Modelling driving behaviour using hybrid automata

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Abstract: The Authors present a new approach to the modelling of human driving behaviour, which describes driving behaviour as the result of an optimization process within the formal framework of hybrid automata. In contrast to most approaches, the aim is not to construct a (cognitive) model of a human driver, but to directly model driving behaviour. We assume human driving to be controlled by the anticipated outcomes of possible behaviours. These positive and negative outcomes are mapped onto a single theoretical variable - the so called reinforcement value. Behaviour is assumed to be chosen in such a way that the reinforcement value is optimized in any given situation. To formalize our models we use hybrid automata, which allow for both continuous variables and discrete states. The models are evaluated using simulations of the optimized driving behaviours. A car entering a freeway served as the scenario to demonstrate our approach. First results yield plausible predictions for car trajectories and the chronological sequence of speed, depending on the surrounding traffic, indicating the feasibility of the approach.

1 Introduction

In the domain of driver modelling assumptions are made about the factors controlling driver behaviour. Among these variables are for example attitudes, personality, experience, driver state, task demand and situation awareness [1]. In the literature different types of driver models can be found. One familiar classification is the one of Michon who distinguishes four basic types of driver behaviour models [2]: task analyses, trait models, mechanistic/adaptive control models and motivation/cognitive models. These are organized in a two-way classification table distinguishing input-output (behaviour oriented) and internal state (psychological/motive oriented) firstly and taxonomic and functional secondly [3]. Driver behaviour models can further be located on a dimension ranging from specific to unspecific [2].

Driver models fulfil different purposes, leading to another distinction, e.g. conceptual and computational models [4]. Conceptual models are developed in order to understand the processes involved in driving. Computational models are constructed in order to compute, simulate and predict individual driving behaviour or to rebuild interactions among several

road users. Driver models are used in research as a tool for Rapid-Prototyping, reducing the need of extensive experiments with real subjects [4-6]. The dominant paradigm for the modelling of human driving behaviour is information processing in the cognitive domain in the tradition of cognitive architectures (e.g. ACT [7, 8]). The cognitive approach tries to model the relevant cognitive processes of a driver in order to explain and to predict his driving behaviour in certain situations. There are a large number of cognitive processes possibly involved in driving behaviour, for example perceiving, evaluating, goal-setting, deciding, etc. [9, 10]. Therefore, many existing modelling approaches use cognitive architectures (e.g. ACT [7, 8]). Because the description of dynamic processes is difficult within these modelling frameworks their application poses considerable problems in the domain of driver simulation. An alternative approach is the use of models for vehicle guidance that focus on the interaction between driver and vehicle and are conceptualized according to cognitive action theories [11, 12]. In this framework driver behaviour is described as the result of extensive internal planning and decision processes [13, 14]. These approaches focus on the specification of processes and structures underlying cognition [15]. The cognitive approach – although intuitively convincing – does not only suffer from heavy methodological problems (cognitive processes are intrinsically unobservable [8]), but also leaves open the question whether it is actually necessary to model internal processes in order to predict behaviour.

In contrast to this approach, we propose a new modelling framework for driving behaviour, which uses theoretical concepts from Behavioural Psychology [16]. In Behavioural Psychology the focus lies on observing apparent behaviour and analysing its relations to situational stimuli. Theories of inner processes are not of primary interest [17, 18].

The core idea is that in a pragmatic setting what is needed is not a *driver-model* but a model of *human driving* – that is, a formal description of how controllable external variables influence the movement of a car in traffic. The fact that this is mediated by the cognitive processes (and of course by the physical actions) of a living driver sitting inside the car is not essential to questions concerning car movement. Therefore the approach put forward takes driver and car to be one single agent in a traffic scenario, rather than modelling the interaction between them. The theoretical background used in the present approach is an application of optimization theory and rests on the assumption that human behaviour is gradually adapted to the environment (this may include physical environment, as well as social factors or the behaviour of other organisms) [19-21]. In our models we are neither interested in the internal processes that lead to the observed behaviour, nor in those that mediate the process of adaptation. Instead, we start with the general assumption that driving behaviour is to find out what is "optimal" in a given situation [22]. How the optimization process is implemented in the organism is not relevant for our models.

2 Behavioural approach

To formalize the concept of optimization we introduce a theoretical variable which will be called "reinforcement value" (due to its theoretical roots in operant behaviour theory). This reinforcement value plays an essential role in our models and simulations, because we assume behaviour to be chosen in order to maximize a theoretical reinforcement value. The reinforcement value of a behaviour in a given situation is taken to be a mapping of all anticipated positive and negative consequences of this behaviour onto a single dimension (Fig. 1).



Fig 1: Origin of the reinforcement value [23]

Thus, in any given situation, all possible behaviours can be assigned a reinforcement value by means of specific evaluative functions.

Behaviour is assumed to be the result of this evaluation against positive and negative outcomes, in the way that in each situation the behaviour with the highest expected reinforcement value (with regard to a specific time horizon and a specific set of possible behaviours) is chosen. We would like to stress that this approach – although situated in the domain of behavioural psychology – does not take behaviour to be determined by external

factors alone, but to be the result of the specific reinforcement values of a person with respect to the possible behaviours in a given situation. In fact, the notion of reinforcement value maximization is very similar to the basic idea of Expected Utility Theory [24]. In contrast to Expected Utility Theory, however, the current approach does not assume driving behaviour to be the result of a rational decision process. Moreover, for reasons of parsimony, we omit the concepts of expectancies and subjective probabilities, which results in a slightly different formalization.

2.1 Hybrid automata

We use the 'Theory of Hybrid Automata' as a formal background to implement these assumptions into a quantitative model. Hybrid automata provide a helpful framework for our models, because they allow both for continuous variables as well as discrete states to describe a system [25]. Within a single state the change of each variable is described by a differential equation. Between states there are certain criteria which specify the transition from one state into another. This way it is possible to specify simple if-then-rules as well as continuous functions and even their interaction.

To apply this formal framework to the aforementioned theory of optimal behaviour we break up the timeline into distinct situations and identify these with the states of a hybrid automaton. The driving behaviour in each situation changes continuously over time – thus we identify the corresponding variables (namely speed and trajectory) with the continuous part of the automaton. Thus, driving behaviour is described by a different set of continuous functions of time in each situation. To incorporate the concept of reinforcement maximization, these continuous functions are not specified a priori but modelled as unknown functions, which are to be maximized against a reinforcement value which depends upon suitably chosen functions of relevant external variables (e.g. distance to other cars, lateral position, steering angle etc.).

2.2 Exemplary scenario

As an exemplary scenario to apply our modelling approach we take a car entering the freeway. Merging onto the freeway is a rather complex driving task, as several factors have to be considered by the driver. The driver has to adapt his speed according to several factors, e.g. the road geometry, the speed limit and the car ahead, he has to control the distance to the car ahead, the lane markings and the end of the acceleration lane, before a lane change can be conducted he has to find an appropriate gap on the freeway, he has to adjust his driving speed to the traffic on the motorway, change lane and finally reach travelling speed [26]. Instead of modelling all these tasks and making assumptions about the related internal processes like perception, decision and response selection, and response execution [27] or

taking into account every variable that might have an influence in the situation, like personality, experience, task demand, driver state and situation awareness [1], our model focuses on observable behaviour, namely trajectory and speed of the ego car. Furthermore, as mentioned before, we model the driver and the car as one unit, omitting intermediate steps like steering or breaking. As long as these driver behaviours are causally dependent on external factors, it is not necessary to include them in the model, since they do not enhance predictive power. It is important to indicate that we do not doubt that the mentioned variables and interactions may have an impact on driving performance. However, we want to evaluate the predictive and explanatory power of a parsimonious model which is deduced from another scientific paradigm.

The model is based on the assumption that the driver starts at a given velocity and has a desired travelling speed on the freeway. Moving onto the freeway he tries to minimize forces due to acceleration or trajectory change (trying to avoid unpleasant jerks, as well as possible threat associated with sudden car movements), to stay as far to the right as possible (resulting in a tendency to drive on the rightmost lane, which is also stipulated by the German road traffic regulations) and, of course, avoid collisions with other vehicles. The minimization of forces, accomplished by gradual braking and accelerating, results in smooth movements. It is supposed that drivers pursue smooth movements due to biological adaptation. Since abrupt movements are associated with aversive stimulus situations like stumbling, running into something or being hit, they are assumed to be aversive per se. Any departure from smooth movements are therefore taken to be the result of restricting factors in the environment (e.g. cars that get into the way of the ideal - that is smooth - trajectory). To formalize our assumptions we assigned corresponding reinforcement values to high forces, collisions etc. The resulting hybrid automaton is depicted in Fig. 2. Note that the timeline is divided into three functionally distinct parts - each being visualized by a circle containing the continuous functions controlling behaviour in this state. The first state stands for the time just before it is possible to enter the freeway. The second state describes the process of filtering into the traffic. The third state is just an exit-state, which corresponds to the fact that filtering onto the freeway is now accomplished. In a more elaborated model, of course, there would have to be a number of new states describing the task of driving on the freeway – possibly completed by additional states corresponding to changes in the environment like new cars entering or overtaking manoeuvres. The states contain a description of both the ego car and external factors relevant to driving behaviour. The ego car is assigned a position (x, y), a current velocity v, and an angle α to the lane. Our model considers the variables v and α to be controlled by the driver via the functions f and g, representing acceleration and steering, respectively. These two functions are optimized for maximal reinforcement value q. We add another car to our model, which is driving on the right lane of the freeway – with position (x_2, y_2) and velocity w. To transform steering and acceleration behaviour into absolute car

position the model uses the trigonometric functions $v \times \sin(\alpha)$ and $v \times \cos(\alpha)$. State transitions are determined by the position on the x-axis, which corresponds to how far the ego car has proceeded on its way onto the freeway. The most important part of the model is given by the two evaluative functions Δ_1 and Δ_2 , which assign a reinforcement value q to every possible steering and acceleration behaviours for each momentary state of the ego car. These functions are formalizations of the aforementioned theoretical assumptions made about the effects of certain external variables on the driver. During the process of filtering in the evaluative function is given by

$$\Delta_1 = -f(x)^{\omega} - \tan(g(x)^{\omega})v^2 - \frac{\lambda}{(x - x_2)^2 + (y - y_2)^2 + k}$$

The parameters ω and λ are person specific values which express force aversion and the aversiveness of collisions, respectively. Forces are assumed to be only moderately aversive when small, but increasingly unpleasant when high – we model this by the use of a power function with $\omega > 1$. To formalize the avoidance of car crashes we took the squared distances in both dimensions to construct a hyperbolic function with $\lambda \ge 1$ contributing to the steepness of the curve. This results in extremely negative values for small distances and values close to zero for large distances. Because the term under the fraction line must not be zero we add a constant *k*, which is to be set to a very small number.

As soon as the ego car has arrived on the freeway, the evaluative function changes to

$$\Delta_2 = -\tau (v - v_{des})^2 + \sigma [\min(0, y - bound_r) + \min(0, bound_1 - y)] - \rho \times y$$
$$-\frac{\lambda}{(x - x_2)^2 + (y - y_2)^2 + k}$$

The collision term is the same as in the previous state. Instead of force aversion, however, we include the squared deviation from the desired speed v_{des} , which is weighted by person and situation specific factor τ . This parameter stands for the relative importance of reaching the desired travelling speed and can be interpreted as time pressure. We further modelled the tendency to avoid leaving the road by assigning positive values, if the ego car is within the boundaries of the road *bound*_r and *bound*₁. These terms are weighted by another parameter, σ , which stands for the threat posed by an accident due to deviations from the road. The last factor is a general tendency to drive on the right. The corresponding weighting parameter ρ stands for the threat posed by the German traffic law, which demands to drive on the rightmost lane, whenever possible.



Fig. 2 Model of driver moving onto a freeway with another vehicle already on it

 \mathbf{x}, \mathbf{y} : position, \mathbf{v} : velocity, $\boldsymbol{\alpha}$: angle to freeway direction, $\mathbf{x}_2, \mathbf{y}_2$: position of car 2, \mathbf{w} : velocity of car 2, $\mathbf{f} \in [0, 2]^{\mathbb{R}}$: acceleration (optimized), $\mathbf{g} \in [-0, 1, 0, 1]^{\mathbb{R}}$: steering (optimized), \mathbf{q} : reinforcement value (measured at $\mathbf{x} = \mathbf{140}$)

3 Evaluation of the model

In order to evaluate the basic properties of the model, we conducted a series of numerical simulations. For this reason we assigned exemplary values to the variables specifying the scenario. The dimensions of the road were given by three driving lanes, each 5 metres wide and an acceleration lane of 55 metres length. The starting speed of the ego car was set to 40 km/h and the desired travelling speed was fixed at 120 km/h (the unit used in the simulations was actually 10 km/h – the reason for this is that dividing velocity by 10 enabled us to keep the remaining parameters simple, resulting in more comprehensive formulas). The velocity of the second car was varied between 70 km/h and 80 km/h. The person specific parameters of the evaluative functions Δ_1 and Δ_2 were estimated within a simplified model which did not contain other cars on the freeway but was identical to the original model in every other respect. We used an iterative estimation procedure to find estimates which resulted in a smooth movement from the acceleration lane onto the freeway. To accomplish the estimation of λ without having an car to collide with we set it equal to σ . This seems reasonable because both parameters represent the same anticipated consequence: the threat for death due to either collisions with other cars or leaving the road. The resulting values were:

- $\omega = 1$ for the force aversion parameter
- $\tau = 1$ for the weighting of reaching desired speed
- $\rho = 5$ for the tendency to drive on the rightmost lane
- $\lambda = \sigma = 1000$ for the avoidance of crashes

The constant k was set to 0.01, representing an arbitrary small number to prevent division by zero. The resulting evaluative functions are

$$\Delta_1 = -f(x)^2 - \tan(g(x)^2)v^2 - \frac{1000}{(x - x_2)^2 + (y - y_2)^2 + 0.01}$$

for state number two and

$$\Delta_2 = -(v - 12)^2 + 1000[\min(0, y - 5) + \min(0, 15 - y)] - 5y$$
$$-\frac{1000}{(x - x_2)^2 + (y - y_2)^2 + 0.01}$$

for state number three, respectively.

To test the plausibility of the specified model we entered the estimated parameter values into the complete model (including the other car on the freeway) and observed the resulting optimal behaviour when another car "gets in the way". For reasons of computational resources we did not calculate the complete state space of the automaton but executed montecarlo approximations to estimate the expected value of the reinforcement value. The

optimization was accomplished by a genetic algorithm.

3.1 Results of the evaluation process

First results of the simulation show the feasibility of our approach. Depending on the traffic on the freeway, our model predicts different driving manoeuvres, which are rather complex in nature. If there are no cars on the freeway, the ego car "drifts" smoothly to the driving lane. If, however, there is another car on the lane, the ego car either enters the freeway in front of the other car or slows down and filters in behind the other car to overtake it after having entered the driving lane (see Fig. 2). The behaviour is chosen depending on the speed of the other car – a car that "gets in the way" of the preferred trajectory changes the optimal behaviour in this situation and thus results in a trajectory that can be described as a best alternative to what would have been done if there had been no other car. The behaviour of the ego car underwent an abrupt change between w = 7.8 and w = 7.9: When the other car If, however, the other car travelled at 79 km/h the ego car stayed slow until it has passed and enters after the other car. If, however, the other car travels just a little bit more slowly (78 km/h), the ego car overtakes and enters the freeway in front of the second car.



Fig. 2: Two simulation results with differing velocities of the other vehicle.

We took the evaluation a step further by varying some of the person parameters which determine the driver's preferred behavior. The aim was to explore how changes of preferences could lead a driver to engage in a risky overtaking manoeuver in a situation where he would otherwise have filtered in after the other car has passed. Therefore we set the other car's velocity to w = 79km/h, resulting in the safer behaviour depicted in the lower panel of figure 2. We then changed the weighting factor of deviation from the desired travelling speed from $\tau = 1$ to $\tau = 10$, representing a situational change in preference (for example the event of taking a look at a watch and noticing that one has to hurry). As one might expect, the ego car's behavior switched to the risky overtaking manoeuver (resulting in a trajectory very similar to that of the upper panel of fig.2). Another parameter we were interested in was whether a higher tendency to drive on the rightmost lane. The question we were interested in was whether a higher tendency to drive on the rightmost lane could result to riskier behavior – although it is mostly considered to prevent car accidents by enhancing traffic flow. We therefore doubled the corresponding parameter ($\rho = 10$). Indeed, this change resulted in the riskier overtaking behaviour, as well.

4 Conclusions and outlook

We presented a model of driving behaviour based on assumptions from Behavioural Psychology. Internal processes are neglected in favour of a parsimonious behavioural approach which takes behaviour to be the result of a subjective optimization process. In order to formalize this idea, a theoretical variable (reinforcement value) is introduced to represent the evaluation and summation of consequences of possible behaviours. We chose the driving task "merging onto the freeway" as an exemplary scenario to apply the model.

At least on a qualitative level, the model generates plausible predictions for driving behaviour in this situation. We would like to stress that although our model predicts qualitatively distinct manoeuvres, we did not model a decision process. Neither did we attempt to model a learning process. What our model does is to find an optimal driving trajectory for a given situation, provided a valid evaluation of anticipated consequences. The rationale behind this approach is that behaviour can be best understood if one starts with theoretical assumptions about how an organism would behave, if there were no restrictions from the environment. Formalizing these theoretical assumptions within a behavioural model allows for the deduction of specific instances of behaviour from the underlying principles. Variation in behaviour is understood as the result of external disturbances, which lead to deviations from the optimal behaviour. In the exemplary scenario given above behaviour is "optimal" with respect to the specific preferences (incorporated in the model as reinforcement values) of a driver. The reinforcement value of acceleration forces, for example, may vary considerably between

drivers, depending on age, experience or gender. Thus, the model allows for differences in the behaviour of different drivers and proposes a quite simple explanation for them. The critical point of our modelling approach is to determine the "correct" reinforcement values for a given class of drivers. Whilst in the present model the corresponding functions are merely plausible assumptions based on a very general behavioural hypothesis ("high forces are aversive"), it would be desirable to derive the exact distributions of the parameters empirically. This would also allow for the exploration of different driving styles (e.g. "sportive" vs. "play-it-safe"). A differential approach to modelling driving behaviour within the current theoretical framework arises naturally from the fact that variation in reinforcement values leads to systematic variation in driving behaviour. Differences in driver behaviour can therefore be incorporated by letting the reinforcement parameters vary between drivers. Although our approach may seem rather technical, paying little attention to what happens "inside" the driver, the principle of reinforcement maximization does say a lot about the agent in the car. Since the reinforcement values in our model reflect (possibly unconscious) driver preferences, they might as well be interpreted as motivational factors. Shifting the focus away from the information processing occuring in a driver, the proposed model presents a way to formalize a functional approach to driving behaviour. Instead of modelling how a person accomplishes driving, the reinforcement maximization approach gives an account for why people drive the way they do. This perspective can give new insights in the driving process and provide a promising ground for the development of advanced driver assistance systems that take into account both external factors and their interaction with behavioural preferences. Knowledge about drivers' preferred behaviour may as well lead to predictions about optimal (that is safe) road construction As our approach does not only allow for the deduction of qualitative hypotheses but leads to specific quantitative hypotheses that can be compared to empirical data, it should be possible to derive a more valid simulation using an adequate experimental setting.

5 References

- Cacciabue, P.C., Carsten, O.: 'A simple model of driver behaviour to sustain design and safety assessment of automated systems in automotive environments', Appl. Ergon., 2010, 41, (2), pp. 187–197
- 2 de Winter, J.C.F., Happee, R.: 'Modelling driver behaviour: a rationale for multivariate statistics', Theor. Issues Ergon. Sci., 2012, 13, (5), pp. 528–545
- 3 Michon, J.A.: 'A critical view of driver behavior models: what do we know, what should we do?', in Evans, L., Schwing, R.C., (Eds.): 'Human behavior and traffic safety' (Plenum, 1985), pp. 485–520

- 4 Salvucci, D.D.: 'Modeling driver behavior in a cognitive architecture', Hum. Factors, 2006, 48, (2), pp. 362–380
- 5 Deml, B., Neumann, H., Müller, A., Wünsche, H.J.: 'Fahrermodellierung im Kontext kognitiver Automobile', Automatisierungstechnik, 2008, 56, pp. 601–608
- Plöchl, M., Edelmann, J.: 'Driver models in automobile dynamics application', Veh.
 Syst. Dyn., 2007, 45, (7–8), pp. 699–741
- 7 Anderson, J.R.: 'Act: a simple theory of complex cognition', Am. Psychol., 1996, 51,
 (4), pp. 355–365
- Anderson, J.R.: 'ACT: a simple theory of complex cognition', in Polk, T.A., Seifert,
 C.M., (Eds.): 'Cognitive modelling'(Bradford, 2002), pp. 49–66
- 9 Möbus, C., Eilers, M., Garbe, H., Zilinski, M.: 'Probabilistic, and empirical grounded modeling of agents in partial cooperative (traffic) scenarios'. Conf. Proc. Second Int. Conf. Digital Human Modeling, San Diego, 2009, pp. 423–432
- Salvucci, D.D., Boer, E.R., Liu, A.: 'Toward an integrated model of driver behavior in a cognitive architecture', Transport. Res. Rec.: J. Transp. Res. Board, 2001, 1779, pp. 9–16
- Hochstädter, A., Zahn, P., Breuer, K.: 'Ein universelles Fahrermodell mit den Einsatzbeispielen Verkehrssimulation und Fahrsimulation'. 9. Aachener Kolloquium Fahrzeug- und Motorentechnik 2000 (Aachen, 2000), pp. 1343–1360
- Krajzewicz, D.: 'ACME Driver ein kognitives Fahrermodell'. 5. Fachübergreifende
 Konferenz Mensch und Computer (Wien, 2005), pp. 105–110
- 13 Toledo, T., Koutsopoulos, H.N., Ben-Akiva, M.: 'Integrated driving behavior modeling', Transp. Res. C, Emerg. Technol., 2007, 15, (2), pp. 96–112
- Weber, L., Baumann, M., Lüdkte, A., Steenken, R.: 'Modellierung von Entscheidungen beim Einfädeln auf die Autobahn.'. 8. Berliner Werkstatt Mensch-Maschine-Systeme: Der Mensch im Mittelpunkt technischer Systeme (Berlin, 2009), pp. 27–28
- 15 Smith, E.E., Nolen-Hoeksema, S., Fredrickson, B.L., Loftus, G.R.: 'Atkinsons und Hilgards Einführung in die Psychologie' (Spektrum, 2007)
- 16 Skinner, B.F.: 'The behaviour of organisms' (Appleton-Century, 1938)
- 17 Anderson, J.R.: 'Kognitive Psychologie' (Springer, 2007)
- 18 Zimbardo, P.G., Gerrig, R.J.: 'Psychologie' (Pearson Studium, 2008)
- 19 Baum, W.M.: 'Understanding Behaviorism' (Blackwell, 2005)

- 20 Fryerand, M.J., Greenman, J.V.: 'Optimisation theory. Applications in or and Economics' (Arnold, 1987)
- 21 Kappeler, P.: 'Verhaltensbiologie' (Springer, 2009)
- Parker, G.A., Smith, J.M.: 'Optimality theory in evolutionary biology', Nature, 1990, 348, pp. 27–33
- 23 Schwarze, A., Borgstede, M., Eggert, F., Schicke-Uffmann, J., Goltz, U.: 'Modelling driving behaviour using hybrid automata'. European Conf. Human Centred Design for Intelligent Transport Systems, Valencia, 2012
- von Neumann, J., Morgenstern, O.: 'Theory of games and economic behavior' (Princeton University Press, 1947)
- 25 Borgstede, M., Schicke, J., Eggert, F., Goltz, U.: 'Hybrid automata as a modelling approach in the behavioural science'. HAS Workshop, Saarbrücken, 2011
- 26 Schwarze, A., Leske, K., Eggert, F.: 'Acceptance of a new driver assistance system for merging into motorway traffic'. Proc. Fifth Int. Conf. Traffic and Transport Psychology, Groningen, The Netherlands, 2012
- Ranney, T.A.: 'Models of driving behavior: a review of their evolution', Accident Anal.Prevent., 1994, 26, (6), pp. 733–750