Towards a Refinement Type System for Hybrid Synchronous Program Verification

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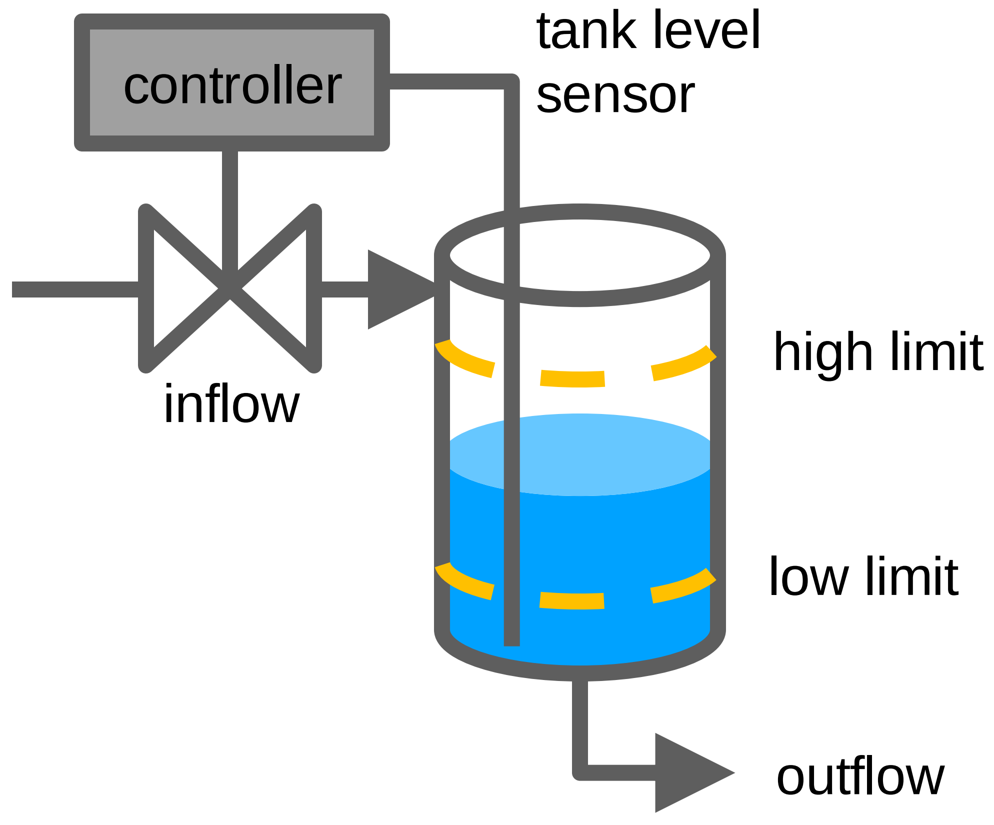
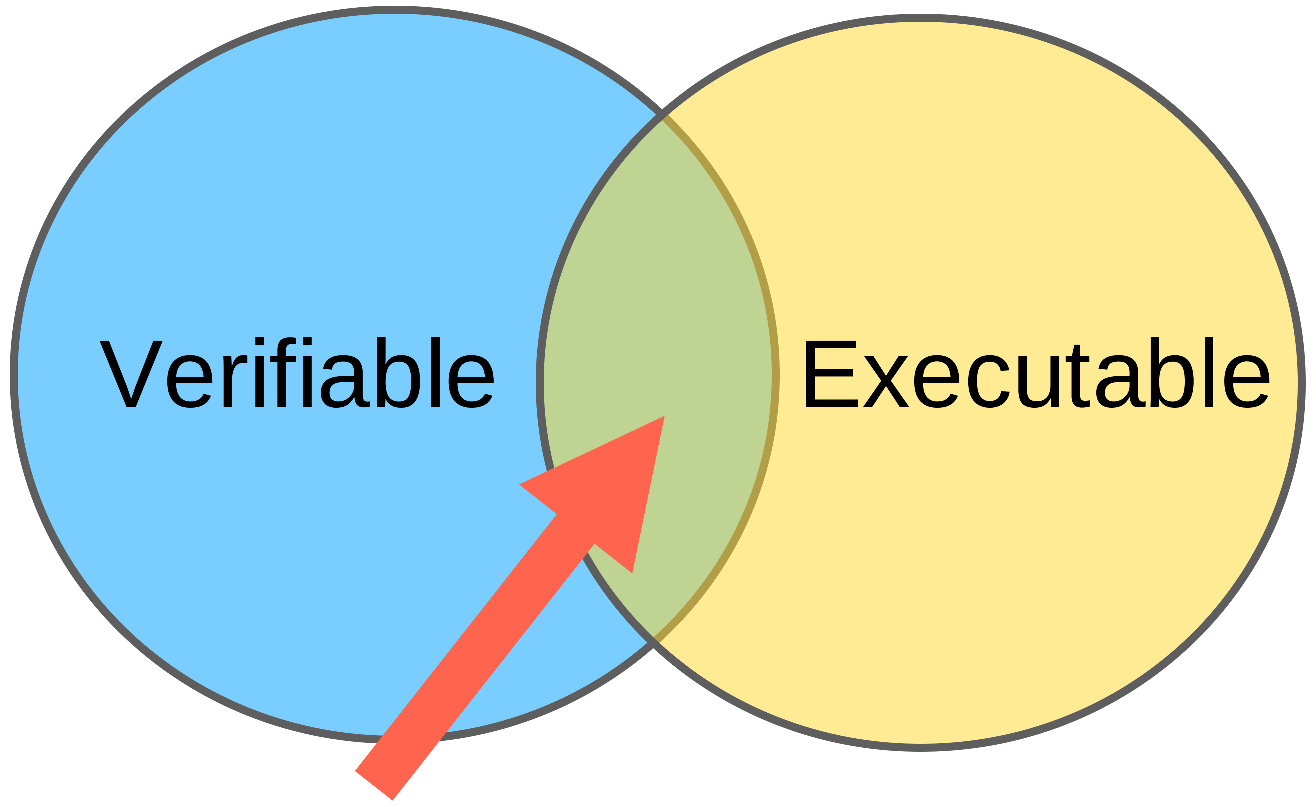
Context

- OCaml-based robotics platform with verification and real-time execution
- Preliminary work published at 2022 FTSCS workshop
- Full paper at ICFP 2024
  - Available, Reusable artifact

Background

- Cyber-Physical Systems (CPS): Software that **interacts with the physical environment**
  - Stringent safety requirements
- **CPS Verification**: Implementation?
- **CPS Implementation**: Verification?
- CPS designers **shouldn't need to choose** between **verification** and **implementation**
- Language with **both**?

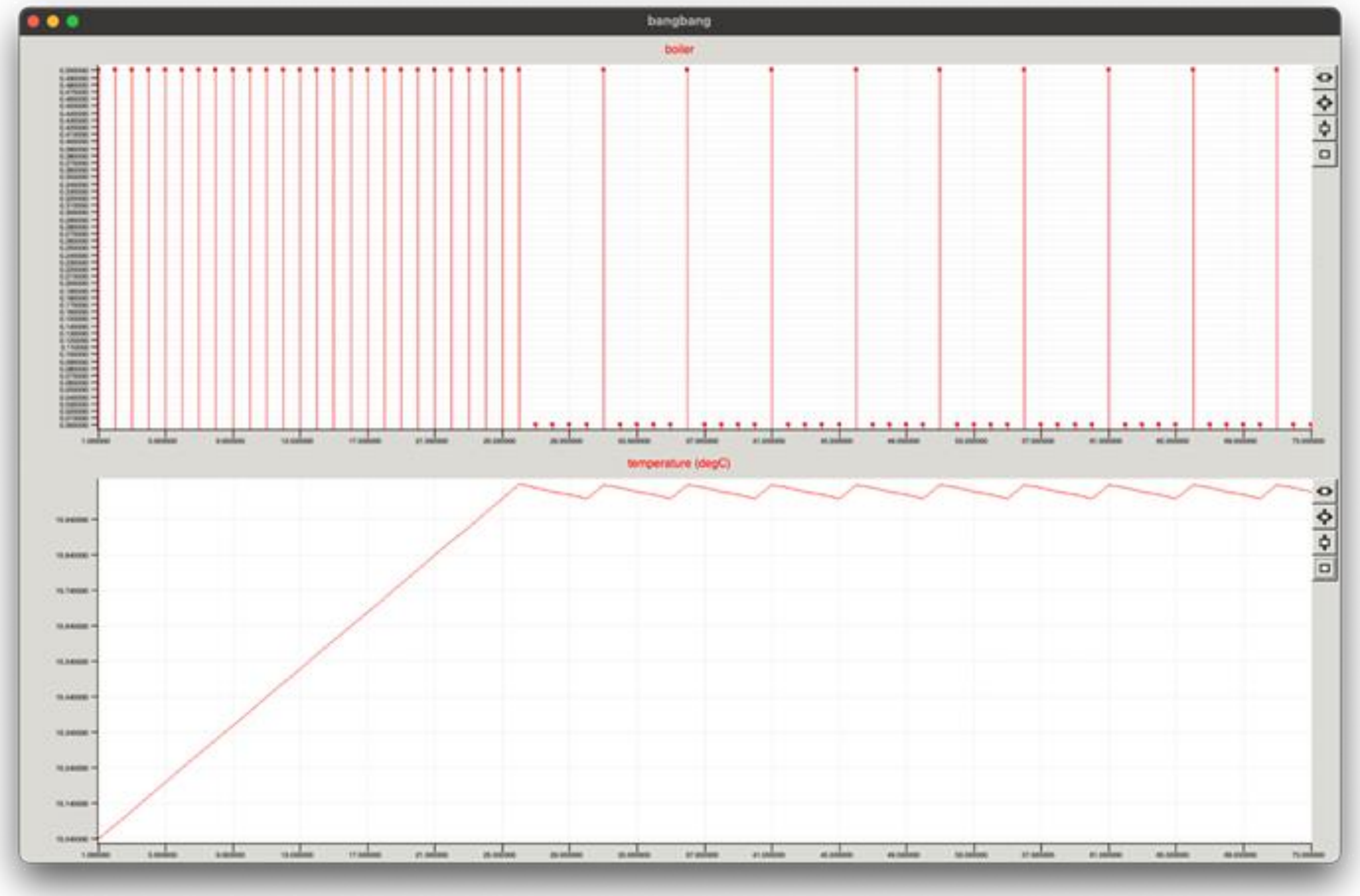


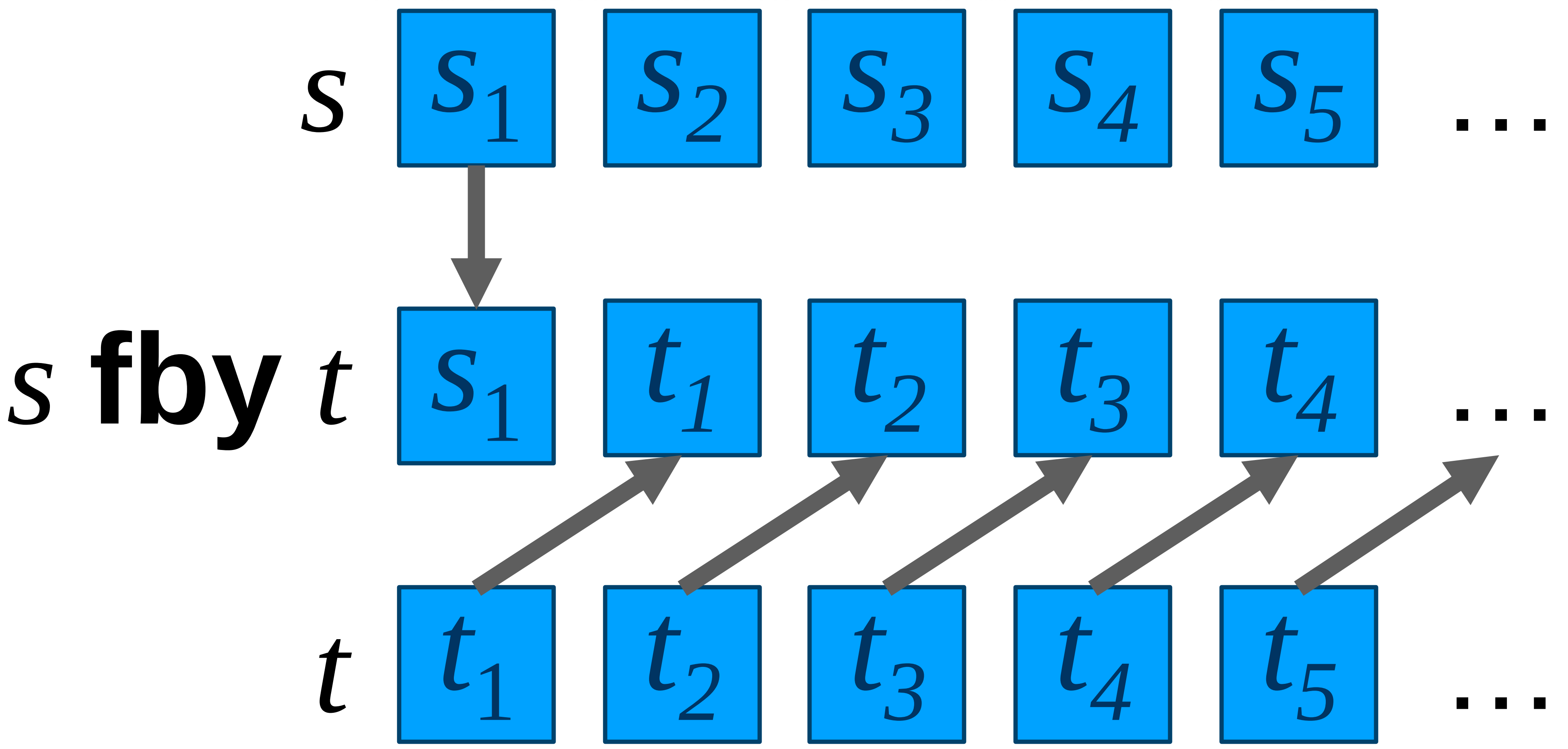


Synchronous Programming

[Caspi et al., POPL ’87; Bourke et al., HSCC ’13; Colaço et al., TASE ’17]

- **Proven track record** in industry
  - Lustre, SCADE, Esterel, Signal, etc.
- Data as **streams** (over time), programs as **stream manipulations**
- Our work is based on Zélus
  - Hybrid program **modeling and simulation**
  - Streams built using **unit delay** and **recursion**
  - Eventually: **hybrid systems verification**





Refinement Types

[Freeman and Pfenning, PLDI ’91; Rondon et al., PLDI ’08; Vazou et al., ICFP ’14; Jhala and Vazou, FTPL ’21]

- Inspired by Liquid Haskell
- Decidable **SMT-based** type checking and subtyping
- Type refinements on streams = **temporal properties**
- Support a **subset of LTL**
  - Interested in **safety properties**

Refinement Predicate

```
let x:{v:float| v >= 0.} = 3.14
```

Base Type

Term Variable

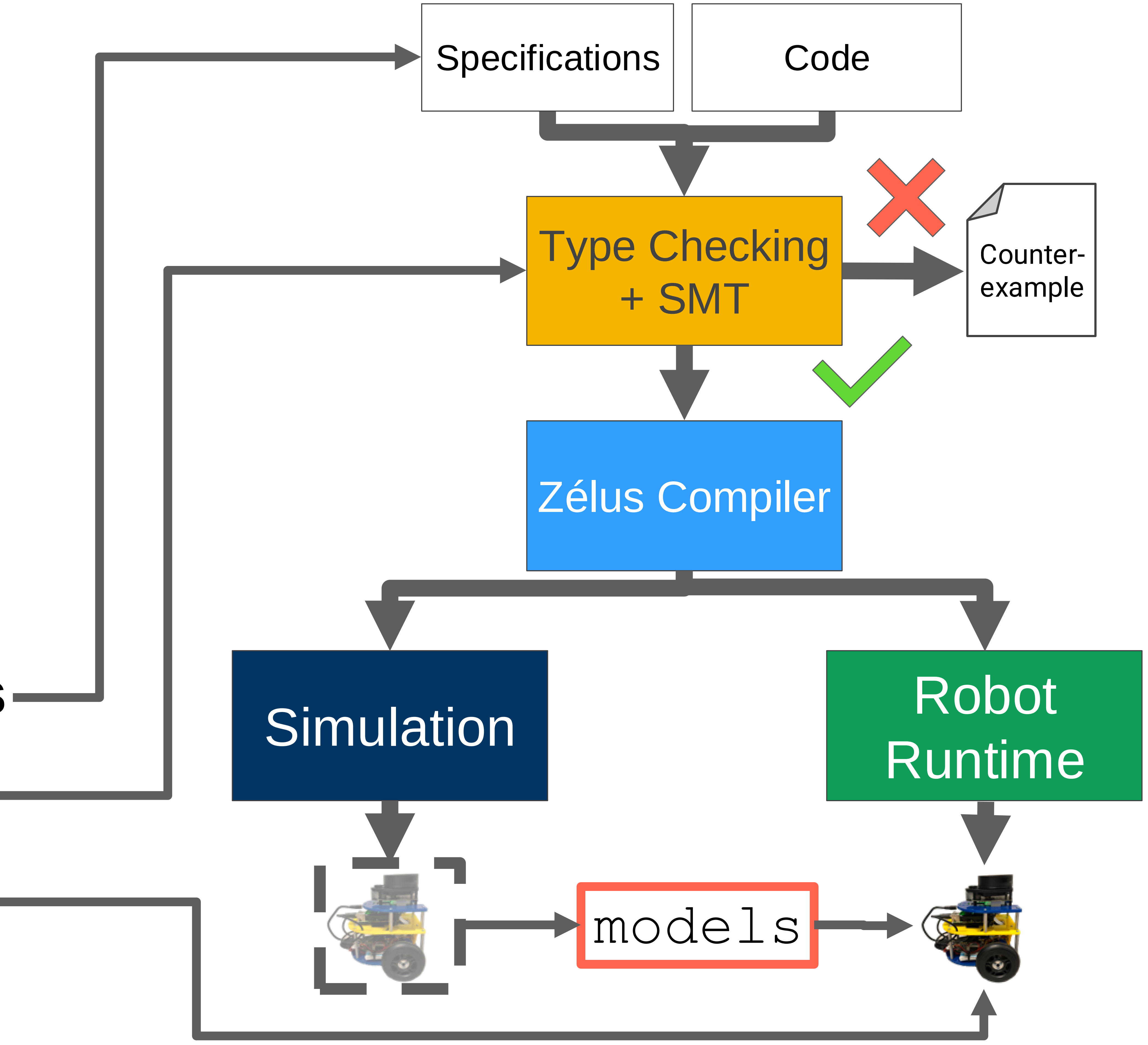
$$p, q ::= \text{true} \mid \text{false} \mid x \mid e_1 = e_2 \mid e_1 > e_2 \mid p \wedge q \mid \neg p$$

$$\varphi, \psi ::= p \mid \boxed{\Box\varphi \mid \bigcirc\varphi} \mid \varphi \wedge \psi$$

We formalize **refinement types** for a **synchronous** language, prove **type safety**, and implement verified programs on **physical robots**.

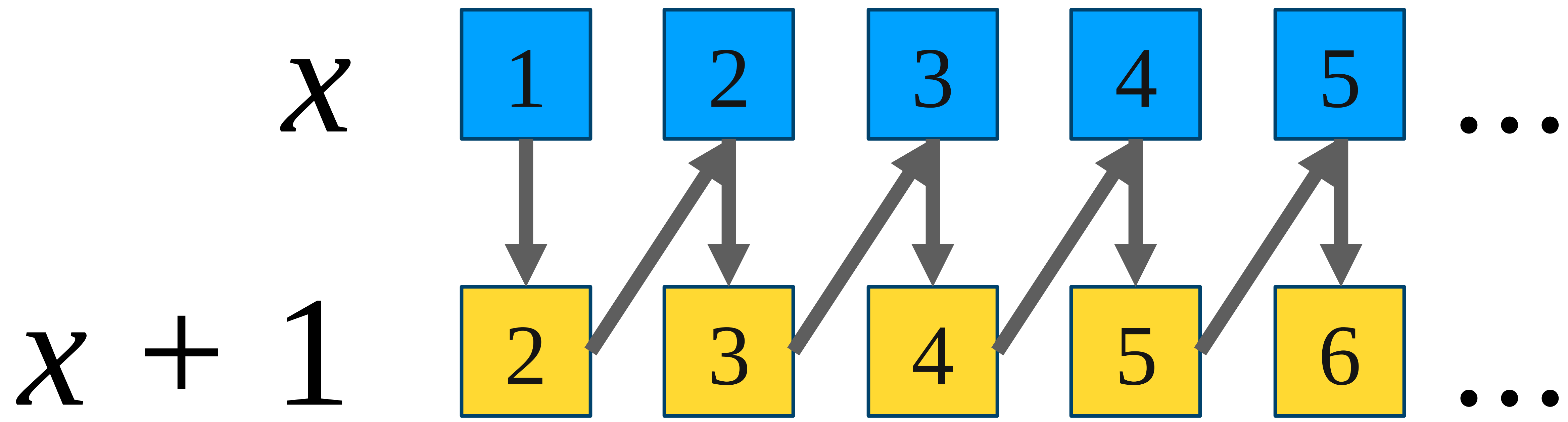
MARVeLus

- Method for Automated Refinement-Type Verification of Lustre
- **Separate compilation paths** for **simulation** and **execution**
- Verify a **discrete-time** subset of Zélus
- Our contributions:
  - Formal refinement **type system** and **semantics**
  - Type checker inside Zélus (+ artifact)
  - Demonstration of real-time execution

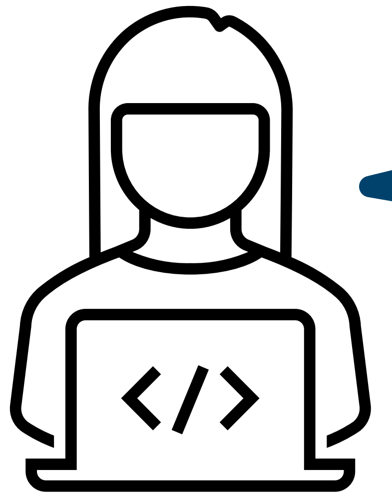


A Simple Example

```
let rec x
  = 1 fby (x + 1) in x
```

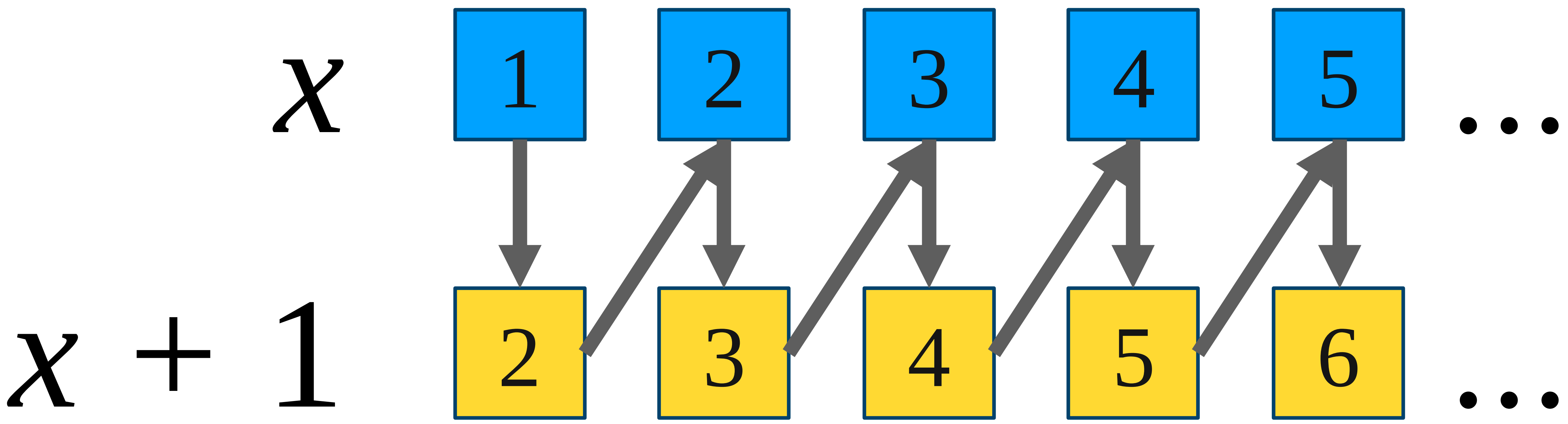


A Simple Example



"x should always be positive"

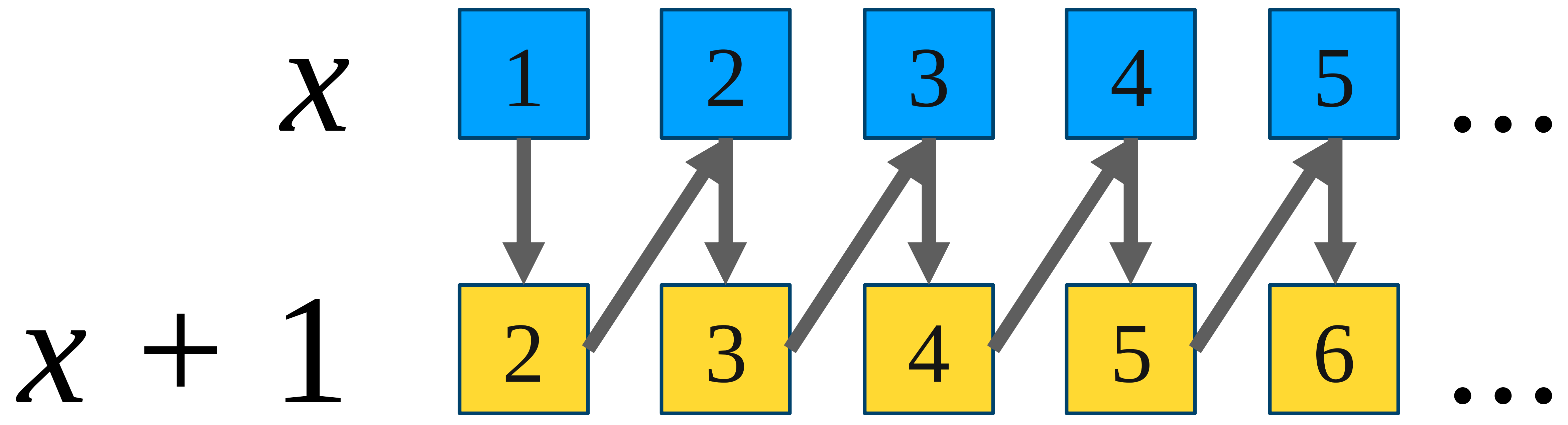
```
let rec x
  = 1 fby (x + 1) in x
```



A Simple Example

"x should always be positive"

```
let rec x:{v:int | □(x>0)}
  = 1 fby (x + 1) in x
```



A Simple Example

"x should always be positive"

```
let rec x:{v:int | □(x>0)}
  = 1 fby (x + 1) in x
```

$$1 > 0 \quad \land \quad \forall\, x{:}\mathrm{int}\,.\; x > 0 \Rightarrow (x+1) > 0$$

$$\forall\, x{:}\mathrm{int}\,.\; x > 0$$

SMT

Syntax-guided

MARVeLus Semantics

[Caspi and Pouzet, ICFP ’96; Caspi and Pouzet, CMCS ’98]

- Like Lustre semantics…
  - But adapted for type safety proofs with refinements
- Terms emit a **value** and **rewrite**
- One step **per unit time**

Environment **Term** **Value** **New Term**

$$\boxed{\boxed{S;\ \sigma} \vdash \boxed{e} \xhookrightarrow{\boxed{v}} \boxed{e'}}$$

Functions Terms

$$\frac{S;\ \sigma \vdash \boxed{e_1 \xhookrightarrow{v_1} e_1'}}{S;\ \sigma \vdash \boxed{e_1 \text{ fby } e_2} \xhookrightarrow{\boxed{v_1}} \boxed{\text{delay}(e_2)}}\ \text{(S-FBY)}$$

Semantics Rules

$$\boxed{S;\ \sigma \vdash e \xhookrightarrow{v} e'}$$

$$\frac{}{S;\ \sigma \vdash c \xhookrightarrow{c} c}\ \text{(S-CONST)} \qquad \frac{}{S;\ \sigma, x = h :: v \vdash x \xhookrightarrow{v} x}\ \text{(S-VAR)}$$

$$\frac{S;\ \sigma \vdash e_1 \xhookrightarrow{v_1} e_1'}{S;\ \sigma \vdash e_1 \text{ fby } e_2 \xhookrightarrow{v_1} \text{delay}(e_2)}\ \text{(S-FBY)} \qquad \frac{S,\ \text{prev}(\sigma) \vdash e \xhookrightarrow{v} e'}{S,\ \sigma \vdash \text{delay}(e) \xhookrightarrow{v} \text{delay}(e')}\ \text{(S-DELAY)}$$

$$\frac{S;\ \sigma, x = h :: \text{true} \vdash e_t \xhookrightarrow{v_t} e_t' \qquad S;\ \sigma, x = h :: \text{true} \vdash e_f \xhookrightarrow{v_f} e_f'}{S;\ \sigma, x = h :: \text{true} \vdash (\text{if } x \text{ then } e_t \text{ else } e_f) \xhookrightarrow{v_t} (\text{if } x \text{ then } e_t' \text{ else } e_f')}\ \text{(S-IF-T)}$$

$$\frac{S;\ \sigma, x = h :: \text{false} \vdash e_t \xhookrightarrow{v_t} e_t' \qquad S;\ \sigma, x = h :: \text{false} \vdash e_f \xhookrightarrow{v_f} e_f'}{S;\ \sigma, x = h :: \text{false} \vdash (\text{if } x \text{ then } e_t \text{ else } e_f) \xhookrightarrow{v_f} (\text{if } x \text{ then } e_t' \text{ else } e_f')}\ \text{(S-IF-F)}$$

$$\frac{S;\ \sigma, x = h :: \text{nil} \vdash e_1 \xhookrightarrow{v} e_1' \qquad S;\ \sigma, x = h :: v \vdash e_2 \xhookrightarrow{w} e_2'}{S;\ \sigma \vdash \text{let rec}_h\ x : \tau = e_1 \text{ in } e_2 \xhookrightarrow{w} \text{let rec}_{h::v}\ x : \text{tl}(\tau) = e_1' \text{ in } e_2'}\ \text{(S-LETREC)}$$

$$\frac{S;\ \sigma \vdash e_1 \xhookrightarrow{v} e_1' \qquad S;\ \sigma, x = h :: v \vdash e_2 \xhookrightarrow{w} e_2'}{S;\ \sigma \vdash \text{let}_h\ x : \tau = e_1 \text{ in } e_2 \xhookrightarrow{w} \text{let}_{h::v}\ x : \text{tl}(\tau) = e_1' \text{ in } e_2'}\ \text{(S-LET)}$$

$$\frac{}{S, f(x) = e; \sigma, y = h :: v \vdash f(y) \xhookrightarrow{e[x \mapsto v]} f(y)}\ \text{(S-APP)}$$

$$\frac{S, \sigma \vdash e \xhookrightarrow{v} e' \qquad \neg \text{ robot mode}}{S, \sigma \vdash e \text{ models } r \xhookrightarrow{v} e' \text{ models } r}\ \text{(S-MODELS)}$$

MARVeLus Types

- Like **refinement types**…
  - But with streams and temporal predicates
- **Syntax-guided** type safety…
  - But must also account for streams
  - Modified **progress and preservation**

Environment **Term** **Type**

$$\underbrace{\boxed{\underbrace{G}_{\text{Functions}};\ \underbrace{H}_{\text{Terms}}}}_{\text{Environment}} \vdash \underbrace{\boxed{e}}_{\text{Term}} : \underbrace{\boxed{\tau}}_{\text{Type}}$$

$$\frac{G;\ H \vdash e_1 : \{w : b \mid \boxed{\mathrm{hd}(\psi_1)}\} \qquad G;\ H \vdash e_2 : \{w : b \mid \boxed{\psi_2}\}}{G;\ H \vdash e_1 \text{ fby } e_2 : \{w : b \mid \underbrace{\boxed{\mathrm{hd}(\psi_1)}}_{\text{Beginning of } \psi_1} \wedge \underbrace{\boxed{\bigcirc \psi_2}}_{\text{All of } \psi_2 \text{ later}}\}}\ \text{(T-FBY)}$$

Selected Typing Rules

$$\boxed{G;\ H \vdash e : \tau}$$

$$\frac{}{G;\ H \vdash c : \{w : b \mid \square(w = c)\}}\ (\text{T-CONST}) \qquad \frac{G;\ H(x) = \{w : b \mid \psi\}}{G;\ H \vdash x : \{w : b \mid \psi \wedge \square(w = x)\}}\ (\text{T-VAR})$$

$$\frac{G;\ H \vdash e_1 : \{w : b \mid \mathsf{hd}(\psi_1)\} \qquad G;\ H \vdash e_2 : \{w : b \mid \psi_2\}}{G;\ H \vdash e_1 \text{ fby } e_2 : \{w : b \mid \mathsf{hd}(\psi_1) \wedge \bigcirc\psi_2\}}\ (\text{T-FBY})$$

$$\frac{G;\ \mathsf{prev}(H) \vdash e : \tau}{G;\ H \vdash \mathsf{delay}(e) : \tau}\ (\text{T-DELAY})$$

$$\frac{G;\ H \vdash \tau_1 \qquad G;\ H \vdash e_1 : \tau_1 \qquad G;\ H, x : \tau_1 \vdash e_2 : \tau_2}{G;\ H \vdash \mathsf{let}_h\ x : \tau_1 = e_1 \text{ in } e_2 : \tau_2}\ (\text{T-LET})$$

$$\frac{G;\ H \vdash \tau_1 \qquad G;\ H, x : \tau_1 \vdash e_1 : \tau_1 \qquad G;\ H, x : \tau_1 \vdash e_2 : \tau_2}{G;\ H \vdash \mathsf{let}_h \text{ rec } x : \tau_1 = e_1 \text{ in } e_2 : \tau_2}\ (\text{T-LETREC})$$

$$\frac{G;\ H \vdash x_c : \mathsf{bool} \qquad \begin{array}{c} G;\ H \vdash e_t : \{w : b \mid \mathsf{impl}(x_c, \psi)\} \\ G;\ H \vdash e_f : \{w : b \mid \mathsf{impl}(\neg x_c, \psi)\} \end{array}}{G;\ H \vdash \text{if } x_c \text{ then } e_t \text{ else } e_f : \{w : b \mid \psi\}}\ (\text{T-IF})$$

Typing Rules, Continued

$$\frac{G, f : (x : \{w_1 : b_1 \mid \varphi_1\} \to \{w_2 : b_2 \mid \varphi_2\});\ H \vdash x : \{w_1 : b_1 \mid \Box\varphi_1\}}{G, f : (x : \{w_1 : b_1 \mid \varphi_1\} \to \{w_2 : b_2 \mid \varphi_2\});\ H \vdash f\ y : \{w_2 : b_2 \mid \Box\varphi_2[x \mapsto y]\}}\ \text{(T-APP)}$$

$$\frac{G;\ H \vdash e : \tau}{G;\ H \vdash e \text{ models } r : \tau}\ \text{(T-MODELS)} \qquad \frac{}{G; H \vdash \text{nil} : \tau}\ \text{(T-NIL)}$$

$$\frac{G; H \vdash e : \tau \qquad G; H \vdash \tau \leq \tau' \qquad G; H \vdash \tau'}{G; H \vdash e : \tau'}\ \text{(T-SUB)}$$

Type Safety

Under some assumptions about the environment, **well-typed terms**

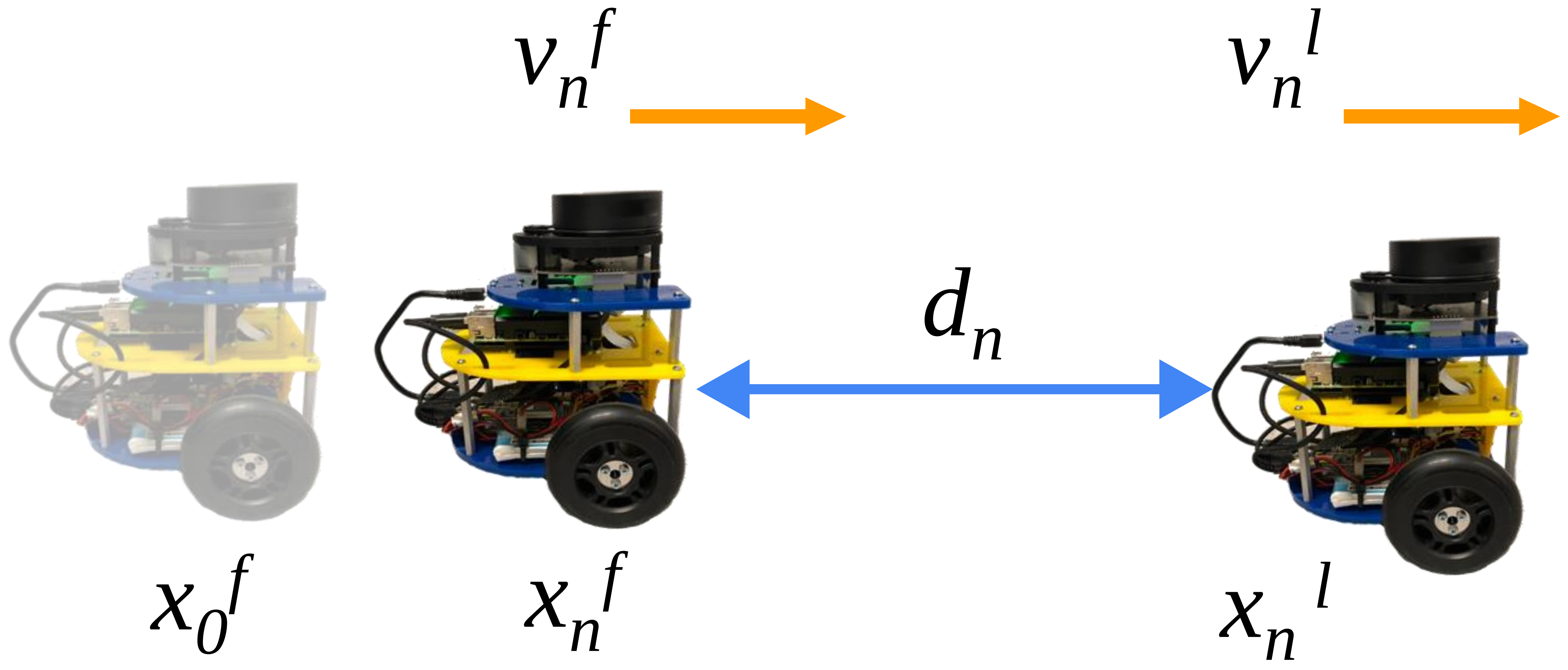
THEOREM 1 (TYPE SAFETY). $\boxed{\textit{If } H \textit{ corresponds with } \sigma \textit{ and } G \textit{ corresponds with } S,}$ *and* $\boxed{\textit{if } G; H \vdash e : \tau,}$ *then* $\boxed{\exists e' \textit{ such that } S; \sigma \vdash e \overset{v}{\hookrightarrow} e' \textit{ where } v \textit{ is a value,}}$ $\boxed{G, H \vdash v : \mathsf{hd}(\tau), \textit{ and } G, \mathsf{tl}(H) \vdash e' : \mathsf{tl}(\tau)}$

always step to terms **which are well-typed in the <u>next time step</u>**

Verified Adaptive Cruise Control

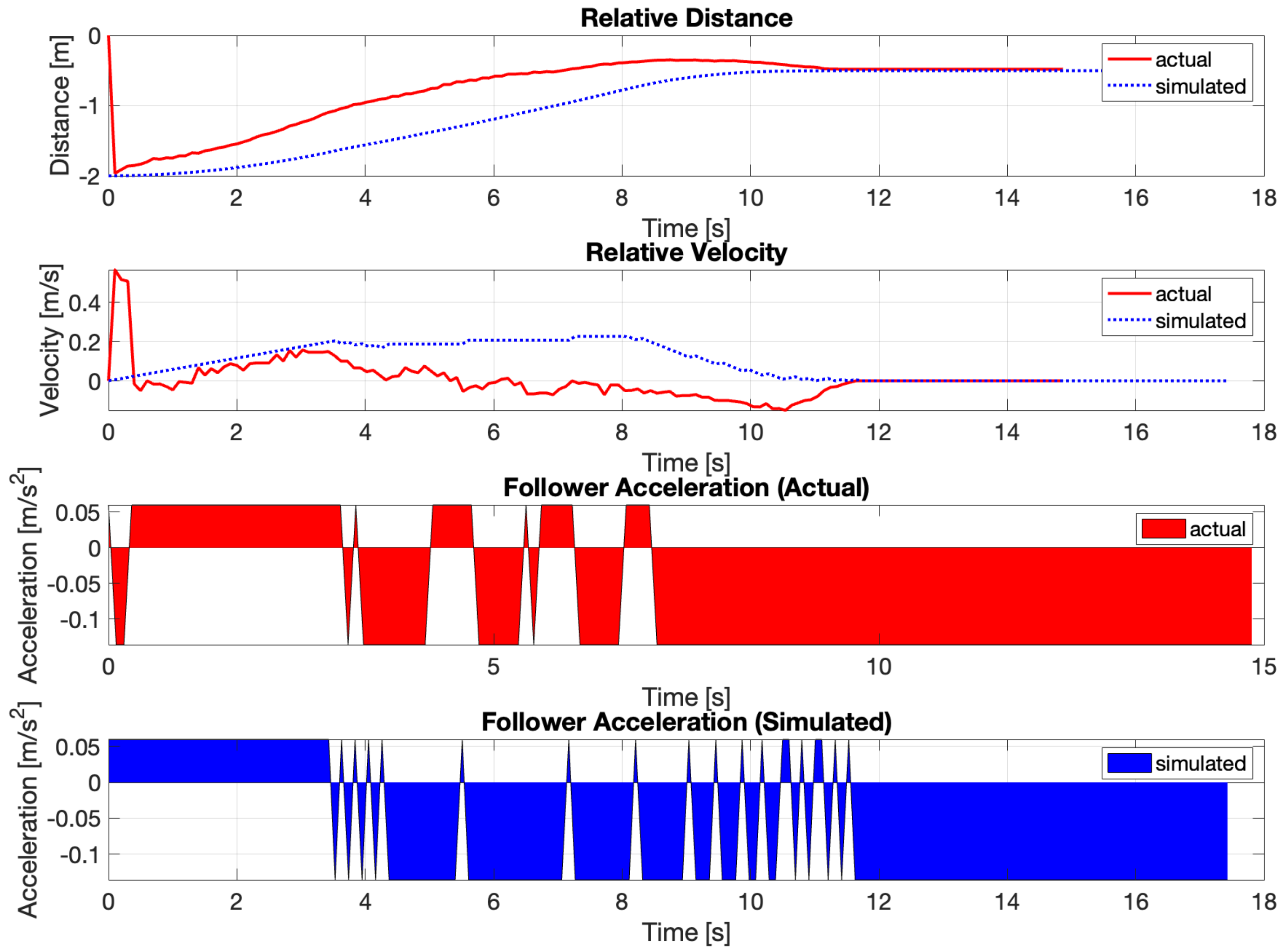
[Loos et al., FM ’11]

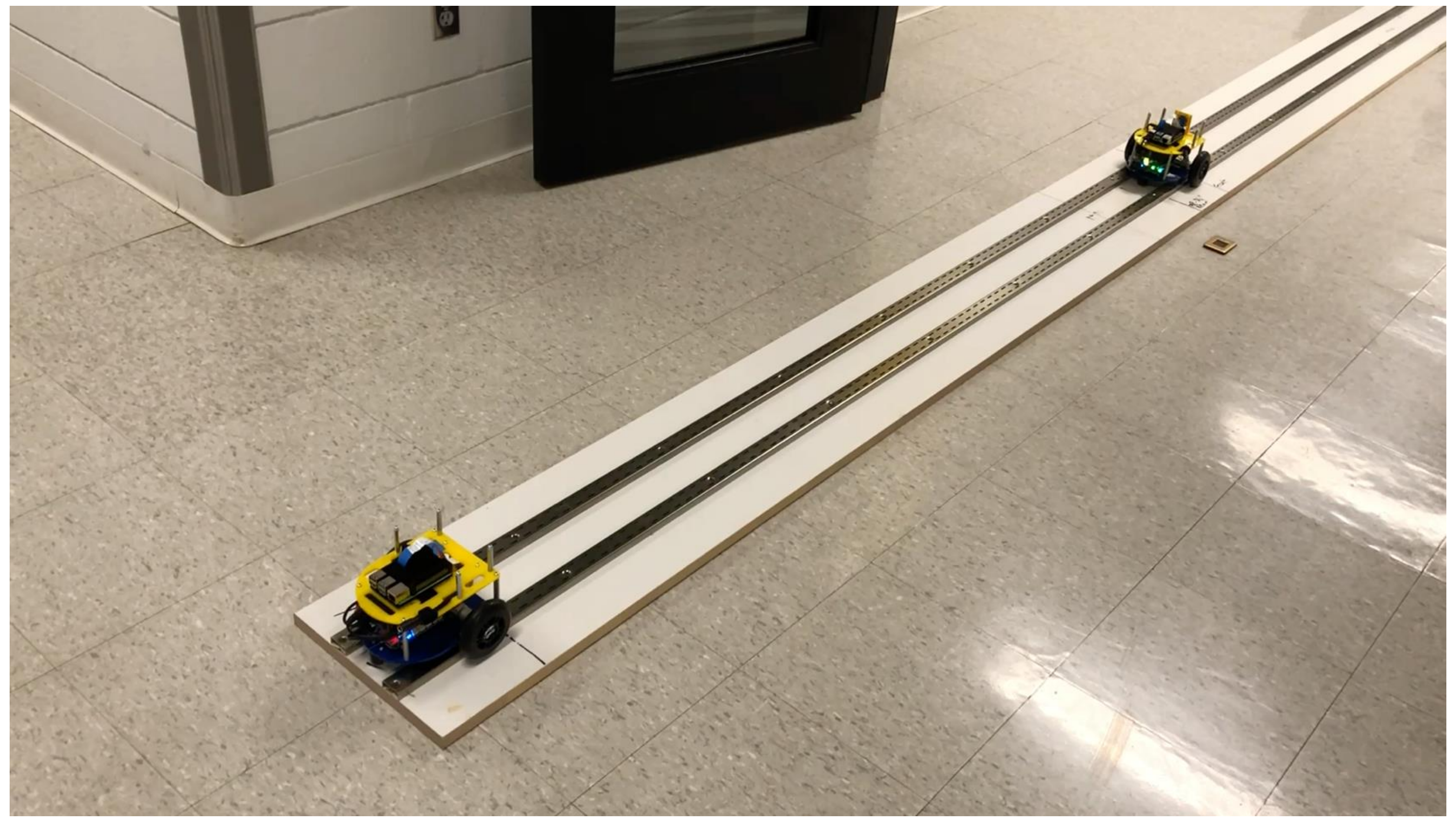
- Verified **autonomous braking controller** for Adaptive Cruise Control
- **Safety Property**: Never crash into the obstacle: $\square(d > 0)$
- Hardware abstractions and sensors trusted



Experiment

- Verified autonomous braking demonstrated on **physical robots**
- Can follow **moving vehicles**
- Collision-free in **all runs**





Future Work

Specification

- Hybrid programs
- Full LTL specifications

Verification / Compilation

- Invariant generation
- Verified compiler

Execution

- More hardware support
- Runtime monitoring

Future Work: Specifications

- Currently we support a limited subset of LTL
  - Necessary for “predicate splitting”
  - May be tricky to extend to hybrid
- Must have:
  - Type safety proof
  - Predictable compile-time verification
- Possible ideas:
  - Synchronous Observers

$$p, q ::= \mathsf{true} \mid \mathsf{false} \mid x \mid e_1 = e_2 \mid e_1 > e_2 \mid p \wedge q \mid \neg p$$

$$\varphi, \psi ::= p \mid \boxed{\Box\varphi \mid \bigcirc \varphi} \mid \varphi \wedge \psi$$

$$\frac{G;\ H \vdash e_1 : \{w : b \mid \boxed{\mathsf{hd}(\psi_1)}\} \qquad G;\ H \vdash e_2 : \{w : b \mid \boxed{\psi_2}\}}{G;\ H \vdash e_1\ \mathsf{fby}\ e_2 : \{w : b \mid \boxed{\mathsf{hd}(\psi_1)} \wedge \boxed{\bigcirc\psi_2}\}}\ \text{(T-FBY)}$$

Beginning of ψ_1 **All of** ψ_2 **later**

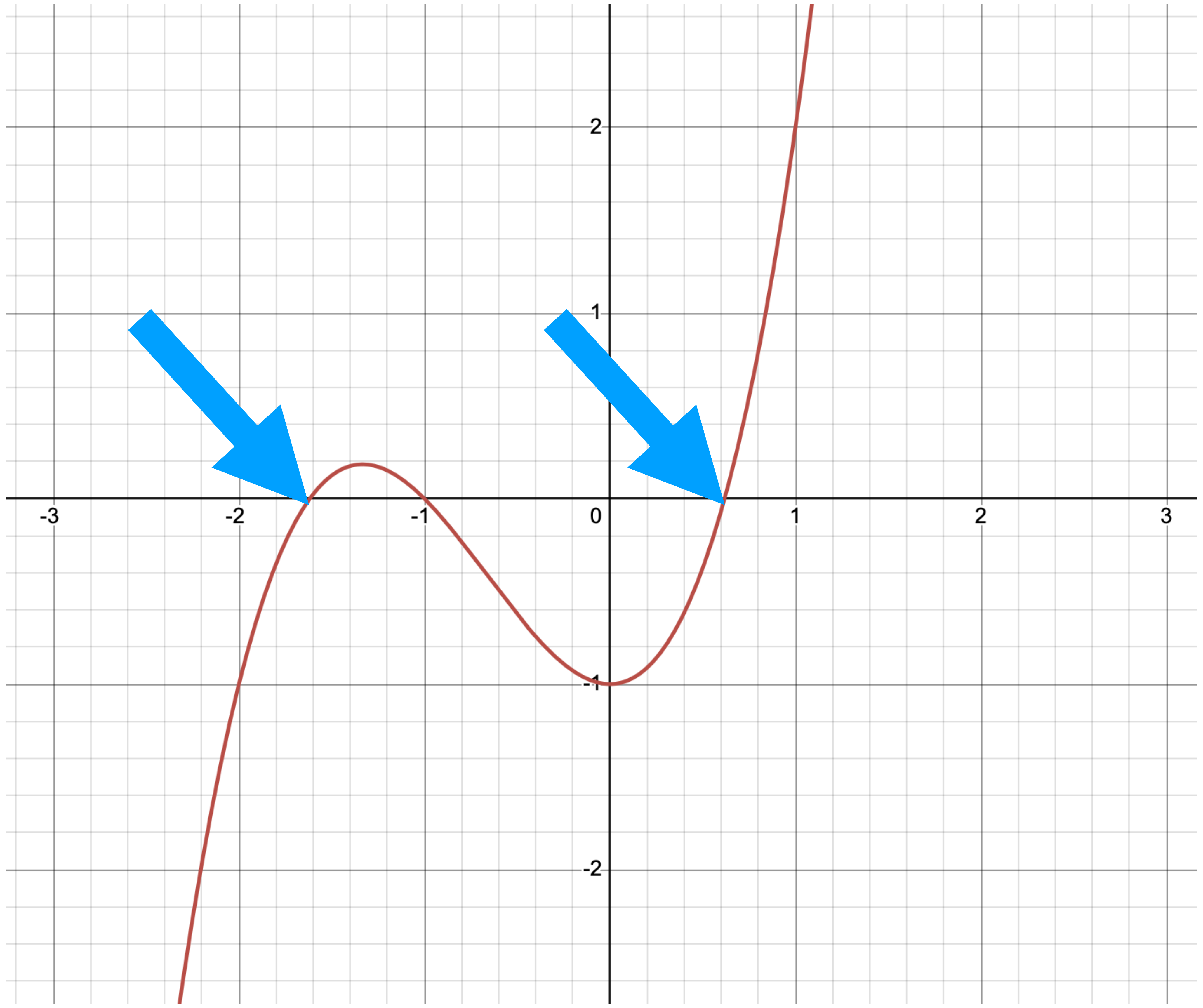
$\psi[0]$	$\psi[1]$	$\psi[2]$	$\psi[3]$	$\psi[4]$	…

$\equiv$

$\psi_1[0]$	$\psi_2[0]$	$\psi_2[1]$	$\psi_2[2]$	$\psi_2[3]$	…

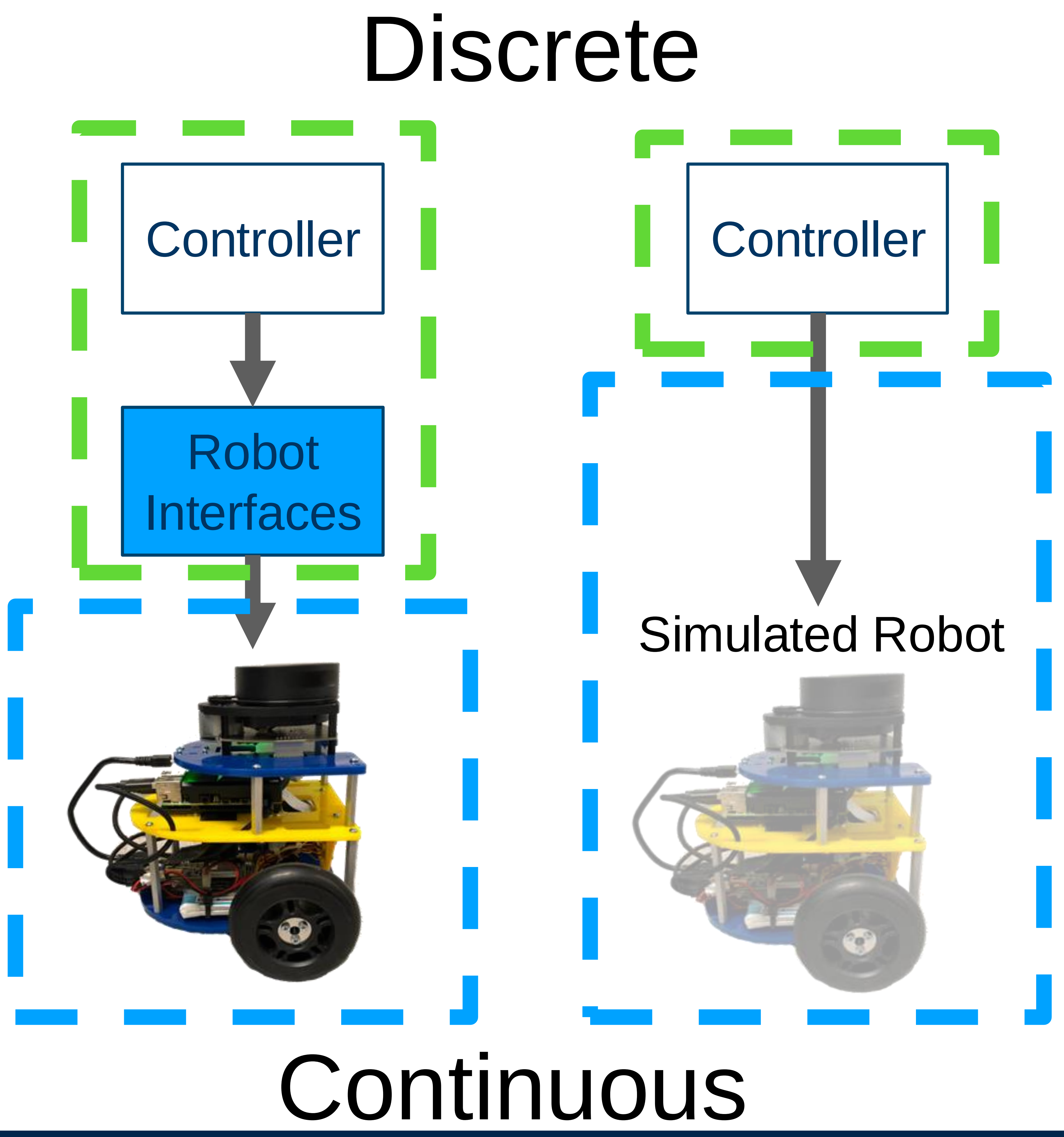
Future Work: Specifications (cont'd)

- Hybrid
  - Model real systems more accurately
  - Upwards Zero Crossings
- Possible Ideas:
  - Differential Dynamic Logic (dL)
    - Parallelism?
  - Barrier Certificates



Future Work: Robotics Integrations

- Provide an intuitive interface for system designers
- Current implementation:
  - Getter and setter functions to access robot variables
  - Deterministic model for verification
- Re-use as much verified code as possible
  - Ideally, reuse the entire verified controller



Automated Braking in MARVeLus

$$\mathrm{needToBrake}(d,\ v) = \left(d - \frac{v^2}{2b} - v \cdot dt - \frac{v \cdot dt}{2}\right) \leq 0$$

“Are we too close to do anything other than brake?”

```
let vfi  :   {v: float | v > 0.} = 0.8;
let dt   :   {v: float | v > 0.} = 0.1;
let b    :   {v: float | v > 0.} = 0.136;
let xl   :   {v: float | v > 0.} = 5.;
```

Constants

```
let rec (df, vf, af):
  {(d:float)*(v:float)*(a:float)|
```

Base Type Spec

$(\ (d > 0)$

Safety Condition

$\wedge ((\neg \mathrm{needToBrake}(d,\ v) \wedge (a \leq 0)) \vee (\mathrm{needToBrake}(d,\ v) \wedge (a = -b)))$

Follower Decisions

$\wedge \left(d - \frac{v^2}{2b} - \frac{v \cdot dt}{2}\right) > 0 \wedge (v \geq 0)$

Dynamics Invariant

```
}) = (xl, vfi, 0.) fby
```

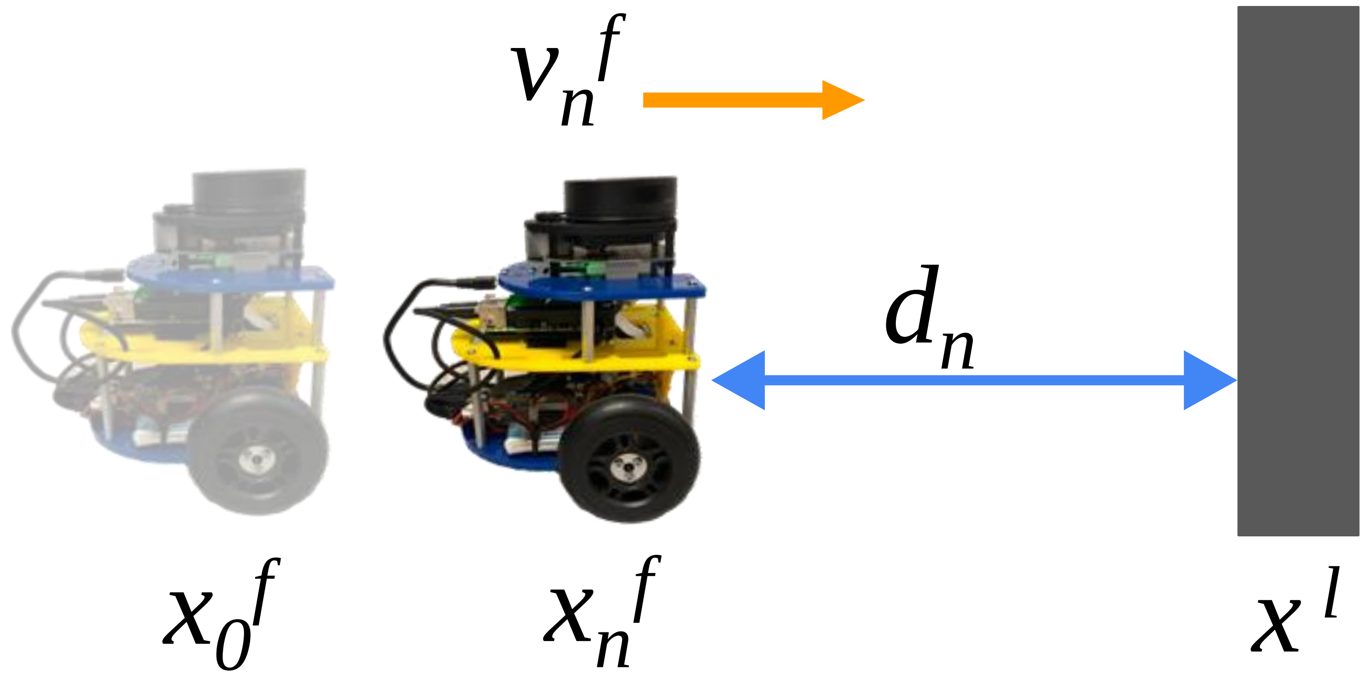
Initial State

```
  let v_next = max(vf + (af * dt), 0.)   models   (robot_get “vel”)    in
  let d_next = df -. (v_next * dt)       models   (robot_get “dist”)   in
```

Dynamics Evolution

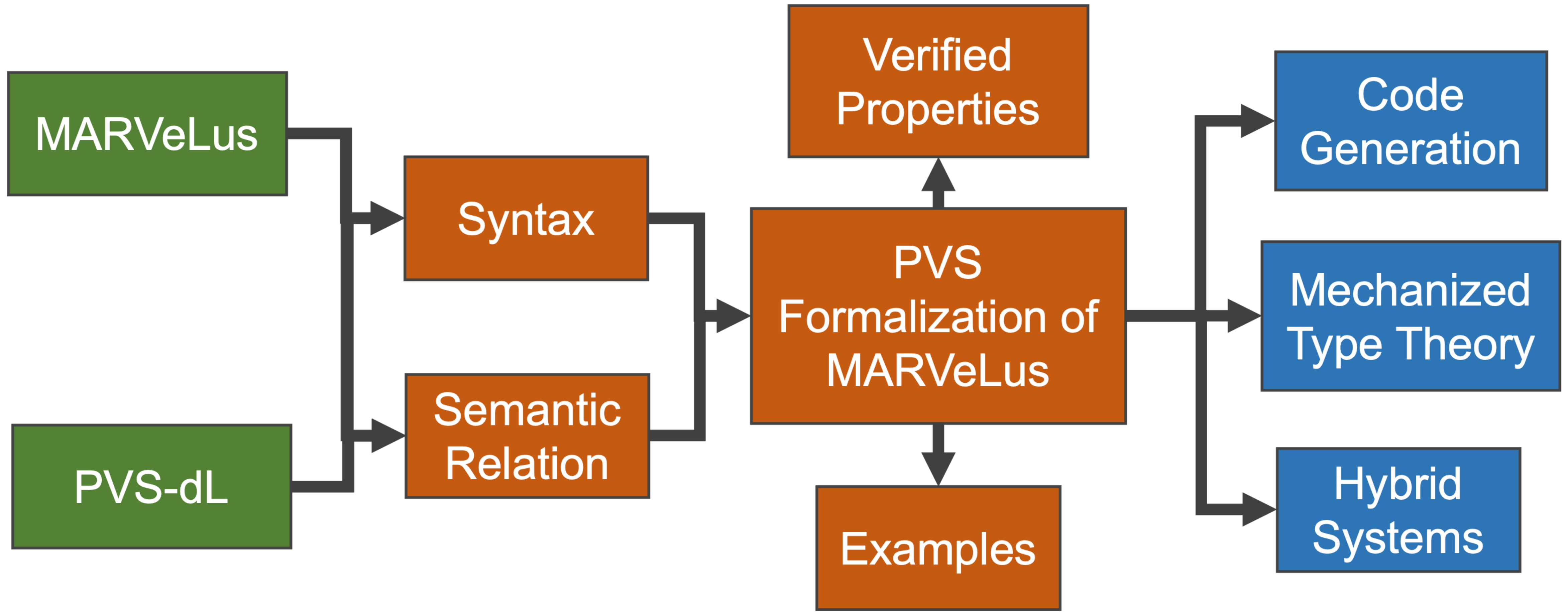
```
  (d_next, v_next,
      if needToBrake(d_next, v_next) then -b else amax)
  in (robot_str “brake” af); (df, vf, af)
```

Follower Control



Future Work: Proof Assistant Embedding

- Goal: Embed MARVeLus in a proof assistant such as PVS
  - Build off existing dL embedding for hybrid
  - Mechanize the type system
  - Enable code generation from specifications (inspired by PRECiSA)
- Existing work embeds Lustre in Coq and PVS



Summary

- Robots and other CPS need **formal verification**
- MARVeLus provides **formal verification** and **execution** in a **unified robotics platform**
- **Synchronous programs** can be enhanced with **refinement types**
- Verified MARVeLus programs can execute on **real hardware**

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- Many more

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