delta + s_f(1 - s_c)j(\lambda)\lambda - \gamma_\lambda\lambda}{\sigma\pi - \gamma_u} = u(\lambda^2) \quad (6)$$

According to the *Keynesian stability condition*, a stable adjustment process requires that the denominator of (6) is positive, which means that investment reacts less sensitive to changes of utilization than saving (Bhaduri and Marglin, 1990)

$$\sigma\pi > \gamma_u.$$

Given that capacity utilization is a quadratic function of the firms debt-asset ratio, capital accumulation gives rise to be quadratic in λ too

$$g = \gamma_o + \gamma_u u(\lambda^2) - \gamma_\lambda\lambda = g(\lambda^2). \quad (7)$$

For the same reasoning, the non-linear nature applies also for the profit rate net of interest payments

$$f = \pi u(\lambda^2) - \delta - j(\lambda)\lambda = f(\lambda^2). \quad (8)$$

Accordingly, changes in the financial position of firms affects the utilization rate, the capital accumulation rate and net profits in a non-linear manner. Whether an increase in λ may or may not raise effective demand depends on the specific parameter values of eq. (6)-(8).

2.4 Debt Dynamics

The interaction between the financial and the real sector is linked through the financing of firms investment projects. Firms can either finance their investment by retained earnings or by taking out loans. Accordingly, the level of firm's indebtedness is assumed to be residually determined. Debt financing becomes the closure of the spread between firm's planned level of investment and their accumulated retained earnings. It should be noted that banks are always ready to provide firms the required amount of loans that is needed. Hence, "credit rationing" does not play any crucial role. The only restriction on credit supply is the varying mark-up on the lending rate

(3). The finance identity is

$$pI = s_f(\Pi - j(\lambda)\Lambda) + \dot{\Lambda}$$

where $\dot{\Lambda}$ is the increment of debt (where the dot above a variable indicates its time derivative). The law of motion of the leverage is then obtained by normalizing the financing constraint by the value of the capital stock and by a logarithmic differentiation of the resulting expression. Its evolution is governed by

$$\dot{\lambda} = (1-\lambda)g(\lambda^2) - s_f f(\lambda^2) - \hat{p}\lambda = \dot{\lambda}(\lambda^3) \tag{9}$$

where $\hat{p}$ is the inflation rate which is assumed to be constant over time. In contrast to the model provided by Ryoo (2013*b*), the dynamics of λ give rise to a cubic function.

2.5 Calibration and Accounting Consistency

The equilibrium position of the utilization rate and the debt-asset ratio can be explicitly determined. For this purpose, we will assume that utilization as well as the debt-asset ratio achieve their respective target rate in steady state. Accordingly, we will explicitly derive possible restrictions on specific parameters which permit utilization and the debt ratio to reach fully adjusted positions in the long-run, i.e. the realized rates (u, λ) adjust to the exogenous given desired rates u^d and λ^d (Amadeo, 1986; Duménil and Lévy, 1999). Taking the dynamic equilibrium condition $\dot{\lambda} = 0$ into account, the parameter values can be determined from those steady state values.

Proposition 1 *Let a steady state position of the model framework determined by the desired or target values, i.e.* $u^o = u^d$ *and* $\lambda^o = \lambda^d$*, where the o in the superscript indicates the steady state. A fully adjusted position in the long-run can be thus reached with*

$$s_f = \frac{(1-\lambda^o)g^o - \hat{p}\lambda^o}{f^o} \tag{10}$$

and

$$s_c = \frac{\gamma_o + \gamma_u u^o - \gamma_\lambda \lambda^o - s_f f^o}{(1-s_f)f^o + j^o\lambda^o} \tag{11}$$

with

$$
\begin{aligned}
g^o &= \gamma_o + \gamma_u u^o - \gamma_\lambda \lambda^o \\
u^o &= \frac{\gamma_o + \sigma\delta + s_f(1-s_c)j^o\lambda^o - \gamma_\lambda\lambda^o}{\sigma\pi - \gamma_u} \\
f^o &= \pi u^o - \delta - j^o\lambda^o \\
j^o &= i
\end{aligned}
$$

Table 1: Numerical parameter values

Parameter	Description	Value
u^d	Desired and steady state utilization rate	0.750
λ^d	Desired and steady state debt-asset ratio	0.300
γ_o	Constant sales expectations	0.000
γ_u	Sensitivity investment react to changes in the utilization rate	0.090
γ_λ	Sensitivity investment react to changes in the debt-asset ratio	0.080
π	Profit share	0.300
ω	Wage share	0.700
δ	Depreciation rate	0.100
$\hat{p}$	Inflation rate	0.025
ι_j	Average loan rate	0.050

For the calibration and the numerical simulations, we employ the parameter values depicted in table 1. The depreciation rate δ and the profit share π are broadly compatible with the values in Franke (2015*b*). The parameter value for γ_u and the steady state utilization rate are from the empirical estimates of Skott (2012) and the target debt-asset ratio of the firms is taken from Flaschel et al. (1997). The values of γ_o, γ_λ, ι_j and ι_λ are arbitrarily but plausibly chosen. Under the numerical parameter values depicted in table 1, we obtain

$$
\begin{gathered}
s_f = 20.86\% \qquad s_c = 20.14\% \qquad f_o = 11.00\% \\
\sigma = 0.3680 \qquad g^o = 4.35\%.
\end{gathered} \tag{12}
$$

Without putting too much emphasis on the particular magnitudes, despite of the high value of the steady state accumulation rate g^o, most of the above listed values are convincing and broadly compatible with those of the literature. Unfortunately, the model lacks on the calibration of a convenient short-run investment multiplier. It amounts to $\partial u/\partial g = 49.02$ which is far too high. This specific shortcoming constitutes not just the present model set-up, instead, it afflicts many frameworks of this kind. For the following discussion on this model, we will accept the distortions coming from the multiplier.[1]

3 Debt Dynamics and the Paradox of Debt

The specific dynamics of firm's indebtedness strictly depend on the numerical parameter calibration. In this subsection, the relation between corporate debt and the mark-up commercial banks impose on loans should be emphasized. For this purpose we analyze the adjustments of some key variables for different parameter scenarios. The scenarios differ in the sensibility the banking system updates their lending rates in response to changes in firms external financing structure. The *first* case covers the scenario where the loan rate corresponds to the average lending rate $j = \iota_j$ ($\iota_\lambda = 0$). The debt dynamics become familiar to those provided by Ryoo (2013*b*) where the increment of debt is quadratic in the debt-asset ratio $\dot{\lambda}(\lambda^2)$. The *second* case represents probably the most convincing case, where commercial banks adjust the loan rate moderately with the sensitivity $\iota_\lambda = 0.7$. The *third* scenario illustrates the case where banks respond in a very aggressive manner $\iota_\lambda = 1.5$ so that the amplification mechanism of the financial accelerator is very large.[2]

Figure 1 illustrates the debt dynamics as well as other key variables for parameter variations in the range of $\lambda \in [0, 0.8]$. The first panel shows the shape of the increment of debt $\dot{\lambda}$ for each parameter configuration of ι_λ. The case without any financial accelerator $\iota_\lambda = 0$ (the dotted line) yields qualitatively similar results as those obtained by Ryoo (2013*b*). The dynamics of the debt-asset ratio take on the shape of a parabola with two distinct equilibrium points, a stable one at the target rate λ^d, the other (unstable) at a higher leverage ratio ($\lambda = 0.59$). The other scenarios with different choices of ι_λ generate cubic shaping functions featuring three-equilibria whose locations depend on the respective parameter value.[3]

The second panel in the north-east of the figure represents investment $g(\lambda^2)$ as quadratic function of λ. The graph clearly illustrates that the value of the sensitivity parameter ι_λ is important in determining the shape and thus the dynamic adjustments of real accumulation. The first case (the dotted line) displays a negative linear relationship between the capital stock

[1] This specific insufficiency was already pointed out by Flaschel et al. (1997, ch. 12). For an elaborated and detailed discussion see Franke (2015*b*). He also provides a fruitful way to get it under control by incorporating an active government which levies taxes (automatic stabilizers). This solution requires to add a further dimension, i.e. the issuing of sovereign debt.

[2] In model frameworks that incorporate uncertainty, such financial markets participants would refer to beeing risk averse or even "highly risk averse" comparable to those in Caballero and Farhi (2014).

[3] The MATLAB codes for the generations of all graphs of this paper are available from the author upon request.

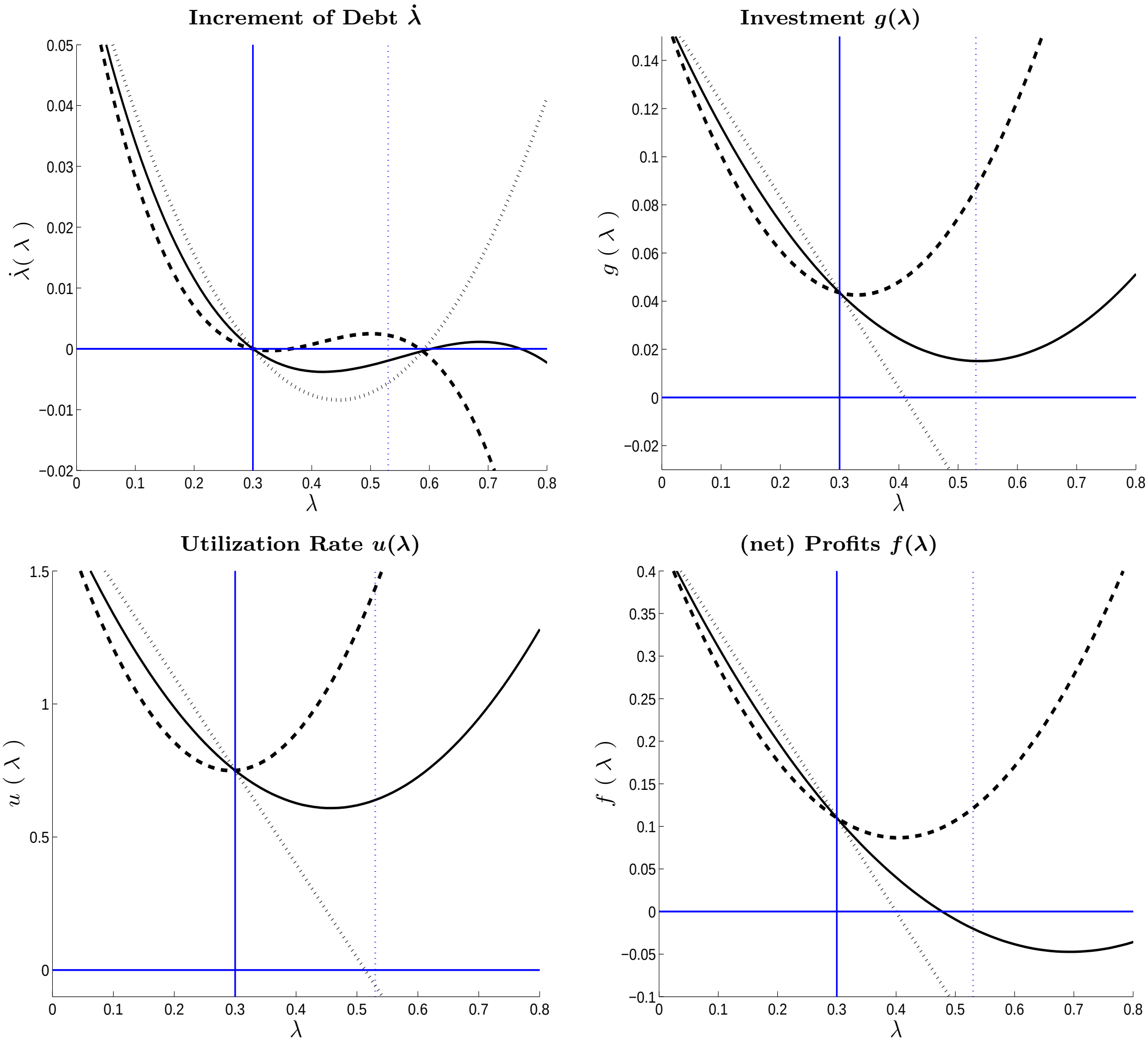

Figure 1: Economic key variables in relation to the firm's debt-asset ratio under different parameter values for the debt sensitivity of the lending rates risk premium $\iota_\lambda = [0, 0.7, 1.5]$. The dotted black line is the Ryoo (2013*b*) type specification $\iota_\lambda = 0$, the thick black line refers to the benchmark calibration $\iota_\lambda = 0.7$ and the thick dashed line describes the case of $\iota_\lambda = 1.5$. The vertical straight blue line displays the target level of indebtedness λ^d and the dotted blue line the turning point of the investment function from which on the paradox of debt appears.

growth rate and the debt-asset ratio. This point was already pointed out by several authors including Lavoie (1995), Hein (2006, 2007) among others and refers to the "paradox of debt", which constitutes a phenomenon in which firms are trying to reduce their leverage ratio by cutting investment which in turn leads paradoxically to an increase in firms indebtedness. The reduction in effective demand and internal funds makes firms to rely more on external than on internal finance. The debt-asset ratio will be ever increasing while firms will cut investment. It exhibits an unique stable equilibrium at λ^d.[4]

[4]The paradox of debt also refers to being a critique on Minsky's financial instability hypothesis. But following the arguments of Ryoo (2013*a*,*b*), the intuition behind the paradox of debt differs in several aspects from Minsky's theory on endogenoeus financial practices. Probably the main departure from Minsky's theory in the present context where monetary policy does not play any role is that firms financial behavior, i.e. lending and borrowing decisions, takes place without any reference to margins of safety. Instead, firms determine their liability structure

The second case, displayed by the U-shaped thick black line, shows that the paradox of debt arises in a certain range of λ. At some value of the leverage $\lambda = 0.53$ (illustrated by the vertical blue dotted line in each panel), the paradox of debt disappears and investment becomes then an increasing function of the debt-asset ratio $g'(\lambda) > 0$. From this point on, firms leverage increases with capital accumulation. Accordingly, increasing real accumulation or internal accumulation (profits) boosts effective demand and raises firms debt ratio. The same mechanism works the other way around, where investment demand decreases due to de-leveraging. Note that the turning point where the investment function switches sign depends critically on how aggressive the banking sector adjusts the loan rate in response to changes in debt accumulation. For higher values of ι_λ, the turning point where the slope of the investment curve gets positive moves to the left in the λ-direction and is therefore associated with lower leverage ratios. As represented by the thick dashed line, for $\iota_\lambda = 1.5$ the paradox of debt vanishes instead at a ratio close to the target rate at $\lambda = 0.33$.

The panel in the south-west illustrates the utilization rate and the last panel net profits out of interest payments both functions of firms indebtedness. Focusing on the $\iota_\lambda = 0.7$ case (the thick black line), effective demand and net profits are U-shaped as well. From the figure we can infer that the utilization rate has it's trough earlier than investment (at $\lambda^o = 0.46$) and firms net profits. Accordingly, a rising debt ratio decreases firms investment, at least for low levels of the leverage, but increases the rentiers' dividend income which in turn boosts consumption and thus effective demand. This effect continues until the debt ratio reaches a relatively high level ($\lambda = 0.69$).

4 The Extended Framework

4.1 Incorporation of Endogeneous Sentiment Dynamics

The present paper aims to provide a stylized model which is able to generate endogenous long-term "Minsky cycles" from the interaction of corporate debt and the investors sentiment. Therefore, it remains outstanding to provide a law governing the dynamics of the firms business sentiment. In this section, we introduce a mechanism that enables us to differentiate between two types of firms which differ in their perceptions of future sales.

Let us turn back to the assumptions concerning firms investment decisions. The constant

as residual of planned investment and retained earnings. He further states that the paradox of debt refers to short-term debt financing whereas Minsky's hypothesis can be rather understood as a long-term phenomenom.

term in equation (1) γ_o, firms capital stock growth expectations, often refers to being "animal spirits" or even to a general business sentiment (Skott and Zipperer, 2012; Franke, 2012). Assumptions that this term may increase (decrease) in a case of over-(under-)utilization leads to the *Harrodian* destabilizing mechanism first raised by Skott (2008). The consequences of permanent deviations of actual from desired utilization were often discussed and many authors argued that it refers rather to a temporary phenomenon and cannot persist systematically. Amadeo (1986) and Lavoie (1995) among others, raise the issue whether it seems to be convincing to assume that utilization may probably differ from the desired or targeted utilization rate. They argue that the variability of utilization can reconcile actual and desired utilization if the desired rate itself varies endogenously. Harrodian instability, in contrast, is a process of cumulative causation. In the case of a permanent mismatch of actual and desired utilization of productive capacity, entrepreneurs would eventually revise their investment plans and respond with adjustments of their expected secular rate of sales, which in turn raises investment and thus utilization again. This unstable process leads to ever rising rates of capacity utilization.

In order to cope with that instability problem, some authors have introduced endogenous adjustment processes for γ_o and the normal or desired utilization rate u^d (Schoder, 2012; Franke, 2015*a*). Correspondingly, in a case of rising sales expectations, firms reduce their target rate in order to respond more spontaneously to demand fluctuations. However, this part of the present paper aims to endogenize the business sentiment term in a convenient manner where Harrodian instability can nevertheless be ruled out. Doing so, the formalization of the dynamic law governing the adjustments of the animal spirits are in the line of Franke (2012) who provided a behavioral foundation for the agent's attitudes.

Following Franke (2012) aggregate sales expectation can be written as

$$\gamma_o = g^\star + \beta a = \gamma_o(a) \tag{13}$$

where $g^\star$ is the trend capital stock accumulation rate and a refers to the sentiment term (whose law of motion should be defined below). In contrast to the literature on this type of models, where planned investment depends directly on utilization (its gap respectively) and the financial position of firms, within the actual framework investment is also determined indirectly via the firms sentiment as in Franke (2012). They determine their investment by taking into account their beliefs about the future prospects of the economy. Accordingly, they can either be optimistic or pessimistic where the sentiment index a is a sort of average among the shares of firms with the respective attitude. The sentiment index is bounded at $a = -1$, the state where all investors are pessimistic about the return of an investment, and $a = +1$, the state where firms are optimistic. In a balanced state $a^o = 0$, the share of optimistic firms corresponds exactly to the share of pessimistic firms which in turn indicates a steady state environment.

Using such a sentiment dependent investment function, the associated utilization rate becomes

$$u = \frac{g^\star + \sigma\delta + \beta a + s_f(1 - s_c)j(\lambda)\lambda - \gamma_\lambda \lambda}{\sigma\pi - \gamma_u} = u(a, \lambda^2) \tag{14}$$

where due to the implementation of the sentiment variable, the effect of changes in the utilization rate on investment is not without ambiguity.

Changes in the sentiment index can be formalized using the "transition probability approach" which has its origin in statistical mechanics and was first published in social sciences by Weidlich and Haag (1983). The first contribution adapting this approach in the context of financial markets is Lux (1995). Franke (2012) put it forward into a business cycle model and derived a differential equation for the sentiment index without having invoked the statistical mechanics apparatus. Therefore, he shows that the law governing the sentiment dynamics can be represented by

$$\dot{a} = \nu[(1 - a)\exp(s) - (1 + a)\exp(-s)]. \tag{15}$$

The parameter ν measures the adjustment speed and $\exp(\cdot)$ is an exponential function.[5] The variable $s = s(\cdot)$ refers to a switching index function that captures the key variables which are responsible for the firms subjective evaluation of the current state of the business cycle. An increase indicates a rise in the general business sentiment.

Suppose that this function captures *self-reference* effects, i.e. a positive feedback of the sentiment a on itself. In particular, it constitutes a concept of contagion or herding where the parameter ϕ_a measures the degree of group pressure and can therefore be understood as a "herding parameter". The remaining terms within the function refer to *hetero-reference* effects, i.e. the feedback of external norms or more objective factors that induce firms to change their attitude. The *first* external norm is the current state of the business cycle expressed in terms of deviations of utilization from firm's desired benchmark rate. It is assumed to have a positive impact on the general sentiment, which is quite intuitive, since if firms observe an increase in

[5]Note that this formula can be reformalized by using hyperbolic functions. Accordingly, it can be illustrated as $\dot{a} = 2\nu[\tanh(s) - a]\cosh(s)$.

effective demand, they believe that their sales will increase as well. The *second* component refers to the return differential $f - i - \rho^\star$ where i is the real rate of interest which is supposed to be exogenous. Specifically, firms (or their entrepreneurs) compare their (net) profits with the returns from the safer investment in government bonds (whose return is assumed to be constant within the present framework). Accordingly, they match the spread between the return on profits and the real interest rate against a desired return differential $\rho^\star$ that in turn reflects what they think would be a suitable benchmark measure. Hence, the sentiment index increases if $f - i$ exceeds $\rho^\star$. In contrast, it decreases if the spread falls short of $\rho^\star$, indicating that the return on investment is no longer profitable enough (Franke, 2015*a*). The *third* building block consists of the financial accelerator component, i.e. the spread commercial banks impose on the lending rate to cover potential losses (3). In particular, consider that higher corporate leverages indicate greater future "default risk" which in turn exerts a downward pressure on the state of confidence. Therefore, the last term enters negatively. Consequently, the switching index function becomes

$$s = \phi_a a + \phi_u(u - u^d) + \phi_r(f(u, j, \lambda) - i - \rho^\star) - \phi_\lambda \iota_\lambda(\lambda - \lambda^d) \tag{16}$$

where ϕ_a, ϕ_u, ϕ_r and ϕ_λ are sensitivity parameter.

Proposition 2 *Consider that capital accumulation is determined by* (1) *and* (13). *A fully adjusted position can be then reached in steady state by equation* (10) *and*

$$s_c = \frac{g^\star + \gamma_u u^o - \gamma_\lambda \lambda^o - s_f f^o}{(1 - s_f) f^o + j^o \lambda^o} \tag{17}$$

with

$$\begin{aligned} g^o &= g^\star + \gamma_u u^o - \gamma_\lambda \lambda^o \\ u^o &= \frac{g^\star + \sigma\delta + s_f(1 - s_c) j^o \lambda^o - \gamma_\lambda \lambda^o}{\sigma\pi - \gamma_u} \\ \rho^\star &= f^o - i. \end{aligned}$$

For simplicity, assume that the real rate of interest corresponds to the average loan rate $i = \iota_j$. With the numerical parameter values depicted in table 1, the reference value in the return differential amounts $\rho^\star = 6\%$.

4.2 Long-Term Fluctuations and the Paradox of Debt Under a Constant Loan Rate

In this section, the dynamic interaction between firms leverage ratio and the general business sentiment should be analyzed. Using the formal specification of the debt-asset ratio (9) and the sentiment index (15) we obtain an autonomous two-dimensional system which can be solved recursively. We will proceed similarly as before by distinguishing between different parameter settings of the financial accelerator coefficient ι_λ. First of all, let us consider the case where the loan rate is fixed at its average ι_j implying $\iota_\lambda = 0$. The system exhibits a stable solution at (a^o, λ^o) for a certain range of the parameters in the switching index function (16). Probably the most interesting dynamic feature of the model from a heterodox perspective is the configuration which generates limit cycles. For $\beta = 0.05, \phi_a = 0.2, \phi_u = 0.1$ and a sufficient large value of the sensitivity parameter the sentiment index reacts to spreads of the return differential $\phi_r = 0.9$, the point of rest looses its stability and a limit cycle arises.[6]

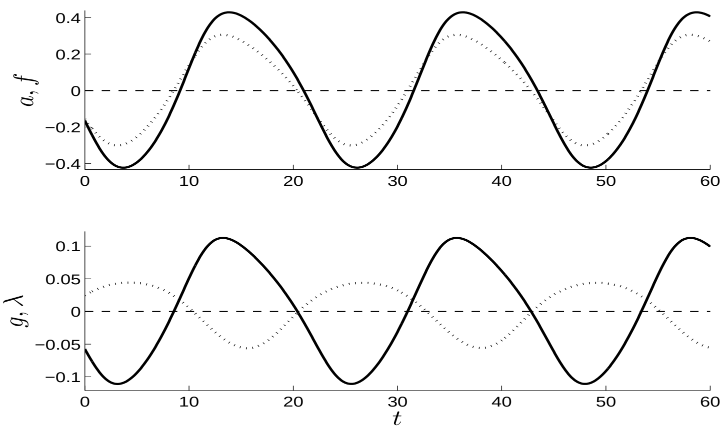

Figure 2: Cyclical (detrended) movements of the sentiment index (upper panel thick line), the firms net profit rate (upper panel dotted line), the capital stock growth rate (lower panel thick line) and the debt-asset ratio (lower panel dotted line) for $\iota_\lambda = 0.0$, $\beta = 0.05, \phi_a = 0.2, \phi_r = 0.9$ and $\phi_u = 0.1$.

The adjusted cyclical pattern of the sentiment index a, firms internal funds f, the capital stock growth rate g and the debt-asset ratio λ are illustrated in figure 2.[7] Concerning the co-movements, the figure suggests that firm's (net) rate of return is a leading variable. During phases where the general business climate increases firms respond with the accumulation of capital and internal funds. At some point, firms profits reach a maximum and begin to fall

[6]Note that the arising limit cycle is attractive and locally stable. Unfortunately, for large values of λ, global stability which can be proofed using the Poincaré-Bendixson Theorem is not satisfied.

[7]The representation of the evolution of effective demand should be neglected due to the distortions coming from the huge multiplier effect. For the same reasoning, the stylized facts emerging from the synthetic time series', such as the period length and the cycle's amplitude, are biased.

thereafter. While the decline in profits initiate firms to reduce their investment demand, the general business sentiment still increases for a while due to the effect of the increasing leverage ratio, which in turn boosts rentiers households' property income and thus consumption and effective demand. This process is backed up by the self-reference effect (herding) defined in the switching index function (16). The fall in firms investment demand is then reinforced by a declining business sentiment. Note that the pattern of the debt-asset ratio moves almost countercyclical.[8] Thus, despite of the decreasing capital stock growth rate, the feedback of firms leverage ratio is still positive for a relatively long period of time which in turn indicates that the paradox of debt unfolds its long-term effects. During such times, firms rather rely on debt than on internal financing so that most of the internal sources are distributed to the shareholders. At some point, where corporate debt reaches its peak, firms start to de-leverage their financing structure which, through the paradox of debt, decelerates the reduction of investment demand until capital accumulation becomes again profitable.

4.3 Long-Term Fluctuations and the Emergence of Co-Existing Limit Cycles

The discussion of the long-term dynamics above is based on the assumption of a fixed loan rate. In the following, the implications of the financial accelerator on the system's stability should be highlighted. For this purpose, we set the financial accelerator coefficient to $\iota_\lambda = 0.7$ which in turn adds some further non-linearities to the model. The dynamics behind the business sentiment and the debt ratio are then discussed using a phase diagram.

Figure 3 represents the trajectories of the differential equations of the debt-asset ratio (9) and the sentiment index (15) for the numerical parameter values depicted in table 1. We also use the parameter values of the switching index function which were employed in the last section and set the parameter the sentiment index responds to changes in the mark-up to $\phi_\lambda = 0.1$. This configuration yields quite interesting results. Since the explicit analytical solution of both isoclines is not transparent enough to draw any conclusion from it directly due to the non-linearities within both dynamic laws, we employ instead the contour plots of the reduced form exhibiting a height of zero to illustrate our results. The thick gray line in the figure represents the ($\dot{a} = 0$)- and the dashed black line the ($\dot{\lambda} = 0$)-isocline. The points where both isoclines intersect are the equilibrium positions. The isoclines of the system give rise to multiple equilibria.

[8]Reasons responsible for such co-movements are probably the biases arising from the strong investment multiplier. An elaborate discussion of the cyclical characteristics of firm's debt-asset ratio is provided by Flaschel et al. (1997, ch. 13.3).

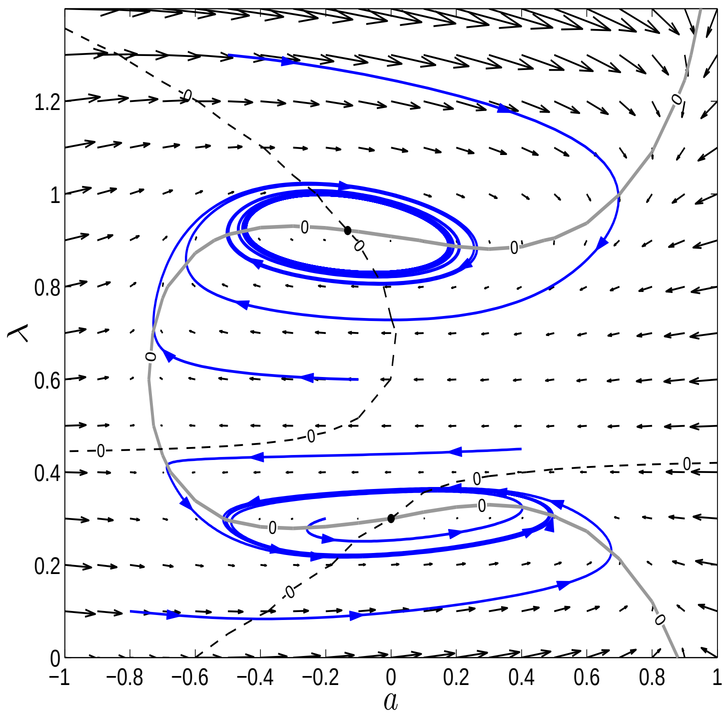

Figure 3: Sentiment-debt interaction for $\iota_\lambda = 0.7$. The other parameters are $\beta = 0.05, \phi_a = 0.2, \phi_r = 0.9$, $\phi_u = 0.1$ and $\phi_\lambda = 0.1$. The thick gray line illustrates the $(\dot{a} = 0)$ - and the dashed black line the $(\dot{\lambda} = 0)$-isoclines.

A vector field is used to indicate the off-equilibrium dynamics.

For the present parameters, and taking into account proposition 2, the equilibrium position $E' = \{a^o, \lambda^o\}$ is unstable and gives rise to anti-clockwise oscillations around the point of rest which is illustrated by the lower closed orbit within the phase plane. The cycle is attractive for a wide range of leverage ratios. But at some point, the limit cycle becomes repelling so that global stability can be ruled out. Instead, the manifolds converge to a co-existing limit cycle with clockwise agitation around the equilibrium position $E'' = \{-0.138, 0.921\}$. The emergence of a second orbit clusters the dynamics into two regimes, a “normal-” and a “high-indebted” regime.[9] The fact that the trajectory of the high-indebted regime oscillates in a clockwise manner leads obviously to fairly different economic adjustment mechanisms. The processes that are working over the periodic orbits can be best described by the (adjusted) cyclical pattern mapped in figure 4.

Before going into detail, note that the time series' in figure 4 are detrended due to illustrative purposes. The left panel of figure 4 represents the evolution of the business sentiment, firm's (net) profit rate, the capital accumulation rate as well as the debt-asset ratio around their steady

[9]It would be an interesting challenge to identify the basins of attraction of each periodic orbit. The approach provided by Giesl (2004, 2007), which elaborates “Borg's criterion”, could be a useful tool for this purpose.

states which were determined in proposition 1 and 2. In contrast, the evolution of the sentiment variable within the high-indebted regime, the right panel, swing around a steady state value of $a^o_{HI} = -0.138$; the profit rate around $f^o_{HI} = -0.097$; the capital stock growth rate fluctuates in a vicinity of $g^o_{HI} = 0.053$ and the debt ratio around $\lambda^o_{HI} = 0.912$. The HI in the subscript indicates the magnitudes of the high-indebted regime. The business regime arising in the right panel of figure 4 is not just characterized by highly-indebted, non-profitable enterprises, it also suffers from permanent over-utilization $u^o_{HI} = 1.475$ ($u^o_{HI} - u^d = 0.725$, not shown in the figure) despite of the mean reverting process of the sentiment index specification which invalidates the Harrodian instability problem. These observations obviate the propositions made before, since sufficiently large disturbances may drive the economy to the high-indebted regime so that it cannot longer be ensured, even in the long-run, that the variables reach a fully adjusted position in steady state. In the present context, the plausibility for the emergence of two different regimes where one of the economies endures permanent over-utilization is not without ambiguity.

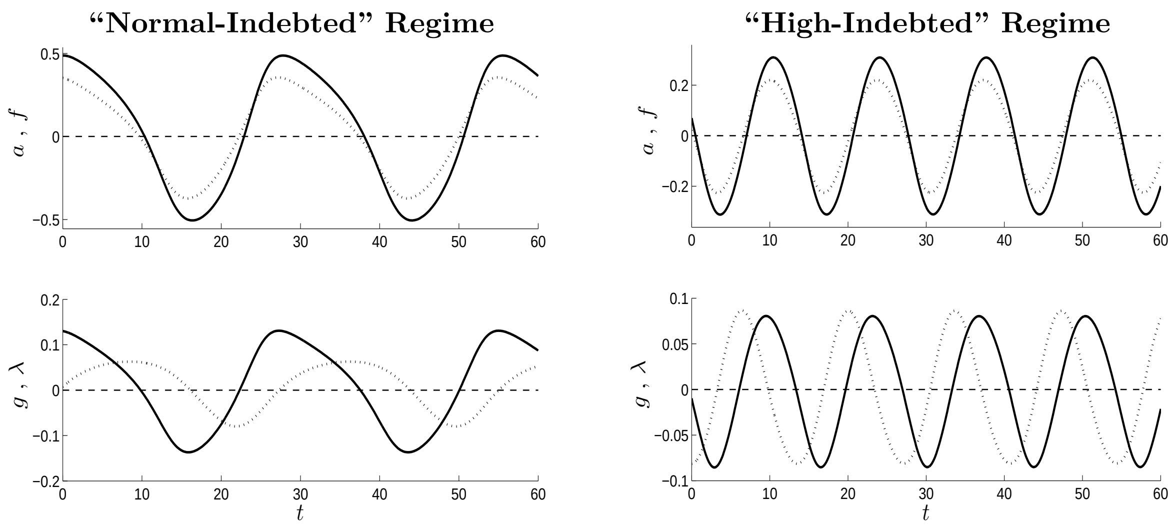

Figure 4: Cyclical (detrended) movements of the sentiment index (upper panel thick line), the firms net profit rate (upper panel dotted line), the capital stock growth rate (lower panel thick line) and the debt-asset ratio (lower panel dotted line) for $\iota_\lambda = 0.7$, $\beta = 0.05, \phi_a = 0.2, \phi_r = 0.9$, $\phi_u = 0.1$ and $\phi_\lambda = 0.1$. The time unit corresponds to years.

However, turning back to figure 4, the cyclical motions of the normal-indebted regime in the left panel exhibit a longer business cycle length than for the high-indebted regime. Furthermore, it turns out that firm's leverage ratio is lagging and possesses a larger amplitude. Firm's profits are leading whereas the capital accumulation rate and the sentiment index roughly swing synchronously. In particular, as long as corporate earnings increase, firm's or their owner's perceptions about future sales induce investment demand to increase as well. While the capital stock growth rate increases the debt-asset ratio declines due to the paradox of debt. Once, when the actual net rate of return out of interest payments exceeds f^o (indicated by the dashes zero line), the leverage ratio experiences its turning point and starts to increase thereafter.

The cyclical motions of the high-indebted regime (the right panel of figure 4) in contrast show that the paradox of debt is no longer valid for the high-indebted regime which coincides with the conclusions made discussing the general properties of the parabola shaped function of the capital stock growth rate (figure 1). Although firms already act at high leverages, an increase in the debt ratio raises the mark-up in the loan rate and thus rentiers' property income. The boost in income increases rentiers household's consumption and thus utilization and corporate profits. The increase in effective demand as well as in firm's internal financing structure is backed up by a swing of optimism and thus firm's subjective sales expectations which in turn raises investment.

One may argue that such environment is not economically reasonable at all, since highly indebted enterprises that permanently work over their production capacity indicate that the economy overheats. But, the dynamic insights of the present framework are nevertheless worth discussing. Large counter-moving disturbances, e.g. policy actions, may probably bring the trajectories from the upper regime down to the normal indebted environment. Our simulations suggest that the high-indebted regime moves down along the ($\dot{\lambda} = 0$)-isocline with increasing values of the financial accelerator coefficient ι_λ. For high values, the upper periodic orbit in the phase diagram 3 vanishes so that merely the normal-indebted regime remains. This fact in turn stresses the importance of firms lending and the banking sector's borrowing decisions on the system's stability. If we consider hypothetically that the mark-up within the lending rate and thus firm's external financing structure is governed by the public banking supervision, debt-regulation becomes a key in stabilizing the unstable economy. Regulation, or macroprudential policy, closely links the corporate financing structure to macroeconomic policies and is therefore worth to consider in such models of the business cycle. According to the empirical findings of e.g. Graham et al. (2014), the leverage of enterprises in the US increased enormously during the last century. This increase occurred mainly in the unregulated industrial sector and affected firms of all sizes while the development of the leverage ratio within the regulated sector evolved in a stable manner.

5 Concluding Remarks

This paper presented a stylized modeling strategy that reflects the dynamic interaction of corporate debt and the general business sentiment. Both dynamic laws are configured in a strong non-linear manner where the notion of the sentiment index accounts for heterogeneity among firms in their sales expectations. It is shown that the relationship between firm's leverage and investment can either be positive or negative, strictly depending on the numerical calibration, in particular, on the strength of the financial acceleration effect. The observation that the paradox of debt occurs for small leverages and vanishes for higher ratios entails some interesting dynamic implications. Numerical simulations suggest that for large disturbances in the firm's external financing structure, the model contemporaneously produces two co-existing business regimes, one in a normal-, the other in a high-indebted environment. The high-indebted regime is characterized by firms which permanently operate over their capacity, whereas the normal-indebted regime is subject to the paradox of debt. The location of the regimes depend on bank's borrowing conditions. It is argued that the more sensitive commercial banks react to changes in the enterprise's leverage, implying high financial acceleration, the greater is the reduction of indebtedness in the high-indebted regime. This in turn raises the importance of policies preventing the possibility of converging to the high-indebted regime, or at least, to execute forces that aim to bring the business regime back to the more favorable environment. Natural candidates are macroprudential policies that either constraint firm's demand or bank's supply of credits. Accordingly, future research on this topic should aim to consider macroprudential policies as macroeconomic stabilization tools and its effects on boom-bust cycles. By doing so, commercial bank's behavior has to be elaborated in a more rigorous way, so that the issue of regulation can be analyzed and illustrated in a clearer manner.

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