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Analogical Representation of RCC-8 for Neighborhood-Based Qualitative Spatial Reasoning

Diedrich Wolter[1] and Arne Kreutzmann[2]

[1] Smart Environments, University of Bamberg diedrich.wolter@uni-bamberg.de
[2] University of Bremen arne@sfbtr8.uni-bremen.de

Abstract. Qualitative representations of spatial knowledge aim to capture the essential properties and relations of the underlying spatial domain. In addition, conceptual neighborhood has been introduced to describe how qualitative spatial relations may change over time. Current qualitative representations mainly use symbolic constraint-based languages that are detached from the underlying domain with the downside that a well-formed sentence is not necessarily consistent. This makes it difficult to design efficient knowledge manipulation techniques that consistently advance a representation with respect to conceptual neighborhood. In this paper we argue for *analogical spatial representations* that inherently obey domain restrictions and, as a result, are consistent per se. We develop a graph-based analogical representation for RCC-8, the construction of which is based on neighborhood transitions realized by efficient graph transformations. The main benefit of the developed representation is an improved efficiency for neighborhood-based reasoning tasks that need to manipulate spatial knowledge under the side condition of consistency, such as planning or constraint relaxation.

1 Introduction

Qualitative Spatial and Temporal Representation and Reasoning (QSTR) [2] aims at capturing human-level concepts of space and time using finite sets of relations over a particular spatial or temporal domain. Existing qualitative representation approaches define symbolic constraint-based languages to encode spatio-temporal knowledge using the relations from a particular so-called qualitative calculus as constraints. An important reasoning problem is that of deciding consistency, i.e., deciding whether a set of constraints can be realized in the given domain.

Aside from reasoning about consistency, there exists another class of reasoning tasks, which is concerned with the evolution of qualitative spatial configurations over time, e.g., qualitative planning or simulation tasks as well as retrieval or relaxation problems based on a notion of similarity of spatial configurations. Given qualitative descriptions of start and end configurations S and E, a question could for example be "What is the simplest way to get from S to E?" which calls for an as-short-as-possible sequence of configurations such that consecutive configurations are connected only by elementary changes of the spatial relations. See Fig. 1 for an example using the well-known RCC-8 calculus [14] for topological relations. Answering this question can provide helpful information for planning manipulation tasks in robotic applications [17].

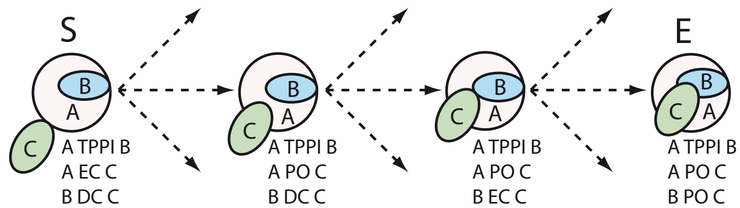


Fig. 1. A simple qualitative planning problem: finding a sequence of qualitative scenarios (here illustrated by concrete example depictions) that connects S and E.

To describe spatial change on a qualitative level, the concept of *conceptual neighborhood* between spatial relations has been introduced [6] and later been extended from individual relations to complex spatial configurations [13]. Difficulties of handling neighborhoods in complex configurations arise from the fact that several relations may constrain one another. For example, consider situation E in Fig. 1: there is no way to detach region C from A by continuous movement without either affecting the relation holding between C and B or B and A. Neighborhood-based reasoning tasks require the modification of a qualitative spatial representation under the side condition of consistency, i.e., to respect such interdependent relation changeovers. In context of the aforementioned planning task, maintaining consistency ensures that the individual steps are valid sub-goals for motion planning. Further neighborhood-based reasoning tasks are discussed in [5,9].

Algorithmically, existing approaches to neighborhood-based reasoning either ignore interdependent relation changeovers [5] (which is acceptable in context of qualitative similarity assessment but yields an upper approximation) or employ a generate-and-filter approach [3]. The latter employs tree search to identify a sequence of changes that transforms one representation into another, using consistency checking to filter out nodes that represent inconsistent representations. As a result the search space grows exponentially with respect to the number of relations that need to be changed. Already identifying a conceptually neighbored configuration gives rise to this problem if multiple relations need to be changed at once. It may indeed be necessary to alter several relations between one object and all other objects at once, for example in configurations in which the spatial extent of all n objects are equal, $n-1$ relations change in the next neighborhood transition. As a consequence, $O(n)$ levels of the search tree involving $O(2^n)$ nodes would have to be explored. This triggers the following research question: Is there a more efficient way of determining conceptual neighborhood among spatial configurations?

The contribution of this paper is to show that identifying and performing neighborhood transitions is possible in polynomial time. Our approach is based on the idea of employing a data structure that is *analogical* in the sense of [12], i.e., a representation that retains important domain structures. Our graph-based representation of RCC-8 scenarios retains topological structure to the level of detail captured by RCC-8 relations, not allowing consistency among RCC-8 relations to be violated. Neighborhood transitions, including construction of the representation, are then realized as polynomial time graph

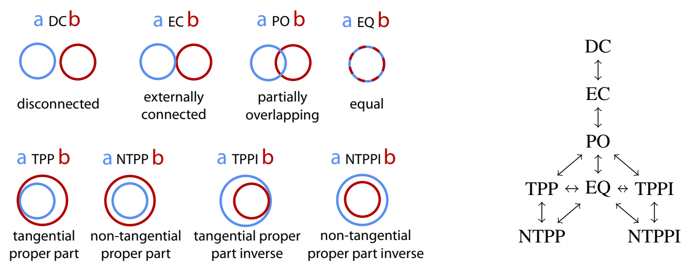


Fig. 2. Left: The eight base relations of the RCC-8 calculus. Right: The RCC-8 conceptual neighborhood graph if size persistency is not enforced.

transformations. We show that our representation provides a model for the RCC-8 theory and give algorithms that operationalize the formal approach to RCC-8 presented in [10,16] by integrating it with the concept of conceptual neighborhoods.

The paper is structured as follows. Sect. 2 contains background information on qualitative spatial representations. In Sect. 3, we present our analogical representation for RCC-8 as well as neighborhood transition and construction procedures. In Sect. 4, we discuss the algorithmic realization and analyze the computational properties of our approach.

2 Qualitative Representation of Space

Qualitative representations define a set $\mathcal{R}$ of spatial relations over a domain of objects D. For every pair of objects from the domain, exactly one relation $R \in \mathcal{R}$ holds, i.e., the set of relations (also called *base relations* or *atomic relations*) is jointly exhaustive and pairwise disjoint (JEPD). This approach generalizes to higher arity relations, but this paper is only concerned with the set of binary relations defined in the RCC-8 calculus, which are shown in Fig. 2.

A qualitative representation is a set of constraints expressed in a quantifier-free constraint language based on a set of relations. Technically speaking, we have a *constraint network* $N = (X, D, C)$ with variables $X = \{X_1, X_2, \ldots, X_n\}$ over the domain D whose valuations are constrained by binary relations given in the constraint matrix C.

Using the set-based semantics of relations and classical set-operations $\{\cup, \cap, \cdot^C\}$, one obtains a Boolean set algebra over the set of so-called *general relations*, the set of all possible unions of base relations. By employing unions of relations as constraint relations, one can express uncertainty. The constraints in a qualitative constraint network are written in the form $\{X_1 \;\; c_{12} \;\; X_2, X_1 \;\; c_{13} \; X_3, \ldots\}$ with $c_{11}, c_{12}, \ldots, c_{nn} \in 2^{\mathcal{R}}$ representing unions of base relations. Constraint networks defined by qualitative relations are assumed to be complete in the sense that there exists a constraint between every pair of variables. This is no limitation since the union of all base relations can serve as a non-restrictive constraint. A constraint network in which all constraints are atomic relations is called a *scenario*.

A qualitative constraint network is *consistent* if there exists a valuation of variables with objects from the domain that satisfies all constraints. A prominent approach in qualitative spatial reasoning is based on a symbolic method that builds on relation-algebraic operations defined on the set of general relations. For this approach, operations for *composition* and *converse* of relations are required. The structure comprising base relations, converse, and composition is known as a *qualitative calculus* [11,4]. Using these operations the so-called algebraic closure algorithm enforces a local consistency called algebraic closure or path-consistency in $O(n^3)$ time, which already decides consistency of RCC-8 scenarios [1]. The class of RCC-8 constraint networks that can be handled with this method has later been extended, but deciding consistency of arbitrary RCC-8 constraint networks remains NP-complete [15].

Neighborhood-based reasoning tasks are based on the notion of *conceptual neighborhood* by Freksa [6]: A base relation is said to be a *conceptual neighbor* of a second base relation if there exists a continuous transformation that brings two objects from the second relation to the first with no other relation holding in between. Galton [7] defines conceptual neighborhood similar, but allows the relation to be reflexive while Freksa considers it as being irreflexive – here, this difference does not matter though. Conceptual neighborhoods have been used to describe how qualitative relations evolve over time when the objects in the domain are subject to continuous transformations such as movement or deformation. Depending on which kind of transformations are considered, the neighborhood relation may be symmetric if transformations are reversible. We write $R \rightsquigarrow R'$ to denote that R' is a conceptual neighbor of R and we use $\leftrightsquigarrow$ to denote symmetric neighborhood relations. The conceptual neighborhood relation is commonly visualized in a so-called conceptual neighborhood graph as shown in Fig. 2 for RCC-8, assuming that regions can move, deform, grow, or shrink [8].

As most neighborhood-based reasoning tasks such as planning involve more than just two objects, one needs to generalize the notion of conceptual neighborhood from a relation between two objects to an entire scenario, i.e., a matrix of atomic constraints. This can be done in a straightforward way, saying that two scenarios are neighbored if a continuous transformation changes one scenario into another with no other scenario holding in between. Changing one relation in a qualitative constraint network may lead to an inconsistent network or, put differently, one change can entail other, simultaneous changes. To this end, [13] have introduced generalized (n, l)-neighborhoods to aggregate individual neighborhood transitions, where n determines the total number of variables considered and l is the number of objects that can be transformed simultaneously. They however assume this structure to be computed beforehand. In our work we investigate generalized $(n, 1)$-neighborhoods for arbitrarily many variables n and we aim to compute all direct neighborhood transitions together with their implications. Depending on the context in which our representation is used, different approaches to measure the degree of change may be useful, e.g., whether to count implications as separate changes or not. Our approach provides the basis to define such measures, but this is not further addressed in this paper.

3 Analogical Representation for RCC-8

RCC-8 describes the connectivity among regions, differentiating between connectivity of interiors (e.g., relation PO) and connectivity of closures of a region (e.g., relation EC). Thus, our analogical representation identifies the parts required to describe a given scenario and links them by containment information. Our representation constitutes the decisive finite fragment of a model for the RCC-8 calculus (a strict model in the sense of [16]). Roughly speaking, the analogical representation is the directed graph of region nestings from now on called the *inclusion graph* (see Fig. 3 for an example) and later defined formally. We employ the approach to RCC-8 based on Boolean connection algebras introduced in [16] to show that our representation is a model for RCC-8. Our inclusion graph can be interpreted as a partially ordered set which, equipped with a notion of connectivity, constitutes a Boolean connection algebra as noted in [10].

Definition 1 *A* Boolean connection algebra *is a Boolean algebra* $< A, \bot, \top, ^{\smile}, \vee, \wedge >$ *equipped with a connection relation* C *that satisfies four axioms:*

1. *C is symmetric and reflexive on* $A \backslash \{\bot\}$
2. $\forall x \in A \ : \ x \neq \top \rightarrow C(x, x^{\smile})$
3. $\forall x, y, z \in A \backslash \{\bot\} \ : \ C(x, y \vee z)$ *iff* $C(x, y)$ *or* $C(x, z)$
4. $\forall x \in A \backslash \{\top\} \ : \ \exists y \in A \backslash \{\bot\} \ : \ \neg C(x, y)$

Our definition of inclusion graphs is similar to that of *maptrees* which have been proposed as topological representations by considering embeddings of connected graphs in closed surfaces [18,19]. We proceed differently since maptrees aim at a more fine-grained topological representation than captured by RCC-8. Using maptrees would complicate defining transition on the (coarser) level of RCC-8 relations and possibly affect efficiency of computing transitions.

Let us assume we are given an RCC-8 scenario with variables $X = \{X_1, X_2, \ldots, X_n\}$. Our task is to define the inclusion graph as a directed graph $G = (V, E)$ whose vertices stand for distinct open and closed regions of the topological space in the given scenario. The set of vertices includes three special elements, the universe vertex $\{\mathcal{U}\}$ (the universe being a special region containing all other regions), the void $\varnothing$ not containing any region, and an outside o contained in $\mathcal{U}$ and not containing any other regions. All other elements of V are sets $\{X_{i_1}, X_{i_2}, \ldots, X_{i_k}, \mathcal{U}\}$ or $\overline{\{X_{i_1}, X_{i_2}, \ldots, X_{i_k}, \mathcal{U}\}}$ representing parts of the topological space. The idea is to interpret elements of V by means of set intersection and to use $\overline{\phi}$ to denote closure of an open region ϕ. Thus, the intersection of regions A and B would be represented by a vertex $\{A, B, \mathcal{U}\}$ (see Fig. 3), while $\overline{\{A, B, \mathcal{U}\}}$ represents the closure $\text{CL}(A \cap B)$. To ease the notation, we say that $\overline{A} \in \overline{\{A, B, \mathcal{U}\}}$. The edges of the inclusion graph represent proper containment relations, i.e., $(v, v') \in E$ implies a subset relation between the parts represented by v and v'.

In order to obtain RCC-8 semantics for our graph we define regions (sets of vertices) $r : X \rightarrow 2^V$, their closures CL, and connectivity **C**, as this allows us to apply the standard specification of RCC-8 [1]. The definition, adapted to our notation, is summarized in Tab. 1. The open region represented by variable X is written as $r(X)$, its closure $\text{CL}(X)$ respectively. Given a set of variables $X = \{X_1, X_2, \ldots, X_n\}$ and an inclusion

	A	B	C
A	EQ	PO	NTPPI
B	PO	EQ	NTPPI
C	NTPP	NTPP	EQ

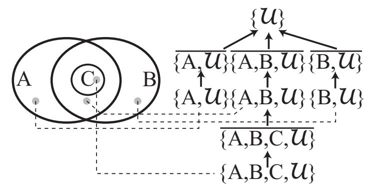


Fig. 3. RCC-8 scenario and a possible visualization with its corresponding inclusion graph.

Table 1. Formal specification of RCC-8 relations, omitting model constraints $\mathbf{C}(r(A), r(A))$ and $\mathbf{C}(r(B), r(B))$ which ensure that regions $r(A)$ and $r(B)$ are non-empty.

Relation	Clauses
A DC B	$\neg\mathbf{C}(\text{CL}(A), \text{CL}(B))$
A EC B	$\mathbf{C}(\text{CL}(A), \text{CL}(B)), \neg\mathbf{C}(r(A), r(B))$
A PO B	$\mathbf{C}(r(A), r(B)), \mathbf{C}(r(A), \text{CL}(B)^C), \mathbf{C}(\text{CL}(A)^C, r(B))$
A EQ B	$\mathbf{C}(r(A), r(B)),\quad \mathbf{C}(\text{CL}(A), \text{CL}(B)),\quad \neg\mathbf{C}(r(A), \text{CL}(B)^C),$ $\neg\mathbf{C}(\text{CL}(A)^C, r(B))$
A TPP B	$\mathbf{C}(r(A), r(B)),\quad \mathbf{C}(\text{CL}(A)^C, r(B)),\quad \neg\mathbf{C}(\text{CL}(A), \text{CL}(B)^C),$ $C(\text{CL}(A), r(B)^C)$
A TPPI B	clauses for TPP(B,A)
A NTPP B	$\mathbf{C}(r(A), r(B)), \neg\mathbf{C}(\text{CL}(A), r(B)^C),$
A NTPPI B	clauses for NTPP(B,A)

graph $G = (V, E)$, we define the RCC-8 interpretation of G as follows:

$$r(X_i) := \{v \in V | X_i \in v\} \tag{1}$$

$$\text{CL}(X_i) := r(X_i) \cup \{v \in V | \overline{X_i} \in v\} \tag{2}$$

$$\widetilde{V'} := V' \cup \{v \in V | \overline{v} \in V'\} \cup \{\overline{v} \in V | v \in V'\} \tag{3}$$

$$\mathbf{C}(V', V'') \leftrightarrow \widetilde{V'} \cap \widetilde{V''} \neq \emptyset \tag{4}$$

The definition of **C** uses an auxiliary function ~to address connectedness of open and closed regions: closed regions are connected to any open region they are contained in, which is achieved by growing the closed region prior to testing for overlap. Note that we distinguish vertices $v \in V$ representing open regions from $\overline{v} \in V$ representing closures. Now we are ready to give the definition of an inclusion graph:

Definition 2 (inclusion graph) *Let $X = \{X_1, X_2, \ldots, X_n\}$ be a set of variables referring to spatial regions. Then we call directed graph $G = (V, E)$ an inclusion graph of X if V is a two-sorted set with elements of type v and $\overline{v}$, $V \subseteq 2^{X \cup \{o, \mathcal{U}, \emptyset\}} \cup 2^{\overline{X \cup \{o, \mathcal{U}, \emptyset\}}}$*

and the following properties are satisfied:

$$\forall v \in V : (\{\mathcal{U}\}, v) \notin E \wedge (\overline{v}, v) \notin E \tag{5}$$

$$\forall v \in V : v \neq \{\mathcal{U}\} \rightarrow (\overline{v} \in V \wedge (v, \overline{v}) \in E) \tag{6}$$

$$\forall v \in V : \forall (v', v) \in E \ : \ X_i \in v \rightarrow X_i \in v' \tag{7}$$

$$\forall X_i : \forall v \in \mathrm{CL}(X_i), v' \in r(X_i) : (v, v') \in E \tag{8}$$

$$\forall \overline{v} \in V : \exists v' \in V \ : \ (\overline{v}, v') \in E \tag{9}$$

$$\forall v \in V : (v, \{o\}) \notin E \tag{10}$$

We now show that the properties required for inclusion graphs to hold reflect the semantics of RCC-8 relations according to Equations 1–4 and Tab. 1. As shorthand notation we write $\mathrm{rcc}_{V,E}(X_i, X_j)$ to denote the RCC-8 relation between variables X_i and X_j indicated by inclusion graph (V, E).

Theorem 1 *The inclusion graph $G = (V, E)$ of a set of variables $\{X_1, \ldots, X_n\}$ is a model of RCC-8.*

Proof. According to [16, Theorems 4 and 5] it suffices to show that our inclusion graph constitutes a Boolean connection algebra (see Def. 1). Since our graph vertices V are sets, $< 2^V, \emptyset, V, ^C, \cup, \cap >$ is a Boolean (set) algebra with more than two elements (we have at least 3 special vertices), i.e., we have $\top = V$, $\bot = \emptyset$, $^\smile = ^C$ (set complement in V), $\vee = \cup$, and $\wedge = \cap$. We now show that our definition of **C** satisfies the axioms. Symmetry of **C** is obvious and **C** is also reflexive since $V' \cap V' \neq \emptyset$ for any $V' \neq \emptyset$. With respect to Axiom 2, observe that in $\mathbf{C}(V', V'^C)$ from $V' \neq \top$ and connectivity of our graph V' contains a vertex but not its successor and predecessor. By Eq. 3, $\widetilde{V'}$ and $\widetilde{V'^C}$ share the common successor or predecessor and, hence, are connected. Axiom 3 directly follows from $\vee = \cup$ and Eq. 4. Axiom 4 states that there is a non-empty region which is not connected to any region $V' \subset V$. Intuitively, the complement of any open set contains another open region. With respect to all sets $r(X_i)$ the outlier vertices $\overline{\{o\}}, \{o\} \in V$ already satisfy the condition due to property Eq. 10.

The inclusion graph involves outlier vertices solely to apply existing theorems in the proof of Theorem 1. In a practical implementation, these vertices are not necessary.

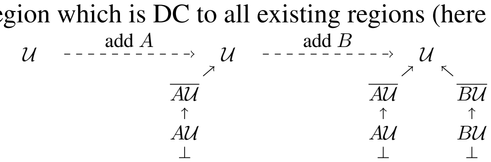
3.1 Conceptual Neighborhood Transitions

We now define conceptual neighborhood transformations that consistently modify inclusion graphs. In addition to the neighborhood transitions shown in Fig. 2, we include an additional transition that allows us to grow a new region. In the following, we will use (V, E) to denote the inclusion graph and $\{X_1, \ldots, X_n\}$ is the set of region variables from which the graph has been constructed. To ease readability we use ϕA as a shorthand notation for an arbitrary vertex $\{A\} \cup \phi$ with $A \notin \phi$. To save space, we omit transformations that are purely symmetrical, e.g., A PO $B \leftrightsquigarrow A$ TPPI B which can be easily obtained from B PO $A \leftrightsquigarrow B$ TPP A.

Definition 3 *A graph transformation is called* admissible *if it preserves Properties 5–10 of inclusion graphs.*

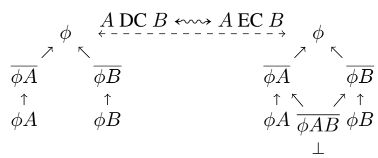
Admissibility of transformation is important as these transformations consistently modify the RCC-8 interpretation since no model properties are violated. In the pictorial representation we add a special edge $\perp$ to indicate if no edge is allowed to end at a node.

Birth Add a new region which is DC to all existing regions (here shown twice).



Lemma 1. *Birth is admissible.*
Proof. It is straightforward to check that the newly introduced vertices agree with the invariances, in particular that Properties 7 and 9 are satisfied.

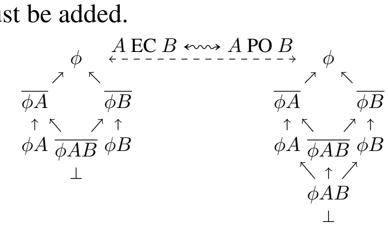
DC $\leftrightsquigarrow$ *EC* If two closures share a common direct ancestor and are currently not connected, then they can be externally connected by inserting a new part which stands for the common closure of the two regions. If two closures share a common *direct* ancestor, their closure is connected exactly once, and their inner regions are not connected, then the connecting closure can be removed.



Lemma 2. *Transition DC* $\leftrightsquigarrow$ *EC is admissible.*

Proof. Property 9 gives us that the common direct ancestor ϕ is an open region and, with respect to EC $\rightsquigarrow$ DC, the preconditions that A and B are not connected other than by $\overline{\phi AB}$ ensures that there can be no ϕAB which we could disconnect violating Property 9. Direction DC $\rightsquigarrow$ EC only affects Property 7, respecting the inclusion relation.

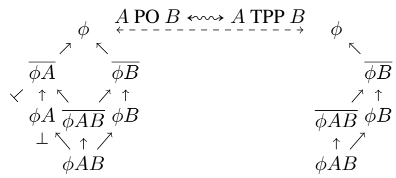
EC $\leftrightsquigarrow$ *PO* Any closure $\overline{AB}$ without inner region can be extended by an inner region AB. The added AB is part of any interior of $r(A)$ and $r(B)$; thus, edges $(\phi AB, v)$ for all $v \in r(A) \cup r(B)$ must be added.



Lemma 3. *The transition EC* $\leftrightsquigarrow$ *PO is admissible.*

Proof. Only Properties 6 and 9 are affected by this transition but it is straightforward to see that they are not violated.

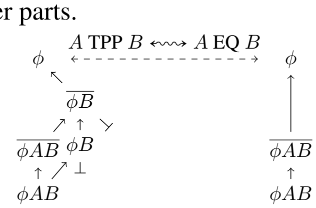
PO $\leftrightsquigarrow$ *TPP* For A PO $B \rightsquigarrow A$ TPP B the parts of A overlapping $\textsc{Cl}(B)^C$ need to be removed; thus, as a precondition for PO $\rightsquigarrow$ TPP, there must not be another vertex v with $(v, \overline{\phi A}) \in E$ or $(v, \phi A) \in E$. The other direction is admissible since we grow a region within another open region.



Lemma 4. *The transition PO $\leftrightsquigarrow$ TPP is admissible.*

Proof. While TPP $\rightsquigarrow$ PO is admissible as the inserted subgraph suits the properties, direction PO $\rightsquigarrow$ TPP is secured by the precondition of having no region within A or $\overline{A}$.

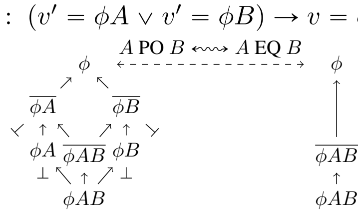
TPP $\leftrightsquigarrow$ *EQ* For A TPP $B \rightsquigarrow A$ EQ B, we remove the part only belonging to B which must not contain other parts.



Lemma 5. *The transition TPP $\leftrightsquigarrow$ EQ is admissible.*

Proof. EQ $\rightsquigarrow$ TPP is admissible since we simply grow a new region within an open one. EQ $\leftsquigarrow$ TPP is admissible since the precondition ensures that $\nexists(v, B) \in E$ with $v \neq AB$.

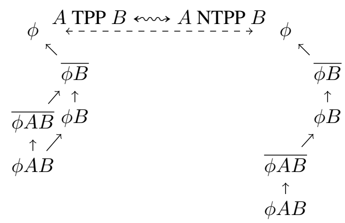
PO $\leftrightsquigarrow$ *EQ* As a precondition for PO $\rightsquigarrow$ EQ, there must be no part contained in either A or B, i.e., $\forall (v, v') \in E \;:\; (v' = \phi A \vee v' = \phi B) \rightarrow v = \phi AB$.



Lemma 6. *PO $\leftrightsquigarrow$ EQ is admissible.*

Proof. While EQ $\rightsquigarrow$ PO is admissible because we can grow a region within an open region. The other direction can, by precondition, only be performed such that one cannot violate Property 6 or Property 9.

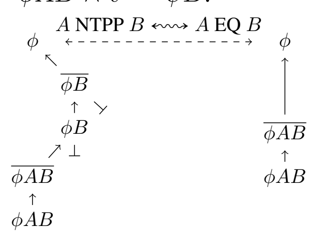
TPP $\leftrightsquigarrow$ *NTPP* As a precondition for TPP $\rightsquigarrow$ NTPP, there must not be another $\overline{\psi A} \in V$ with $\psi \neq \phi$.



Lemma 7. *TPP* $\leftrightsquigarrow$ *NTPP is admissible.*

Proof. Only the direction TPP $\rightsquigarrow$ NTPP bears danger of violating a property as it detaches the closure of A from the closure of B. The precondition ensures that $\overline{AB}$ is not connected to any other region and, hence, not breaking Property 6.

NTPP $\leftrightsquigarrow$ *EQ* As precondition for NTPP $\rightsquigarrow$ EQ, there must not be any other part of B, i.e., $\nexists(v, v') \in E$ with $v \neq \phi AB \wedge v' = \psi B$.



Lemma 8. *The transition NTPP* $\leftrightsquigarrow$ *EQ is admissible.*

Proof. Follows analogously to TPP $\leftrightsquigarrow$ EQ.

These graph transformations are neighborhood transitions that consistently modify the RCC-8 interpretation as shown in the lemmata. The transitions have the desired effect of altering one relation according to the conceptual neighborhood of RCC-8 by construction according to Properties 1–4. Implicit changes of relations involving either object A or B may occur, but the transformations do not affect connectivity between any two regions other than A and B specified in the rules.

3.2 Constructing the Representation

A special feature of our approach is that conceptual neighborhood transitions are employed to construct the representation. We use the transformation *birth* to iteratively create a new region disconnected to all other regions and then move it to its goal position. By doing so, correctness of the algorithm follows from admissibility of the transitions. The complete construction is shown in Algorithm 1.

Theorem 2 *Algorithm 1 computes an inclusion graph G from a consistent RCC-8 scenario S such that the RCC-8 interpretation of the graph is S.*

Algorithm 1 Constructing the inclusion graph

function CONSTRUCT$(\{X_1, X_2, \ldots, X_n\}, C = \{c_{ij}\})$
  $V \leftarrow \{\{U\}, \emptyset, \overline{\{o\}}, \{o\}\}$
  $E \leftarrow \{(\{o\}, \overline{\{o\}}), (\{o\}, \{\mathcal{U}\})\}$
  for $i = 1, 2, \ldots, n$ **do**
    $V \leftarrow V \cup \{\{X_i\}, \overline{\{X_i\}}\}$ ▷ birth of X_i
    $E \leftarrow E \cup \{(\{X_i\}, \{\mathcal{U}\}), (\{X_i\}, \overline{\{X_i\}})\}$
    while $\exists i > j \ : \ \mathrm{rcc}_{V,E}(X_i, X_j) \neq c_{ij}$ **do**
      perform sequence of transitions that moves X_i to its goal relation c_{ij} with X_j
    end while
  end for
  return (V, E)
end function

Proof. (sketch) Correctness of the algorithm directly follows from the fact that only admissible neighborhood transitions are performed. The condition of the "while" loop implies that the algorithm can only terminate with a graph G whose RCC-8 interpretation is identical to C. It remains to be shown that the algorithm terminates. Before a region has reached its goal position, there exists at least one other region to which the region must be connected and which is not contained in any other region. Thus, one can connect these regions and move the current one, if needed, further inside. In other words, we only need to move regions further inside which can only be happening finitely many times.

4 Algorithmic Realization

In this section, we discuss the algorithmic realization of our approach and analyze the computational costs of constructing the representations and performing neighborhood transitions. To facilitate an efficient implementation, we supplement the inclusion graph with a vector indexed by the variables X_i involved in a given RCC-8 scenario. The vector grants access to all vertices in $\mathrm{CL}(X_i)$.

Theorem 3 *For a given RCC-8 scenario with n variables, the corresponding inclusion graph comprises $O(n^2)$ vertices and can be constructed in $O(n^4)$ time.*

Proof. No transformation rule introduces more than two new vertices and every rule needs to be applied at most once for every pair of variables (cp. proof of Theorem 2). Since we have $O(n^2)$ relations to satisfy, no more than $O(n^2)$ vertices can be generated. With respect to time complexity, n regions are processed. In each step of the main loop, $O(n)$ relations need to be satisfied and it takes a constant set of relation transformations to satisfy one relation (the longest path in the conceptual neighborhood graph is 4 steps). Checking applicability of a transformation rule might require to consider all $O(n^2)$ vertices, which results in a total time complexity of $O(n \cdot n \cdot n^2) = O(n^4)$.

Theorem 4 *Given an inclusion graph that involves n variables, all possible neighborhood transitions can be enumerated in $O(n^4)$ time.*

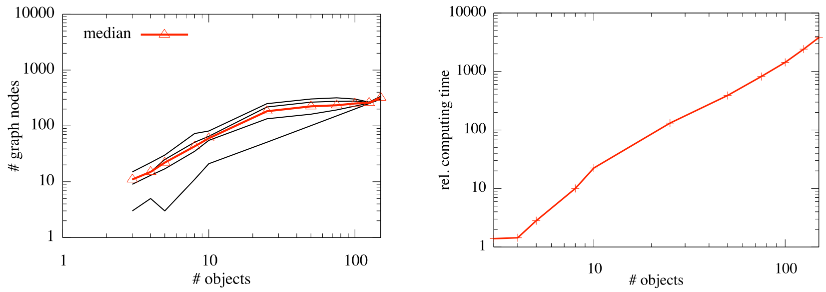


Fig. 4. Left: Size of inclusions graphs in vertices per object. Right: Average relative computation time for performing neighborhood transitions.

Proof. For checking the applicability of a neighborhood transformation, we need to consider at most all $O(n^2)$ vertices for every pair of variables which yields $O(n^4)$ for checking all possible transitions.

4.1 Experimental Evaluation

We implemented the proposed method to study the average computational cost. For our evaluation we generated random consistent RCC-8 scenarios from 3 to 150 variables by performing random neighborhood transitions. We recorded the number of vertices of the inclusion graph, the distribution of which is presented in Fig. 4 left, separated into quartiles. Also, we recorded the average computing time for performing a neighborhood transition. Fig. 4 right shows the development relative to the time required for networks with $n = 3$ variables. While both results accord with the theoretical results, we observe that the distribution of numbers of vertices is concentrated around the median and the coefficient of quadratic growth (slope of the 'line' in log/log-scale) is small.

5 Conclusions

We proposed analogical spatial representations as a novel approach to neighborhood-based reasoning with qualitative spatial representations. The advantage of our approach is that a consistent state of the representation is maintained at any time. While the general idea is applicable to a variety of spatial and temporal representations, this paper is concerned with an analogical representation for the RCC-8 calculus. RCC-8 is of particular interest as it is widely used and since an analogical representation for arbitrary topological spaces is challenging. Our representation is based on the characterization of RCC-8 using on Boolean connection algebras [16]. We operationalize the theoretic foundations in terms of algorithms to construct and to modify the representation. The result is a set of operators that allows us to consistently generate conceptual neighborhoods for a complete scenario in polynomial time, including entailed simultaneous transitions. Our analysis demonstrates that this novel approach is computationally efficient. Thus, analogical spatial representations provide an excellent basis for performing neighborhood-based spatial reasoning.

While this paper focuses on RCC-8, other qualitative calculi could similarly benefit from an analogical representation to speed up neighborhood-based reasoning tasks – investigating such representations is subject to future work.

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