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Abstract

We analyze fiscal consolidations using a New-Keynesian model where agents have finite planning horizons and are uncertain about the future state of the economy. Both consumers and firms are infinitely lived, but only plan and form expectations up to a finite number of periods into the future. The length of agents’ planning horizons plays an important role in determining how spending cuts or tax increases affect output and inflation. We find that for low degrees of relative risk aversion spending-based consolidations are less costly in terms of output losses, in line with empirical evidence. A stronger response of monetary policy to inflation makes spending-based consolidations more favorable as well. Interestingly, for short planning horizons, our model captures the positive comovement between private consumption and government spending observed in the data.

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\textbf{Keywords:} Fiscal policy, Finite planning horizons, Bounded rationality

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1 Introduction

The effects of fiscal consolidations on output and their effectiveness in stabilizing debt have attracted much attention in the empirical literature to date. The recent debt crisis in the Eurozone gave rise to further research as regards the effects of the composition of fiscal consolidations. It appears that a long standing debate is still active regarding this issue. The debate focuses on the distinct effects of tax-based and spending-based consolidations. The theoretical literature has also contributed to that debate, mostly arguing in favor of spending-based consolidations due to their expansionary effect and hence their effectiveness in stabilizing debt faster (Bi et al. (2013)). Apart from the contribution of Blanchard (1985), the theoretical literature has ignored, to a great extent, the implications of households’ finite planning horizons for the effects of consolidations. In this paper, we try to fill this gap and take the analysis a step further by constructing a theoretical model where agents are boundedly rational with finite planning horizons, and by looking at the effects of such behavior on aggregate macro variables. Comparing the model with the rational expectations benchmark, we show that planning horizons are important for the behavior of output and inflation following a fiscal consolidation, with short planning horizons leading to completely different policy implications than the rational expectations paradigm.

Blanchard (1985) examines the dynamic effects of government deficit finance when the horizon of households is finite and is a parameter chosen arbitrarily. Our contribution is different in many respects. First, we assume that agents are infinitely lived but have finite planning horizons. However, they care about the level of their wealth at the end of their planning horizon. This structure generates wealth effects that are not present in Blanchard’s approach. In particular, households derive utility from the level of their wealth at the end of their planning horizon. This implies that households aim to decide upon this level of wealth optimally. In Blanchard’s structure this channel is absent. Moreover, we are interested in the interactions between monetary and fiscal policy. Therefore, we incorporate our approach into a New-Keynesian framework. In our model, households are identical, they do not have different ages, they have the same levels of wealth and the same marginal propensities to consume. This facilitates aggregation in our framework.1 Our approach not only captures

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1 Blanchard introduces a probability of death in order to induce equal horizons across individuals and equal marginal propensity to consume. Otherwise, the relation among different levels and compositions of wealth and different propensities to consume makes aggregation impossible.
the finite horizon aspect, but also the changes in the behavior of households.\(^2\) We capture changes in consumption decisions - equivalent to time inconsistency of consumption plans - by assuming that households are uncertain about the state of the economy after their planning horizon and learn in a recursive manner about the true valuation of their end-of-horizon wealth.\(^3\) Finally, we consider both tax-based and spending-based consolidations and analyze their effects on output, inflation and the debt ratio. Moreover, instead of assuming lump-sum taxes as in Blanchard, we assume distortionary income taxes.

In our model, we assume that the government announces consolidations one period in advance. First, we look at the effects of consolidations for different horizons. We find that when agents are short-sighted - plan for three quarters ahead - consumption falls following spending cuts. This is in line with the empirical literature on the sign of consumption responses following spending cuts (see Blanchard and Perotti (2002), Fatas and Mihov (2001) and Gali et al. (2007) among others). The intuition is that they initially value future wealth too much, and do not fully incorporate how consolidations will decrease the tax burden in the far future. One of the major weaknesses of New-Keynesian models without labor market distortions and rule-of-thumb consumers as argued in Gali et al. (2007), is that they do not capture this positive co-movement between government spending and household consumption. Instead, in these models consumption increases following spending cuts, which is also what we find for longer planning horizons.\(^4\) However, for short planning horizons, our model is able to capture the positive co-movement between private consumption and government spending observed in the data without introducing labor market frictions and/or rule-of-thumb households.

Following tax-based consolidations, private consumption drops, regardless of the planning horizon. However, the decline in consumption is smaller as the planning horizon increases. This is due to the positive wealth effect due to lower expected future taxes once the consolid-\(^2\)In that respect our approach is closer to the life-cycle formulation by Modigliani than to that of permanent income by Milton Friedman. In Blanchard (1985) instead the opposite holds. Moreover, in Blanchard, the assumption of a constant probability of death implies that the objective function of households does not change over time. Therefore, there is no time inconsistency of initial optimal programs.

\(^3\)Our approach is closer to myopic individual consumption decision which is time inconsistent. That is, actual consumption at date \(t\) may be different from what the household had previously planned to consume at that date. This is because at any date \(t\), the household plans its consumption accounting for elements not considered in its plans computed previously. For a more detailed analysis of those issues see Lovo and Polemarchakis (2010) and the references therein.

\(^4\)In fact, when households have long to infinite planning horizons, they anticipate lower future taxes after the consolidation is over due to lower debt. This creates a positive wealth effect which results in higher consumption. Moreover, households with longer planning horizons anticipate the lower future debt service costs which boosts their current consumption further.
idation is over. For short horizons, though, these wealth effects are weaker, since agents do not fully take account of the lower taxes after their horizon, and place too much value on having wealth at the end of their horizon.

The responses of consumption have important implications for the output costs of both spending- and tax-based consolidations. In both cases, when agents have short planning horizons, consolidations can initiate considerably bigger recessions than would have occurred had agents planned for longer or infinite horizons. This is also reflected in the present value multipliers. The effects on inflation are sensitive to the length of planning horizons as well. For shorter horizons, persistently lower demand leads to persistent deflationary pressures, under both types of consolidations. What is interesting, is that this leads the response of inflation under tax-based consolidations to change sign for very short horizons. Even though tax hikes increase marginal costs, which in any standard New-Keynesian model would push inflation up, inflation falls both upon anticipation and upon implementation for short horizons. In this case, the demand effect is stronger than the supply side effect with the drop in household consumption more than offsetting the increase taxes, leading to a persistent drop in inflation. Finally, the debt ratio falls slower under very short horizons, due to the larger fall in output.

Comparing spending- and tax-based consolidations, we find that the former leads to a faster drop in the debt ratio in the short run due to the induced expansion upon anticipation of the imminent consolidation. Under tax-based consolidations, instead, output drops both upon anticipation and in the subsequent quarters. Moreover, inflation and the real interest rate are higher, which delay the fall in debt further. Once consolidations are implemented, under our benchmark calibration, output drops abruptly after spending cuts and the subsequent slower recovery deteriorates the performance of spending-based consolidations in the medium-run, compared to tax-based consolidations. We show however, that this result crucially depends on the degree of relative risk aversion of households. Lower degrees of risk aversion mitigate the adverse effects of spending-based consolidations while they amplify those of tax-based consolidations, leading to deeper recessions. In this case, spending consolidations perform better than tax-based consolidations, both in terms of reducing debt and in terms of the associated output losses.

We find that the monetary policy stance is also important for the way consolidations affect the economy, and for the relative performance of spending- and tax-based consolidations. In
general, as monetary policy becomes more aggressive, the adverse effects of spending-based consolidations are mitigated, while those of tax-based consolidations are amplified. Following spending cuts, a stronger monetary easing following the drop in consumption and inflation can partly offset the negative effects of the abrupt fall in aggregate demand.\(^5\) Under tax-based consolidations instead, the upward pressures on inflation - due to higher marginal costs - do not allow for a monetary easing necessary to counteract the short-run effects of higher taxes. On the contrary, as monetary policy becomes more aggressive, real interest rates are even higher, which leads to deeper recessions, increasing the induced output losses.

All in all, we show that an otherwise standard New Keynesian model, but with households having a finite horizon and being uncertain about the future state of the economy, can capture some key facts observed in the data. That is, the positive co-movement between private consumption and government spending and the milder contractions following spending-based consolidations. In that respect, we show that the monetary policy stance is crucial as well. The virtue of our approach, although computationally challenging, is that it can capture those key facts observed in the data without resting on further complications, like agent heterogeneity or labor market frictions.

The paper is organized as follows. In the next section, we briefly discuss the literature on fiscal consolidations along with that on the existence of non-Ricardian households. Also, we briefly discuss the literature dealing with the optimization problem of boundedly rational agents who form expectations over a finite number of periods, and explain how our approach differs from the existing literature. In Section 3, we present the New Keynesian model with distortionary taxes and finite planning horizons. In Section 4, determinacy and E-stability properties of our model are presented. Section 5 discusses the effects of fiscal consolidations for boundedly rational agents with varying planning horizons. Section 6 concludes.

\(^5\)Canova and Pappa (2011) show that fiscal multipliers are sensitive to the monetary policy stance. However, as Alesina et al. (2015) point out, monetary policy cannot be the main explanation behind the differences between the two types of consolidation.
2 Related literature

2.1 Fiscal Consolidations

In this section we review the literature on the empirical relevance of non-Ricardian households, first, and, second, on the effects of fiscal consolidations in general.

Gali et al. (2007) construct a New-Keynesian model with forward looking and rule-of-thumb households where the latter fraction decides upon its consumption on the basis of their current disposable income only since it is excluded from financial markets. They test the empirical validity of this assumption and find supporting evidence.\(^6\) Such a structure (along with labor market frictions) is able to account for the positive co-movement between consumption and government spending as opposed to traditional neo-classical models, but not under any parametrization. Specifically, they show that with a competitive labor market, a large fraction of rule-of-thumb agents (e.g. more than 65\%) is necessary in order for the model to account for the positive comovement. However, as they show, this fraction falls to plausible levels when the model features a non-competitive labor market. Our model instead, captures this positive comovement, at least in the medium-run, and yields spending multipliers close to the empirical literature without resting on labor market frictions or on agent heterogeneity.

Along the same lines Parker (1999) finds evidence of a high sensitivity of consumption to variations in after-tax income due to anticipated changes in social security taxes, while Souleles (1999) finds evidence of excess sensitivity of households’ consumption to predictable tax refunds. Finally, Campbell and Mankiw (1989) reject the permanent income hypothesis in favor of a model with borrowing constraints or myopic behavior. This empirical evidence motivates our approach of households that have finite planning horizons, but do care about their end of horizon wealth.

In the literature, there is substantial evidence on the effects of fiscal consolidations. In particular, there has been much research over the effects of different types of fiscal consolidations (e.g. spending-based and tax-based). A large empirical literature provides evidence supporting the expansionary fiscal consolidations hypothesis (see Alesina and Perotti (1995), Perotti (1996), Alesina and Ardagna (1998, 2010), Ardagna (2004)). In particular, the key finding is that fiscal consolidations are sometimes correlated with rapid economic growth.

\(^6\)In particular, the existence of rule-of-thumb households leads to an aggregate Euler equation where anticipated changes in taxes have predictive power over private consumption growth.
especially when implemented by spending cuts rather than tax increases. On the other hand, another strand of the empirical literature, using narrative data to identify consolidations, initially introduced by Romer and Romer (2010), finds that output drops following both types of consolidations and that recessions are deeper after tax hikes (Guajardo et al. (2014)).\footnote{Earlier papers using the conventional approach to identify fiscal consolidations argued in favor of the expansionary effects of spending-based consolidations ("expansionary fiscal austerity") Alesina and Ardagna (2010), Alesina et al. (2002), Alesina and Perotti (1996) and Giavazzi and Pagano (1990) among others. However, their measure of identifying consolidations (i.e. the CAPB) suffers from problems like reverse causality or changes in fiscal variables due to non-policy changes correlated with other developments in output. Finally, as Romer and Romer (2010) point out another approach, followed by Blanchard and Perotti (2002) using SVAR analysis and institutional information to identify consolidations, suffers from problems similar to those of the studies above.}

Along the same lines Alesina et al. (2015), using a richer structure for modeling fiscal consolidations, find that spending-based consolidations are less costly, in terms of output losses than tax-based ones. However, as Guajardo et al. (2014) argue, a drawback of contemporaneous estimates is that planned impacts on budgets may tend to be over-optimistic relative to the ex-post outcomes. Consequently, the negative effects of consolidations on output may be understated due to the induced bias. This is the case with spending cuts in many instances, where the announced cuts were stronger than those actually implemented (Beetsma et al. (2016)).

In the theoretical literature, Bertola and Drazen (1993) develop a model where the government satisfies its intertemporal budget constraint by periodically cutting spending, where the latter is inherently unsustainable. A worsening of the fiscal conditions can increase the probability of a beneficial fiscal consolidation which can thus be expansionary. Bertola and Drazen (1993) consider the importance of expectations in the analysis of fiscal consolidations.

Bi et al. (2013) augment the model of Bertola and Drazen (1993) with distortionary taxation and analyze the effects of different types of fiscal consolidations. Moreover, they look at the effects of persistence in those, as well as of the uncertainty of economic agents over the composition of the upcoming fiscal consolidation. Accounting for the monetary policy stance as well, they find that spending- and tax-based consolidations can be equally successful in stabilizing government debt at low debt levels. Nevertheless, at high debt levels, spending-based consolidations are expected to be expansionary and more successful in stabilizing debt, especially when agents anticipate a tax-based consolidation.

Finally, Erceg and Linde (2013) examine the effects of tax-based and spending-based
consolidations in a two country DSGE model for a currency union. They assume agent heterogeneity by introducing fixed fractions of forward looking and "hand-to-mouth" households. They find that tax-based consolidations have less adverse output costs than spending-based ones in the short to medium-run. Moreover, they show that large spending-based consolidations can be counterproductive in the short-run when the zero lower bound in interest rate binds, while they argue in favor of a "mixed strategy" combining both types of consolidations.

2.2 Bounded rationality and bounded optimality

Our model is related to a large literature on bounded rationality and bounded optimality. First of all, under Euler equation learning (see Honkapohja et al. (2013)) agents form expectations only up to one period into the future. When these expectations are not fully rational (and hence do not implicitly take the infinite future into account through a recursive formulation), agents have a one period ahead planning horizon. Resting on this finite horizon learning approach, Evans and McGough (2015) build a framework that formulates the agents’ optimization in a way that is consistent with their short sightedness in forecasting. In their approach, agents learn about the shadow price of their wealth in a recursive way. They show that such a problem can be cast in a dynamic programming setting, which is consistent with the time inconsistency of consumption plans.

Our approach differs from Evans and McGough (2015) in some crucial aspects. First, our agents are uncertain not only about the correct valuation of their end-of-horizon wealth, but also about the state of the economy at the end of their planning horizon. This means that even if our agents might know the true valuation of their wealth, they can still be time inconsistent in their consumption decisions if their beliefs about the state of the economy at the end of their horizon are wrong. As such, even though in a two period setting our approach is similar to theirs, the implied expectations path in our case is different. Moreover, we apply our approach to cases where agents optimize and form expectations for more than two (but a finite number of) periods. Consequently, our approach incorporates important wealth effects which are absent in Evans and McGough (2015).

An alternative to Euler equation learning and shadow price learning is infinite horizon learning (see Preston (2005) and Eusepi and Preston (2017)), where agents fully solve an
infinite horizon optimization problem, given their (possibly boundedly rational) expectations that they have to form over an infinite horizon. We believe however that both the assumption of a horizon of one period and of a planning horizon of infinitely many periods may be too extreme when considering the actions of boundedly rational agents.

Branch et al. (2010) consider the case of finite horizons. In their model, it is however required that agents form expectations about their end of horizon wealth and optimize based on these expectations. In contrast, in our model choosing optimal end of horizon bond holdings is in every period part of the agents’ optimization problem.

3 The model

3.1 Households

Households want to maximize their discounted utility of consumption and leisure over their planning horizon ($T$ periods), and they also value the state they expect to end up in at the end of these $T$ periods (their state in period $T+1$). They are however not able to rationally induce (by solving the model forward until infinity), how exactly they should value their state in period $T+1$. Instead households use a rule of thumb to evaluate the value of their state. Since bond holdings are the only relevant state variable for households, their maximization problem becomes

$$\max \left\{ C_s, H_s, B_t \right\}_{s=t}^{t+T} \sum_{s=t}^{t+T} \beta^{s-t} u(C_s, H_s) + \beta^{T+1} V(t+1, (B_t/(P_t)))$$

(1)

subject to

$$P_s C_s + B_{s+1}^{t+1} \leq (1 - \tau_s)W_s H_{s+1}^t + B_s + P_s \Xi, \quad s = t, t+1, \ldots, t+T,$$

(2)

where $B_t$ are nominal bond holdings from household $i$ at the beginning of period $t$. $\Xi_t$ are real profits from firms, which are equally distributed among households and $W_t$ is the nominal wage rate.

Furthermore, we assume that households have CRRA preferences with relative risk aversion $\sigma$. We also assume that the relative valuation of real bond holdings in period $t + T + 1$ compared to consumption in period $t + T$ as determined in period $t$ is given by $\Lambda_t$. The
functional form of $V^{i,t}(\cdot)$ therefore becomes

$$V^{i,t}(x) = \Lambda^i_t x^{1-\sigma} \frac{1}{1-\sigma}. \quad (3)$$

The parameter $\Lambda^i_t$ is time varying, implying that households can change over time how they plan to trade off future wealth with future consumption. More specifically we assume that households use an adaptive updating mechanism to learn over time how they should optimally value future wealth (see Section 3.5). If households would always choose $\Lambda^i_t$ such that they value future wealth optimally, then, under rational expectations, the above problem would be equivalent to an infinite horizon optimization problem. The bounded rationality in putting an exact value on wealth far in the future is what will drive many of our results.

Dividing the budget constraint by $P_s$ gives

$$C^i_s + \frac{B^i_{s+1}}{(1+i_s)P_s} \leq (1-\tau_s)w_s H^i_s + \frac{B^i_s}{P_s} + \Xi_s, \quad s = t, t+1,...t+T. \quad (4)$$

The first order conditions of the maximization problem are

$$(H^i_s)^\eta = (C^i_s)^{-\sigma} (1-\tau_s) w_s, \quad s = t, t+1,...t+T, \quad (5)$$

$$(C^i_s)^{-\sigma} = \beta \frac{(1+i_s) (C^i_{s+1})^{-\sigma}}{\Pi_{s+1}}, \quad s = t, t+1,...t+T-1, \quad (6)$$

$$(C^i_{t+T})^{-\sigma} = \beta (1+i_{t+T}) (\frac{B^i_{t+T+1}}{P_{t+T}})^{-\sigma} \Lambda^i_t. \quad (7)$$

where $\eta$ is the Frisch elasticity of labor supply.\(^9\)

Next we define a measure of real bond holdings scaled by steady state output: $b_t = \frac{B_t}{P_{t-1}Y}$. Substituting for this expression in (4) and (7) gives

$$C^i_s + \frac{\bar{Y} b^i_{s+1}}{1+i_s} \leq (1-\tau_s) w_s H^i_s + \frac{\bar{Y} b^i_s}{\Pi_s} + \Xi_s, \quad s = t, t+1,...t+T, \quad (8)$$

and

$$(C^i_{t+T})^{-\sigma} = \beta (1+i_{t+T}) (\frac{\bar{Y} b^i_{t+T+1}}{P_{t+T}})^{-\sigma} \Lambda^i_t. \quad (9)$$

where $\Pi_s = P_s/P_{s-1}$ is gross inflation in any period $s$.

\(^9\)Our utility function takes the functional form $u(C^i_s, H^i_s) = \frac{(C^i_s)^{1-\sigma}}{1-\sigma} - \frac{(H^i_s)^{1+\eta}}{1+\eta}$. 

3.2 Firms

There is a continuum of monopolistically competitive firms producing the final differentiated goods. Each firm has a linear technology with labor as its only input

\[ Y_t(j) = AH_t(j), \]  

where \( A \) is aggregate productivity, which is assumed to be constant.

Firms are run by households, and hence will have finite planning horizons. That is, they will form expectations about their marginal costs and the demand for their product for \( T \) periods ahead only. We assume that in each period a fraction \((1 - \omega)\) of firms can change their price. The problem of firm \( j \) that can reset its price is then to maximize the discounted value of its profits for the next \( T \) periods.

\[
\tilde{E}_t^j \sum_{s=0}^T \omega^s Q_{t,t+s}^j \left[ p_t(j) Y_{t+s}(j) - P_{t+s} mc_{t+s} Y_{t+s}(j) \right],
\]

where

\[
Q_{t,t+s}^j = \beta^s \left( \frac{C_{t+s}^j}{C_t^j} \right)^{-\sigma} \frac{P_t}{P_{t+s}},
\]

is the stochastic discount factor of the household that runs firm \( j \).

Using the demand for good \( j \), the firm’s profit maximization problem writes as follows

\[
\max \tilde{E}_t^j \sum_{s=0}^T \omega^s \left( \frac{C_{t+s}^j}{C_t^j} \right)^{-\sigma} P_t \left[ \left( \frac{p_t(j)}{P_{t+s}} \right)^{1-\theta} Y_{t+s} - mc_{t+s} \left( \frac{p_t(j)}{P_{t+s}} \right)^{-\theta} Y_{t+s} \right].
\]

The first order condition for \( p_t(j) \) is

\[
\tilde{E}_t^j \sum_{s=0}^T \omega^s \left( \frac{C_{t+s}^j}{C_t^j} \right)^{-\sigma} P_t \left[ (1-\theta) \left( \frac{p_t^*(j)}{P_{t+s}} \right)^{-\theta} + \theta mc_{t+s} \left( \frac{p_t^*(j)}{P_{t+s}} \right)^{-1-\theta} \right] = 0,
\]

where \( p_t^*(j) \) is the optimal price for firm \( j \) if it can re-optimize in period \( t \). Multiplying by \( \frac{(C_{t+s}^j)^{-\sigma} p_t^*(j)^{1+\theta}}{P_t^{1-\theta}} \) gives

\[
\tilde{E}_t^j \sum_{s=0}^T \omega^s \left( \frac{C_{t+s}^j}{C_t^j} \right)^{-\sigma} Y_{t+s} \left[ p_t^*(j) P_{t+s} - \frac{\theta}{\theta - 1} mc_{t+s} P_{t+s}^{1+\theta} \right] = 0.
\]
This can be written as

\[
\frac{p_t^s(j)}{P_t} \tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \left(C_{t+s}^j\right)^{-\sigma} \left(\frac{P_{t+s}}{P_t}\right)^{\theta-1} Y_{t+s} = \frac{\theta}{\theta-1} \tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \left(C_{t+s}^j\right)^{-\sigma} \left(\frac{P_{t+s}}{P_t}\right)^{\theta} Y_{t+s} m c_{t+s}. \tag{16}
\]

Finally, the aggregate price level evolves as

\[
P_t = \left[\omega P_{t-1}^{1-\theta} + (1 - \omega) \int_0^1 p_t^s(j)^{1-\theta} dj \right]^{1/\theta}. \tag{17}
\]

### 3.3 Government and market clearing

The government issues bonds and levies labor taxes \((\tau_t)\) to finance its spending \((G_t)\). Its budget constraint is given by

\[
B_{t+1} \frac{1}{1 + i_t} = P_t G_t - \tau_t W_t H_t + B_t, \tag{18}
\]

with \(H_t = \int H_t^i di\) and \(B_t = \int B_t^i di\) aggregate labor and aggregate bond holdings respectively. Dividing by \(\tilde{Y} P_t\) gives

\[
\frac{b_{t+1}}{1 + i_t} = g_t - \tau_t w_t \frac{H_t}{\tilde{Y}} + \frac{b_t}{\Pi_t}, \tag{19}
\]

where \(b_t = \frac{B_t}{\tilde{Y}}\) and \(g_t = \frac{G_t}{\tilde{Y}}\) are the ratios of debt to steady state GDP and government expenditure to steady state GDP, respectively.

Market clearing is given by

\[
Y_t = C_t + G_t = C_t + \tilde{Y} g_t. \tag{20}
\]

Monetary policy is defined by a Taylor type interest rule where the government responds to current inflation and output.

\[
\frac{1 + i_t}{1 + i} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_1} + \left(\frac{Y_t}{\tilde{Y}}\right)^{\phi_2}. \tag{21}
\]

We assume that taxes always respond to debt in order to keep the debt ratio close to a (possibly time varying) target set by the government. The coefficient with which taxes respond to debt \((\gamma_t^D)\) is however not very large. If debt lies far above its target, e.g. because
market pressures or political considerations have led the government to lower its debt target, the government may decide to additionally implement consolidations. These consolidations can be tax-based or spending-based, or a combination of both. In the case of a tax-based consolidation, the government temporarily increases the coefficient with which taxes respond to deviations of the debt ratio from its target by $\gamma^\text{cons}_\tau$. If consolidations are instead spending-based, the government temporarily lets spending respond to deviations of the debt ratio from its target with a coefficient $\gamma^\text{cons}_g$. In particular, government spending and taxes, evolve as

$$g_t = g_1 - \zeta \gamma^\text{cons}_g \mathbb{1}_{\text{cons}}(b_t - DT_{t-1}),$$

and

$$\tau_t = \tau^{DT}_{t-1} + (\gamma^0_\tau + (1 - \zeta)\gamma^\text{cons}_\tau \mathbb{1}_{\text{cons}})(b_t - DT_{t-1})$$

Here, $DT_{t-1}$ denotes the target for the debt ratio that the government sets and which we assume to be publicly known. $g_1$ and $\tau^{DT}_{t-1}$ determine the steady state levels of spending and taxes. $\tau^{DT}_{t-1}$ is made time varying, so that the government can make it depend on $DT_{t-1}$ and thereby assure that any debt target can be reached in the long run. $\mathbb{1}_{\text{cons}}$ is an indicator function that is 1 when the government is implementing consolidations, and 0 otherwise. $\zeta$ is the fraction of the consolidation that is spending-based. Below we will only consider the two extreme cases of only spending-based consolidations ($\zeta = 1$) and only tax-based consolidations ($\zeta = 0$).

### 3.4 Log linearized model

The steady state of the non-linear model described in the previous section, assuming zero inflation, is given in Appendix A. We proceed by log-linearizing all the above model equations around this steady state. Starting with the consumers, an optimal consumption decision can be derived by iterating the budget constraint from period $t + T$ backward, and substituting for future labor and future consumption and final period bond holdings, using the first order conditions of the household. This expression can then be aggregated across households to obtain an expression for aggregate consumption.

Combining the resulting aggregate consumption equation with the log-linearized supply side and government budget constraint results in the following system of equations. Details
of the derivations can be found in Appendix B.

\[(1 - \nu_y)\ddot{Y}_t = \frac{1}{\rho} \ddot{b}_t + \ddot{g}_t + \nu_t \sum_{s=0}^{T} \beta^s (\ddot{E}_t \ddot{\hat{\tau}}_{t+s}) + \nu_y \sum_{s=0}^{T} \beta^s (\ddot{E}_t \ddot{\gamma}_{t+s}) + \nu_y \sum_{s=1}^{T} \beta^s (\ddot{E}_t \ddot{\gamma}_{t+s}) \] (24)

\[-\mu \sum_{s=1}^{T} \beta^s \sum_{j=1}^{s} (\ddot{E}_t \pi_{t+j-1} - \ddot{E}_t \pi_{t+j}) + \frac{\bar{b}}{\rho} \sum_{s=0}^{T} \beta^s (\beta \ddot{E}_t \ddot{\tau}_{t+s} - \ddot{E}_t \ddot{\tau}_{t+s}) \]

\[-\beta^{T+1} \frac{\bar{b}}{\sigma \rho} \sum_{j=0}^{T-1} (\ddot{E}_t \pi_{t+j} - \ddot{E}_t \pi_{t+j+1}) - \beta^{T+1} \frac{\bar{b}}{\sigma \rho} \ddot{E}_t \pi_{t+T} - \beta^{T+1} \frac{\bar{b}}{\sigma \rho} \dot{\Lambda}_t,\]

\[\pi_t = \ddot{\kappa}(\eta + \frac{\sigma}{1 - g}) \sum_{s=0}^{T} \omega^s \beta^s \ddot{E}_t \ddot{\gamma}_{t+s} - \frac{\ddot{\kappa} \sigma}{1 - g} \sum_{s=0}^{T} \omega^s \beta^s \ddot{E}_t \ddot{\gamma}_{t+s} \]

\[+ \frac{\ddot{\kappa}}{1 - \tau} \sum_{s=0}^{T} \omega^s \beta^s \ddot{E}_t \ddot{\tau}_{t+s} + \tilde{\kappa} \sum_{s=1}^{T} \omega^s \beta^s \sum_{\tau=1}^{T} \ddot{E}_t \pi_{t+\tau}, \] (25)

\[\ddot{b}_{t+1} = \frac{1}{\beta} \ddot{g}_t - \frac{\bar{w}}{\beta} \left[ \ddot{\tau} \left( (1 + \eta + \frac{\sigma}{1 - g}) \ddot{Y}_t - \sigma \frac{\ddot{g}_t}{1 - g} + \frac{\ddot{\tau}_t}{1 - \tau} \right) + \ddot{\tau}_t \right] + \frac{1}{\beta} \dddot{b}_t + \dddot{b}(\ddot{b}_t - \frac{1}{\beta} \ddot{\tau}_t). \] (26)

The definition of composite parameters \(\nu_y, \nu_x, \nu_t\) and \(\ddot{\kappa}\) are given in Appendix B.3.

The monetary and fiscal policy equations, obtained by log-linearizing (21), (22) and (23) are

\[\ddot{i}_t = \phi_1 \pi_t + \phi_2 \ddot{Y}_t, \] (27)

\[\ddot{g}_t = -\zeta \gamma^\text{cons}_g \dddot{f}_{\text{cons}}(\ddot{b}_t - \ddot{D}T_{t-1}), \] (28)

\[\ddot{\tau}_t = \ddot{\tau}_{t-1} + (\gamma^0 + (1 - \zeta) \gamma^\text{cons} \dddot{f}_{\text{cons}})(\ddot{b}_t - \ddot{D}T_{t-1}), \] (29)

where we let

\[\ddot{\tau}^{DT}_{t-1} = \ddot{\tau}_{t-1} + \frac{1 - \beta}{\bar{w} \left(1 - \frac{\beta}{\eta + \frac{T}{\rho} \frac{1}{1 - \tau}}\right)} \ddot{D}T_{t-1}. \] (30)

The term \(\ddot{\tau}^{DT}_{t-1}\) in the tax rule represents the tax rate consistent with a debt ratio equal to the debt target, \(\ddot{D}T_{t-1}\). When the debt target is equal to its value of the nonlinear steady state around which the model is log-linearized (so that \(\ddot{D}T_{t-1} = 0\)), then the term \(\ddot{\tau}^{DT}_{t-1}\) is zero. In that case, the tax rate converges to its nonlinear steady state value after the debt ratio has been stabilized. However, if the debt target is above this steady state (\(\ddot{D}T_{t-1}\)
is positive), the tax rate needs to be raised proportionately in order to guarantee that the debt ratio will converge to the debt target. Otherwise, debt will converge to a level different from the target. In other words, the term $\tilde{\tau}_{t-1}^{DT}$ guarantees the existence of a (unique) steady state in our linearized model where the debt ratio is equal to its target, for any value of the debt target. Expression (30) is derived by using the linearized government budget constraint under the restriction that the debt ratio is equal to the debt target, and by using the fact that steady state inflation and marginal costs are not affected by the level of the debt target. We hence assume that the government believes that agents behave in a rational way. For agents instead we assume that they know the form and the parameters of the tax rule and that they are always aware of the current values of $\tilde{DT}_{t-1}$ and $\tilde{\tau}_{t-1}^{DT}$, which are announced by the government. However, agents do not know how the term $\tilde{\tau}_{t-1}^{DT}$ is set by the government.

### 3.5 Valuation of future bond holdings

Finally, we turn to the evolution of $\hat{\Lambda}_t^i$. This parameter determines how agents value bond holdings relative to consumption at the end of their horizon. It thereby determines what trade-off agents plan to make between saving and consuming at the end of their horizon. We assume that agents try to learn what trade-off to make in the future based on the actual trade-offs they have been making in the past. Since both the optimal trade-off between consumption and saving and the trade-off agents actually make in each period vary over time, due to changes in their wealth, we assume agents use constant gain learning to update their $\hat{\Lambda}_t^i$.\(^{10}\)

More specifically, agents use first order condition (9) to observe their actual saving/consumption trade-off in the past. In log-linearized form this gives

$$\hat{\Lambda}^{i,\text{realized}}_t = \frac{\sigma}{b} \tilde{b}^i_{t+1} - \sigma \tilde{C}^i_t - \tilde{i}_t,$$

(31)

\(^{10}\)Note that agents could also try to learn how their valuation of future debt exactly depends on their current bond holdings. There are however 2 problems with such an approach. First of all, such a learning algorithm can be extremely slow to converge, leading to unintuitive dynamics. Secondly, if the algorithm would have converged, agents would then know exactly how to value debt, even if their financial position changes. This may not be fully realistic and would bring us close again to an infinite horizon optimization problem. With the approach chosen in this paper, as debt varies, agents are constantly learning in their new environment how exactly they should value future wealth. Therefore we capture the bounded rationality aspect that agents may dynamically make mistakes in how to value wealth and therefore make sub-optimal decisions. However, in the long run, as their bond holdings converge to a steady state, they learn how to value future wealth correctly.
where $\hat{\Lambda}_{i,realized}^t$ is the realized relative valuation of real bond holdings used by the household as opposed to the relative valuation the households planned $T$ periods in advance ($\hat{\Lambda}_{i}^t$).

Agents then use past realizations of $\hat{\Lambda}_{i,realized}^t$ to determine how they plan to trade-off bond holdings and consumption in the future. This results in

$$\hat{\Lambda}_{i}^t = \gamma \sum_{j=0}^{\infty} (1 - \gamma)^j \hat{\Lambda}_{i,realized}^{t-j},$$

with $\gamma$ the gain parameter. This can be written in the following adaptive form.

$$\hat{\Lambda}_{i}^t = (1 - \gamma)\hat{\Lambda}_{i-1}^t + \gamma \hat{\Lambda}_{i,realized}^{t-1},$$

Aggregating (31) and (33) and using market clearing gives a formula for the evolution of $\hat{\Lambda}_{t}$:

$$\hat{\Lambda}_{t} = (1 - \gamma)\hat{\Lambda}_{t-1} + \gamma \left( \frac{\sigma}{b} \dot{b}_{t} - \frac{\sigma}{1 - \tilde{g}} \tilde{Y}_{t-1} + \frac{\sigma}{1 - \hat{g}} \hat{g}_{t-1} - \dot{i}_{t-1} \right).$$

4 Model properties

Before we turn to an analysis of fiscal consolidations, we briefly review the dynamic properties of our model. In particular, we consider different horizons and investigate under what conditions our model is locally determinate under rational expectations and E-stable under adaptive learning.

4.1 Determinacy

First, we assume that, given the boundedly optimal optimization problem with finite horizons, and given the law of motion of $\hat{\Lambda}_{t}$ of Equation (34), expectations are fully rational. We check for determinacy by writing our system in the form $A z_t = B z_{t-1}$ with $z_t = [\tilde{Y}_{t+T}, \tilde{\pi}_{t+T}, \hat{b}_{t+T}, \tilde{\pi}_{t+T-1}, \hat{b}_{t+T-1}, ..., \tilde{Y}_{t+1}, \hat{b}_{t+1}, \tilde{\pi}_{t}, \hat{\Lambda}_{t}]$. The matrices $A$ and $B$ are chosen such that they represent Equations (24) and (25) in the first 2 rows, and Equation(26) forwarded $T - 1$ periods in the third row. Equation (34) is written in the bottom row of $A$ and $B$, and all other rows of these matrices consist only of zeros and ones. Since $z_{t-1}$ contains $2T$ forward looking variables and $T + 3$ predetermined ones, our system is determinate if and only if the matrix $A^{-1}B$ has exactly $2T$ eigenvalues outside the unit
Figure 1: Determinacy and E-stability for different horizons ($T$) and different responses of monetary policy to inflation ($\phi_1$), in case of $\phi_2 = 0$ and passive fiscal policy.

Figure 2: Determinacy and E-stability for different horizons ($T$) and different responses of monetary policy to inflation ($\phi_1$), in case of $\phi_2 = 0$ and active fiscal policy.

circle.

Figure 1 plots the determinacy region for the case where taxes respond sufficiently to debt (passive fiscal policy in the terminology of Leeper, 1991), and monetary policy does not respond to output gap ($\phi_2 = 0$). It can be seen in the figure, that in this case determinacy requires $\phi_1 > 1$, no matter how long or short agents’ planning horizons are. When $\phi_1 < 1$ there are not enough eigenvalues outside the unit circle, and there is indeterminacy. We further find that when the central bank also responds to output gap ($\phi_2 > 0$) the condition for determinacy on $\phi_1$ is slightly relaxed. This occurs (for all horizons) in line with the Taylor principle.

Figure 2 plots the case where taxes respond to debt only very weakly (fiscal policy is active in the terminology of Leeper). In this case the condition for determinacy is reversed and now requires $\phi_1 < 1$ for all horizons. When $\phi_1 > 1$ there are too many eigenvalues outside the unit circle and our system is explosive (there does not exists a rational expectations equilibrium satisfying the transversality condition).

We can conclude that the conditions for determinacy in our model do not depend on the horizon, and are in line with those of models with infinite planning horizons.
4.2 E-stability

Next, we consider the properties of our model when agents do not form expectations rationally, but instead are learning. For this, we turn to the concept of E-stability. When an equilibrium is E-stable, this implies that agents could learn the equilibrium by acting as econometricians and performing regressions on past data (recursive least squares learning).\textsuperscript{11}

We determine E-stability by numerically evaluating the Jacobian of our system at the minimum state variable solution (MSV) of our model. When all eigenvalues of this Jacobian are inside the unit circle then the MSV is E-stable, and when some eigenvalues are outside the unit circle then it is E-unstable. We find that for passive fiscal policy, the conditions for E-stability coincide with those of local determinacy, for all horizons. This is illustrated in Figure 1. When fiscal policy is active, no non-explosive MSV exists for active monetary policy. For passive monetary policy, we find that the MSV is E-stable as well as determinate, as is indicated in Figure 2.

5 Fiscal Consolidations

In this section we solve the model and simulate for two different experiments, namely tax-based and spending-based consolidations. First, we look at the effects of different planning horizons on the ability of each type of consolidation to stabilize debt and on the way each type of consolidation affects the real economy (Sections 5.3 and 5.4). Second, we fix the forecast horizon $T$ of our agents and look at the differences between the two types of consolidations. Additionally, we analyze the relative importance of the fiscal policy stance, as measured by the reaction of taxes or spending to debt ratio fluctuations, as well as the effect of the monetary policy stance (Section 5.5).\textsuperscript{12} Finally, we consider the role of relative risk aversion in determining the relative performance of spending- and tax-based consolidations (Section 5.6).

Before this analysis can be conducted, we need to specify how agents form expectations (Section 5.1), and how we calibrate the model parameters (Section 5.2).

\textsuperscript{11}More specifically, agents are assumed the know the correct functional form of the minimum state variable solution, and try to learn the parameters of the MSV solution by means of OLS. See Evans and Honkapohja (2012) and references therein for details.

\textsuperscript{12}Bi et al. (2013) show that the responsiveness of the interest rate to inflation fluctuations affects the ability of spending-based consolidations to stabilize debt while that of tax-based is unaffected.
5.1 Expectations

In Section 4, we discussed how our model behaves under rational expectations and under adaptive learning. In this section, we will assume that agents form expectations in a forward looking manner and hence stay relatively close to the assumption of rational expectations. However, we introduce some bounded rationality in the formation of agents’ expectations, in a way that is consistent with agents’ finite planning horizons. We assume that agents rationally use the model equations within their horizon to form model consistent expectations, but they are not able to form expectations for variables outside their horizon in a sophisticated manner. Furthermore, we take an anticipated utility approach, by assuming that agents form expectations and make decisions based on their current relative valuation of future wealth. That is, when they make a consumption plan, agents do not consider how they might update their $\hat{\Lambda}_t$ in the future. Because of the above mentioned bounded rationality, our agents know the model equations and the structural form of the minimum state variable solution of the model, but are not always fully correct in computing the parameters of this solution, as we describe below.

Agents start with forming expectations about the final period of their horizon. To do this, they take the model equations of period $t + T$. However, in the IS and Phillips curves of that period (Equations (24) and (25) forwarded $T$ periods), finite sums with expectations about period $t + T + 1$ up to period $t + T + T$ appear. That is, in the model equations they are considering, expectations of variables outside their planning horizon show up. Agents therefore need to give these expectations a value, without being able to solve in a sophisticated manner what will happen in these periods (since they lie outside their planning horizon). Instead, they assume that in periods after their horizon, the model will have converged to a steady state. With this assumption, they are able to solve for period $t + T$ variables in terms of the state variable $b_{t+T}$. They then move to the model equations of period $t + T - 1$. Here they plug in the solution of period $t + T$ variables as expectations, and again assume steady state levels for expectations of variables outside their horizon. They can then solve for period $t + T - 1$ variables in terms of state variable $b_{t+T-1}$. This process goes on until they have solved for all expectations within their horizon in terms of the observed state variable.

Assuming that agents do consider how they might update their relative valuation of bond holdings in the future would make them in a way "hyper-rational", as argued by Branch and McGough (2016).
Expectations of variables for the periods within the horizon can then be obtained from these policy functions by plugging in the value of the current debt level.

It is important to note that the steady state that agents expect the model to be in after their horizon depends on the valuation of future wealth, $\hat{\Lambda}^t_i$. Because of anticipated utility, agents calculate the steady state that would come about if agents value debt with $\hat{\Lambda}^t_i$ also in future periods. Therefore, when their valuation of future wealth, $\hat{\Lambda}^t_i$, is very high, agents expect to end up in a steady state with sub-optimally low consumption and high savings. In Appendix D we derive the steady state value of all variables as a function of $\hat{\Lambda}^t_i = \Lambda$, assuming that this parameter is held fixed.

When, in later periods, agents update their $\hat{\Lambda}^t_i$, they adjust their consumption and labor plans, so that realized dynamics can be quite different from what agents expected initially. When, however, $\hat{\Lambda}^t_i$ converges to its steady state value, the steady state that agents expect to be in after their horizon coincides with the actual steady state of the model. In that case agents will have (near) perfect foresight, as long as the model converges to (values close to) this steady state within the agents’ planning horizon.

Finally, note that it is important that agents know the current value of the parameter $\tilde{\tau}^{DT}_{t-1}$ in the tax rule, but that they do not know (or trust) that this parameter assures that debt equals its target in steady state. As we show in Appendix D, agents compute their perceived steady state using the model’s equations. If agents would know that $\tilde{\tau}^{DT}_{t-1}$ is such that the model has a steady state with debt equal to its target, then they could use this to infer the optimal relative valuation of end of horizon wealth, $\Lambda^t_i$, that is consistent with this steady state. This would allow them to behave almost as fully rational agents, at least as long as the model converges to this steady state relative quickly. Many of our results will however come from the bounded rationality of not being able ex-ante to value wealth outside the planning horizon optimally, and hence not having fully correct expectations about what happens after the horizon.

As we describe below, we consider the case where the government announces a change in its target for the debt ratio $DT_{t-1}$ and in the parameter $\tilde{\tau}^{DT}_{t-1}$ once and permanently. Therefore, once this happens, agents do not expect future changes in these parameters.
5.2 Parameterization

We now calibrate our model. We are interested in analyzing the effects of fiscal consolidations before, during and after implementation. We do not calibrate the model for a specific country. One period corresponds to one quarter. In our baseline calibration, we set the coefficient of relative risk aversion \( \sigma = 1 \), the inverse of the Frisch elasticity of labor supply \( \eta = 2 \), the elasticity of substitution \( \theta = 6 \) and the Calvo parameter \( \omega = 0.75 \). The nonlinear steady state government spending as a share of GDP is set at \( G/Y = 0.21 \), the steady state debt target and debt ratio are set to 0.7, which requires a steady state tax rate of \( \tau \approx 0.26 \). In the benchmark calibration, we set \( \phi_1 = 1.3 \) and \( \phi_2 = 0 \) in the interest rate rule, while we set \( \gamma^0_r = 0.05 \) in the tax rule. This combination of active monetary policy and passive fiscal policy implies that the model is determinate and E-stable, as can be seen in Figure 1. Finally, we set \( \gamma^{cons}_g = 0.3 \) and \( \gamma^{cons}_r = \frac{1}{\bar{w}} \gamma^{cons}_g = 0.36 \). In Section 5.5 we also consider what happens if fiscal and/or monetary authorities respond more aggressively to debt and inflation.

5.3 Spending-based consolidations

In this section we proceed with the following experiment. We assume that initially the model is in the steady state consistent with a positive debt target (\( \tilde{DT}_t = 0.05 \), which corresponds to a debt ratio of 75% of GDP). Suddenly, the government decides that debt is too high, and lowers its debt target to \( \tilde{DT}_t = 0 \) (which corresponds to a debt ratio of 70% of GDP) permanently. In the absence of consolidations, this would imply a very slow transition to a new steady state consistent with the lower debt target. Since the government wants to bring the debt ratio towards its target faster, the government additionally announces that it will start implementing consolidations in the next period, and will continue to do so for 12 quarters. The permanent drop of the publicly known debt target is therefore unanticipated. But once the drop has taken place, the imminent consolidation is anticipated. Moreover, we assume that the government credibly announces the nature of the consolidation (i.e. spending-based in this case) so that there is no uncertainty regarding the composition. In Appendix E.1 we consider what happens if the private sector initially does not know the composition of the

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\[14\] In order to make tax-based and spending-based consolidations comparable we set \( \gamma^{cons}_r = \frac{1}{\bar{w}} \gamma^{cons}_g \) so that the effect on tax income rather than the tax rate is the same as the effect on spending. Spending and tax-based consolidations then have an equal (direct) impact on the government budget constraint, as can be seen in Equation (B.4) in Appendix B.3.
upcoming consolidations.

In Figure 3 we plot the transition paths of the variables from the steady state with $\bar{DT}_t = 0.05$ to the steady state with $\bar{DT}_t = 0$ in case of spending-based consolidations, for different values of the planning horizon. The blue curves in Figure 3 depict the response to a drop in the debt target when $T = 3$, while the green curves depict the case of $T = 8$. The yellow curves show dynamics when $T = 100$, while the thin, black curve depicts the case of infinite planning horizons and fully rational expectations. In this case, the model can be written recursively, which is shown in Appendix C. For all variables, the yellow and thin, black curves overlap and are indistinguishable from each other. This implies that when agents
have planning horizons of 100 periods in our model, their actions are (almost) the same as if they had solved an infinite horizon optimization problem with rational expectations. The effects of consolidations in an economy where agents have long but finite planning horizons are therefore (almost) the same as in an economy with fully rational agents with an infinite planning horizon.

Let us first compare the initial with the new steady state values of debt, consumption, output, inflation and the nominal interest rate, and turn to the transition paths implied by the consolidations later. For $T = 3$ the model does not yet reach the new steady state within the 25 periods that are plotted (as will be discussed below). Therefore, we make a steady state comparison by comparing the initial and final values of the other three curves in Figure 3. The lowering of the debt target implies a steady state with permanently lower debt, and therefore a lower interest rate burden for the government. In the new steady state the government can therefore afford lower taxes. These lower taxes imply that agents have more income and consume more, which also leads to a slight increase in output. Regarding inflation, the following occurs. The lower taxes increase households’ marginal return of labor and labor supply. At the same time, the increase in consumption increases labor demand. Therefore, wages and marginal costs do not change, and the steady state inflation and interest rates are not affected by the drop in the debt target.

Let us now look at the transition paths towards the new steady state implied by spending-based consolidations, for different horizons. Regardless of the horizon, as soon as the debt target is lowered, agents realize that next period spending-based consolidations will start. They furthermore realize that the fall in government spending will lower aggregate demand and induce firms to lower both prices and labor demand. The latter leads to a decrease in wages, which will further lower inflation. Since firms expect low inflation in the periods to come, they already lower their prices in the period where consolidations are announced, hence the immediate drop in inflation.

Meanwhile, consumers expect a decrease in the nominal and real interest rate due to the periods of low inflation. However, the way they react to this knowledge depends crucially on their planning horizon. When agents have a longer planning horizon, their consumption decision is to a large extent driven by expectations about their disposable income in the future and about future real interest rates. If they expect lower future interest rates this leads them
to consume more today. However, for shorter horizons, agents’ consumption depends less on their expectations about periods within their planning horizon, and more on their current wealth and valuation of future wealth. Agents with shorter planning horizons realize that the consolidations will decrease their bond holdings in the coming periods, but do not immediately realize how the drop in government debt should affect their valuation of future wealth ($\hat{\Lambda}_t^i$). The resulting sub-optimally high value of $\hat{\Lambda}_t^i$ leads them to expect to end up in a steady state with relatively lower consumption, higher debt and higher taxes, compared to a fully rational agent (see Appendix D). Therefore, they plan to save more and consume less, not only in the current period, but also in subsequent periods, than agents with a longer planning horizon.

In Figure 3, it can be seen that consumption both in the case of a planning horizon of 100 periods and in the case of an infinite planning horizon (the yellow/black curves), goes up considerably in the period of the announcement. For $T = 3$ (blue) instead, we see that consumption goes up much less. For a planning horizon of three periods, the effects of a fall in next period’s wealth and an overvaluation of future wealth, as described above, thus dominate, leading to relatively low private consumption. For $T = 8$ (green), agents also consume somewhat less than for $T = 100$, but considerably more than for $T = 3$.

As consolidations are implemented and subsequently decrease in magnitude, under all horizons, consumption jumps on impact, and then slowly goes down. Note that since agents do not value future wealth correctly yet during this transition, agents with a planning horizon of $T = 3$ and $T = 8$ expect that in the new steady state, they will consume less. For this reason, the green curve, and especially the blue curve, initially seem to converge to a lower consumption level than the yellow and the thin black ones. Interestingly, after 2 periods the spending cuts imply for $T = 3$ that consumption is lower than its original steady state value for an extended period of time. This is in line with the empirical evidence on the positive co-movement between private consumption and government spending (see Blanchard and Perotti (2002), Fatas and Mihov (2001) and Gali et al. (2007) among others).

Over time, agents start to update their valuation of future wealth, $\hat{\Lambda}_t^i$, and all lines converge to the same steady state eventually. For $T = 3$, the fall in private consumption below the steady state prolongs the recession, harming thereby the performance of the consolidation. As a result, it takes longer (more than 25 quarters) until the debt ratio converges to the new target, which keeps spending persistently low. The persistent consolidation in this case delays
the recovery further and the economy converges slowly to the new steady state.

Output dynamics are mainly dominated by government spending. However, for longer horizons output initially goes up considerably because of the increase in consumption in the anticipation period. Meanwhile, inflation dynamics also differ considerably for different horizons. Expectations about lower future bond holdings strongly affect inflation dynamics for shorter horizons. In particular, because of the wrong value of $\hat{\Lambda}_t$, firms anticipate the economy to reach a state with lower consumption, and lower output, marginal cost and inflation. This implies that firms already lower prices more today. Moreover, the lower consumption path for shorter horizons lowers the inflation path even more. As agents start updating $\hat{\Lambda}_t$, and eventually learn to value debt correctly in their new environment, inflation (just like consumption) starts converging to the same level as under a longer (or infinite) planning horizon.

Finally, we consider debt dynamics. As can be seen in Figure 3 debt is consolidated at the same rate when $T = 8$ as when $T = 100$ or when the horizon is infinite. We can therefore conclude for $T = 8$, that the shrink of the tax base due to relatively lower output is offset by the drop in debt service costs that are implied by the relatively lower level of the real interest rate. This is not the case for $T = 3$, where the fall in output is too large, and debt is reduced more slowly.

5.4 Tax-based consolidations

Now we turn to the case of tax-based consolidations. We conduct the same experiment as above with an unanticipated drop in the debt target followed by an anticipated consolidation after one quarter. Now, consolidations are implemented for 12 quarters through tax hikes only. The transition of the variables towards the new steady state is displayed in Figure 4.

In the case of tax-based consolidations, an increase in taxes leads to a lower marginal return of labor for the household, causing it to decrease its labor supply. Since agents try to smooth consumption and since the tax increases do not lead to a very large loss in disposable income, agents do not lower their consumption by much. Aggregate demand and hence labor demand are therefore lowered by much less than labor supply. As a consequence, the tax increase causes a strong increase in real wages, which puts upward pressure on inflation. In the period before consolidations, consumers already anticipate slightly lower future consumption and
Figure 4: Transitions to new steady state following a drop in the debt target from 75% of GDP ($DT_t = 0$) to 70% of GDP ($DT_t = 0.05$) and tax-based consolidations, for different horizons. In particular, $T = 3$ (blue), $T = 8$ (green), $T = 100$ (yellow) and infinite horizon rational expectations (thin black). The last 2 curves overlap.
decrease their consumption in anticipation of the tax increases. This result holds regardless of the planning horizon.

Comparing the paths for different planning horizons in Figure 4, consumption falls more, the shorter the horizon, both upon anticipation of the imminent consolidation and upon implementation. This is again because short-sighted households start valuing their future wealth too much, in the face of upcoming consolidations. The larger fall in consumption for short horizons more than offsets the upward pressures on inflation due to higher marginal costs, leading to deflation. Since short-sighted firms expect to end up in a steady state where consumers consume less, they also expect to end up in a steady state with lower output and inflation. This leads them to lower prices already today which amplifies and moves forward the drop in inflation. When, in the next period, consolidations are implemented, marginal costs increase considerably, pushing up inflation somewhat. However, as taxes and wages start to go down again, the low expected future inflation and output dominate, and inflation falls.

For longer horizons though, households decrease their consumption to a lesser extent, since they base their consumption decision less on current wealth and on the assumed steady state after the horizon, but instead more on expected disposable income in the future periods within their horizon. Moreover, households better account for the fact that taxes will be permanently lower in the future. This leads them to reduce consumption less than agents with shorter horizons. In this case, the demand channel (lower consumption) is weaker than the supply channel (increase in marginal costs due to higher taxes), causing a jump in inflation before and upon implementation. The different interaction between the demand and the supply channel according to the horizon, also explains the differences in the monetary policy stance. Monetary policy is expansionary for short horizons and contractionary for longer horizons.

As with spending-based consolidations, the yellow curves for $T = 100$ are almost indistinguishable from the responses depicted by the thin, black curves with infinite horizon rational expectations. Furthermore, as short-sighted agents after some periods learn to value wealth correctly, all variables converge to the same steady state levels as when agents have an infinite planning horizon.

Finally, again as in the case of spending-based consolidations, for $T = 3$ output falls by so much that the resulting drop in the tax base causes consolidations to be less effective in
terms of stabilizing debt. For $T = 8$ however, the drop in the tax base is much smaller, and fully offset by lower real interest payments on existing debt. This causes debt to fall at the same rate as in the case of an infinite planning horizon.

### 5.5 Spending- versus tax-based consolidations

In this section, we compare the effects of the two types of consolidations under different monetary and fiscal policy stances. We will calculate present value multipliers of tax-based and spending-based consolidations and compare the impulse responses. We do this for the benchmark calibration used in Sections 5.3 and 5.4, but also for the cases of stronger consolidations.
and more aggressive monetary policy. Because we believe that the multipliers already tell most of the story, we limit the number of figures by only plotting impulse responses for the case of $T = 8$. As we saw in Sections 5.3 and 5.4, this case has representative features of both shorter and longer planning horizons.

Since we are interested in the effects of consolidations and not in the effects that a lowering of the debt target would have even in the absence of consolidations, we control for the latter when calculating multipliers and impulse responses. In particular, we normalize by subtracting in each period the value that a variable would have taken without consolidations from the value that it takes with the consolidation.

5.5.1 Benchmark case

Figure 5 compares the impulse responses of spending-based and tax-based consolidation for the case of $T = 8$ under the benchmark calibration that was used in Section 5.3 and 5.4. In the figure, it can be seen that upon anticipation variables react differently under the two consolidation plans. Under spending-based consolidations output goes up, leading to a drop in the debt ratio, while under tax-based consolidations output contracts, leading to an increase in debt. However, the big recession that follows when spending cuts are implemented makes spending-based consolidations relatively less effective in stabilizing debt in the medium to long run.

This is also reflected in the present value multipliers of tax-based and spending-based consolidations. Following Mountford and Uhlig (2009) and Bi et al. (2013), we calculate present value multipliers as follows

$$
\Gamma^y_{t+k} = \sum_{j=0}^{k} \left( \prod_{i=0}^{j} (r_{t+i} - 1) \right) \frac{\left( \hat{Y}_{c t+j} - Y_{nc t+j} \right)}{\sum_{j=0}^{k} \left( \prod_{i=0}^{j} (r_{t+i} - 1) \right) \left( \tilde{x}_{c t+j} - x_{nc t+j} \right)},
$$

where $r_t$ is the real interest rate, and $x$ denotes the type of fiscal consolidation: $x_t = \tau_t \bar{w} \bar{H}$ for tax-based and $x_t = -G_t$ for spending-based. $15$ $Y^c_{c t}$ and $x^c_t$ indicate values taken when there

15We multiply taxes by $\bar{w} \bar{H}$ to get a change in tax income due to a changed tax rate, rather than the change in the tax rate itself. This facilitates compatibility with changes in government expenditure. Using the definitions of (log)-linearized variables, we can calculate the tax-based multiplier as $\Gamma^y_{t+k} = \sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( \hat{Y}_{c t+j}^c - \hat{Y}_{nc t+j}^c \right) / \sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( \bar{w} (\tilde{\tau}_{t+j}^c - \tilde{\tau}_{nc t+j}^c) \right)$ and the spending-based multiplier as $\Gamma^g_{t+k} = \sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( \hat{Y}_{c t+j}^c - \hat{Y}_{nc t+j}^c \right) / \sum_{j=0}^{k} \left( \prod_{i=0}^{j} r_{t+i}^{-1} \right) \left( \tilde{g}_{c t+j}^c - \tilde{g}_{nc t+j}^c \right)$.
are consolidations, and \(Y_t^{nc}\) and \(x_t^{nc}\) indicate values that would have occurred in the absence of consolidations.

Panel a: \(\phi_1 = 1.3, \gamma_{cons}^{g} = 0.3\)

<table>
<thead>
<tr>
<th>(\Delta Y/\Delta \tau)</th>
<th>1qr</th>
<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T = 3)</td>
<td>-0.82</td>
<td>-0.88</td>
<td>-0.90</td>
<td>-0.87</td>
</tr>
<tr>
<td>(T = 8)</td>
<td>-0.61</td>
<td>-0.58</td>
<td>-0.52</td>
<td>-0.46</td>
</tr>
<tr>
<td>(T = 100)</td>
<td>-0.58</td>
<td>-0.54</td>
<td>-0.50</td>
<td>-0.45</td>
</tr>
</tbody>
</table>

Panel b: \(\phi_1 = 1.3, \gamma_{cons}^{g} = 0.6\)

<table>
<thead>
<tr>
<th>(\Delta Y/\Delta \tau)</th>
<th>1qr</th>
<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T = 3)</td>
<td>-0.83</td>
<td>-0.89</td>
<td>-0.90</td>
<td>-0.87</td>
</tr>
<tr>
<td>(T = 8)</td>
<td>-0.71</td>
<td>-0.68</td>
<td>-0.62</td>
<td>-0.55</td>
</tr>
<tr>
<td>(T = 100)</td>
<td>-0.69</td>
<td>-0.66</td>
<td>-0.61</td>
<td>-0.56</td>
</tr>
</tbody>
</table>

Panel c: \(\phi_1 = 1.8, \gamma_{cons}^{g} = 0.3\)

<table>
<thead>
<tr>
<th>(\Delta Y/\Delta \tau)</th>
<th>1qr</th>
<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T = 3)</td>
<td>-0.65</td>
<td>-0.66</td>
<td>-0.62</td>
<td>-0.53</td>
</tr>
<tr>
<td>(T = 8)</td>
<td>-0.50</td>
<td>-0.44</td>
<td>-0.35</td>
<td>-0.24</td>
</tr>
<tr>
<td>(T = 100)</td>
<td>-0.52</td>
<td>-0.48</td>
<td>-0.52</td>
<td>-0.37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\Delta Y/\Delta \tau)</th>
<th>1qr</th>
<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T = 3)</td>
<td>-0.47</td>
<td>-0.54</td>
<td>-0.59</td>
<td>-0.60</td>
</tr>
<tr>
<td>(T = 8)</td>
<td>-0.32</td>
<td>-0.29</td>
<td>-0.22</td>
<td>-0.12</td>
</tr>
<tr>
<td>(T = 100)</td>
<td>-0.33</td>
<td>-0.32</td>
<td>-0.30</td>
<td>-0.26</td>
</tr>
</tbody>
</table>

**Table 1: Output Multipliers**

In Panel (a) of Table 1, the impact multiplier and the cumulative multipliers after 4, 8 and 12 quarters under the benchmark calibration are displayed for both spending-based and tax-based consolidations and for different horizons. It can be seen that for \(T = 8\) tax-based multipliers are considerably lower than spending-based ones, both on impact and after 4, 8 and 12 quarters.\(^{16}\) This reflects the deeper recession that arises under spending-based consolidations, observed in Figure 5. In this figure, it can furthermore be seen that output recovers quickly relative to government spending and taxes, and even turns positive under tax-based consolidations. This is also reflected in the present value multipliers of both type of consolidations in Table 1, which fall in absolute value as the number of periods over which they are calculated increase (from 1, to 4, to 8, to 12 quarters).

Turning to the case of \(T = 100\), it can be seen in Panel (a) of Table 1 that the impact multipliers of both spending- and tax-based consolidations are slightly less negative than those obtained under \(T = 8\). This is consistent with the smaller drop in output for \(T = 100\) than for \(T = 8\) that can be observed in Figures 3 and 4.

\(^{16}\)Note that the impact multiplier considers the period where consolidations are implemented for the first time. The period where consolidations are anticipated but not yet implemented, in which output goes up under anticipated spending-based consolidations but falls under anticipated tax-based consolidations, is therefore not included in the multipliers.
Finally, turning to the Multipliers for $T = 3$ in Panel (a) of Table 1, they are much more negative than those of $T = 8$ and $T = 100$. Furthermore, unlike in those two cases, present value multipliers now become more negative rather than less negative as they are calculated over more quarters. We can conclude that when agents have very short planning horizons the consolidations are quite harmful to output in both the short- and the medium-run. This is because agents then care more about their current wealth, while being wrong about the effects of consolidations on the present value of their future wealth.

### 5.5.2 Stronger consolidations

Now, we consider the case where consolidations are stronger. In particular, we increase the consolidation coefficients in the spending and tax rule to $\gamma^{cons}_g = 0.6$ and $\gamma^{cons}_\tau = \frac{1}{\bar{w}} \gamma^{cons}_g = 0.72$. We plot impulse responses for the case of $T = 8$ in Figure 6. Comparing this figure with Figure 5, it can be seen that the cut in spending and the increase in taxes when consolidations start are more than twice as large now. For tax-based consolidations this leads to a much faster reduction in debt. When consolidations are spending-based, the larger spending cuts lead to a larger drop in output, which reduces the tax base and makes the consolidation less effective.

Looking at panel (b) of Table 1, we see that for $T = 8$ spending multipliers are more negative than those of the benchmark calibration in panel (a). However, for tax-based consolidations multipliers are less negative than in the benchmark case. Moreover, the present value multiplier after 12 quarters turns positive now. This means that under stronger tax-based consolidations, taxes are increased by more, but that the extra output losses that these higher taxes imply are limited. This is indeed what is observed in Figure 6, and this also explains why debt falls more quickly in this figure.

Next, we turn to the multipliers for the case of $T = 3$ and $T = 100$ in panel (b) of Table 1. For $T = 100$ all multipliers are again slightly less negative than for $T = 8$. For $T = 3$ we see that spending multipliers do not change significantly compared to the benchmark case, but that tax-based multipliers are less negative. In fact, tax-based consolidations become expansionary 8 quarters after the start of their implementation.

We can conclude that when the government implements tax-based consolidations it seems desirable to do this with large tax hikes. For all horizons, this leads to a faster reduction in
Figure 6: Impulse responses of spending-based (green) and tax-based (black) consolidations for $T = 8$, in case of stronger consolidations. We now have $\gamma_{cons}^{g} = 0.6$ instead of $\gamma_{cons}^{g} = 0.3$.
debt without leading to significantly lower output. However, when the government implements spending cuts, doing so too strongly may be quite harmful to the economy, especially for longer horizons. Hence more gradual spending cuts may be preferable.

5.5.3 More aggressive monetary policy

Next, we turn to the case where the central bank responds more aggressively to inflation. Now $\phi_1 = 1.8$ in the Taylor rule. Figure 7 plots the corresponding impulse responses for the case of $T = 8$. Since spending-based consolidations imply deflation, a more aggressive response to inflation implies an even lower real interest rate, which induces agents to consume more and hence limits the drop in output. This in turn implies a smaller fall in the tax base, making the consolidation more effective in reducing debt. In contrast, tax-based consolidations are slightly inflationary, so that more aggressive monetary policy leads to a larger drop in output, making the consolidation less effective, in the short-run, in stabilizing debt. Importantly, we see in Figure 7 that the two types of consolidations are now very similar in terms of the induced output losses, and that the initial benefits that arise under spending consolidations because of the debt reduction in the anticipation period persist for much longer than in the benchmark case.

Turning to the multipliers in panel (c) of Table 1, we see that for all horizons spending-based multipliers are less negative than in the benchmark case, as expected. For tax-based consolidations it can be seen that for $T = 8$ and $T = 100$ multipliers are more negative under more aggressive monetary policy, but that they become less negative when $T = 3$. The reason for that, is that tax-consolidations are deflationary rather than inflationary when agents are very short sighted, as we showed in section 5.4. More aggressive monetary policy hence decreases the real interest rate which allows for a faster recovery of private consumption for very short horizons.

5.6 The role of relative risk aversion

While the calibration of most of model parameters are not very important for our qualitative result, there is a parameter that crucially determines the relative performance of spending- and tax-based consolidations. This parameter is the relative risk aversion, $\sigma$. In this section, we show how dynamics change under both spending-based and tax-based consolidations when
Figure 7: Impulse responses of spending-based (green) and tax-based (black) consolidations for $T = 8$, in case of more aggressive monetary policy. We now have $\phi_1 = 1.8$ instead of $\phi_1 = 1.3$. 
Figure 8: Impulse responses of spending-based (green) and tax-based (black) consolidations for $T = 8$, in case of lower relative risk aversion. We now have $\sigma = 0.157$ instead of $\sigma = 1$.

we change the relative risk aversion.$^{17}$

Figure 8 plots spending- and tax-based consolidations for $T = 8$ under the benchmark calibration with $\phi_1 = 1.3$ and $\gamma_{cons} = 0.3$, but now with a relative risk aversion of $\sigma = 0.157$ (as in e.g. Woodford (1999)) rather than $\sigma = 1$ (as in e.g. Clarida et al. (2000)). First focusing on spending-based consolidations (green), it can be seen that the recession caused by consolidations is much milder than in Figure 5. The reason for this is that agents with a lower relative risk aversion smooth consumption less, and therefore respond more strongly to temporary changes in their expected future disposable income and in expected future real

$^{17}$Note that the changes in dynamics discussed in this section arise almost solely due to the change of the relative risk aversion parameter in the utility function $u(C_{i,t}, H_{i,t})$. When we only change the relative risk aversion parameter in the value function $V^{i,t}(x)$ (given in Equation (3)) dynamics are hardly affected.
interest rates. Hence, agents with a low relative risk aversion increase their consumption much more in response to the anticipated low future real interest rates implied by spending cuts. Output then goes down by less, which causes the tax base to remain relatively high, so that debt drops faster.

Next we turn to tax-based consolidations (black). It can be seen that for tax-based consolidations output falls more and debt is reduced more slowly than in Figure 5. The intuition for this is similar to the case of spending-based consolidators. Agents with lower relative risk aversion care less about smoothing consumption and hence reduce their current consumption more when they anticipate tax increases to imply a lower labor income in the next few periods. This results in a bigger recession, less tax income for the government, and hence slower debt reduction.

Comparing spending- and tax-based consolidations in Figure 8, we see that the recession under spending-based consolidations is now considerably milder than under tax-based consolidations, and that this results in spending-based consolidations being more effective in reducing debt both in the short run and in the medium run.

This is also reflected in the multipliers in Panel a of Table 2. In this table we reproduce the multipliers of Table 1, but with $\sigma = 0.157$ instead of $\sigma = 1$. It can be seen in Panel a for the case of $T = 8$ that the tax-based multipliers are much less negative than in Table 1 and that the present value multiplier after 12 quarters even turns positive. For tax-based

<table>
<thead>
<tr>
<th>Panel a: $\phi_1 = 1.3, \gamma_{cons}^{\text{g}} = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\Delta V}{\Delta g}$</td>
</tr>
<tr>
<td>$T = 3$</td>
</tr>
<tr>
<td>$T = 8$</td>
</tr>
<tr>
<td>$T = 100$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel b: $\phi_1 = 1.3, \gamma_{cons}^{\text{g}} = 0.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\Delta V}{\Delta g}$</td>
</tr>
<tr>
<td>$T = 3$</td>
</tr>
<tr>
<td>$T = 8$</td>
</tr>
<tr>
<td>$T = 100$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel c: $\phi_1 = 1.8, \gamma_{cons}^{\text{g}} = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\Delta V}{\Delta g}$</td>
</tr>
<tr>
<td>$T = 3$</td>
</tr>
<tr>
<td>$T = 8$</td>
</tr>
<tr>
<td>$T = 100$</td>
</tr>
</tbody>
</table>

Table 2: Output Multipliers for low relative risk aversion ($\sigma = 0.157$)
consolidations on the other hand, multipliers are much more negative than in case of a $\sigma = 1$. It can further be seen by comparing the first panel of Table 2 and 1, that for $T = 3$ and $T = 100$ multipliers change in a similar way. Again multipliers of $T = 100$ are slightly less negative than those of $T = 8$ and multipliers of $T = 3$ are more negative than those of both other cases.

Finally, looking at panels b and c of Table 2 and comparing with Table 1, it can be seen that as we make consolidations stronger or monetary policy more aggressive, multipliers change in a similar way as for the case of higher relative risk aversion. These qualitative results therefore are robust to the calibration of $\sigma$.

6 Conclusions

In this paper we analyze the effects of fiscal consolidations when agents are infinitely lived, but optimize over a finite number of periods. This is because agents do not have the cognitive ability to form expectations and make rational inferences over an infinite horizon. Moreover, agents are uncertain about the correct valuation of their wealth at the end of their planning horizon and learn about the correct valuation using a constant gain learning mechanism. The government uses either spending or taxes periodically in order to consolidate debt when the latter is too far above its target. Agents are aware of the upcoming consolidations once this happens and adjust their expectations accordingly. Hence, our model can also capture anticipation effects.

Following a consolidation, we show that the magnitude of the responses of inflation, output and private consumption is sensitive to the agents’ planning horizon. More importantly, we show that for short horizons the effects on output, inflation and consumption change qualitatively as well. In particular, for short planning horizons private consumption falls following spending-based consolidations. This result is in line with empirical evidence on the positive co-movement between private consumption and government spending. On the contrary, as we increase the planning horizon of households, the response of consumption changes sign, resulting in a negative co-movement with government spending, in line with one of the weaknesses of standard neoclassical models with agents optimizing over an infinite horizon.

The virtue of our framework is that it captures the finite horizon aspect of life and the
time inconsistency of consumption plans. Consequently, it does not generate the strong wealth effects inherent in standard neoclassical models where agents optimize and form expectations over the infinite future. In those models, such wealth effects imply correlations between macro variables that are not consistent with the actual data and as such they may lead to the wrong conclusions regarding the effects of consolidations.

We find that in our benchmark calibration tax-based consolidations perform better than spending-based consolidations, both in terms of their effectiveness in reducing debt and in terms of output losses. However, we show that the performance of spending-based consolidations can improve considerably when we increase the aggressiveness of monetary policy in its reaction to inflation fluctuations. Moreover, increasing the strength of consolidations seems to make tax-based consolidations more effective in reducing debt without leading to larger output losses, while the opposite holds for spending-based consolidations. Interestingly, we show that for low degrees of relative risk aversion spending-based consolidations outperform tax-based resulting in lower costs in terms of output losses and a faster reduction of debt. In fact, this is in line with empirical evidence as regards the output costs of fiscal consolidations.
A Steady state

In this section, we derive the steady state around which the model is log-linearized, where gross inflation equals 1.

Evaluating (16) at the zero inflation steady state gives

\[ \bar{mc} = \frac{\theta - 1}{\theta}. \]  
(A.1)

From the first order conditions of the households it follows that in this steady state we must have

\[ 1 + \bar{i} = \frac{1}{\beta}. \]  
(A.2)

Furthermore, normalizing technology \( A \), to 1, it follows from (10) that

\[ \bar{H} = \bar{Y}. \]  
(A.3)

Next, we solve the steady state aggregate resource constraint, (20), for consumption, and write

\[ \bar{C} = \bar{Y}(1 - \bar{g}). \]  
(A.4)

Plugging in these steady state labor and consumption levels in the steady state version of (5) gives

\[ \bar{w} = \frac{\bar{Y}^\eta (\bar{Y}(1 - \bar{g}))^\sigma}{1 - \bar{\tau}} = \frac{\bar{Y}^\eta \sigma (1 - \bar{g})^\sigma}{1 - \bar{\tau}} = \frac{\theta - 1}{\theta}. \]  
(A.5)

Where the last equality follows from \( \bar{mc} = \bar{w} \) and (A.1). We can thus write

\[ \bar{Y} = \left( \frac{\theta - 1}{\theta} \frac{1 - \bar{\tau}}{(1 - \bar{g})^\sigma} \right)^{\frac{1}{\eta + \sigma}}. \]  
(A.6)

Then we turn to the government budget constraint. In steady state, (19) reduces to

\[ \bar{b} = \frac{\bar{\tau} \bar{w} - \bar{g}}{1 - \beta} = \frac{(\bar{\tau} \frac{\theta - 1}{\theta} - \bar{g})}{1 - \beta}, \]  
(A.7)

where we used (A.2) to substitute for the interest rate.
Steady state government spending and taxes are given by

\[ \bar{g} = g_1 - \zeta \gamma_{g}^{con} (\bar{b} - DT), \quad (A.8) \]

and

\[ \bar{\tau} = \bar{\tau}^{DT} + (\gamma_{\tau}^0 + (1 - \zeta) \gamma_{\tau}^{con}) (\bar{b} - DT), \quad (A.9) \]

where we assume that \( \bar{\tau}^{DT} \) is chosen such that steady state debt equals the steady state debt target. It follows from Equation (A.7) that this requires

\[ \bar{\tau}^{DT} = \theta \left( \frac{1 - \beta}{\theta - 1} (1 - \beta)DT + \bar{g} \right). \quad (A.10) \]

We then have \( \bar{g} = g_1 \) and \( \bar{\tau} = \bar{\tau}^{DT} \), as well as \( \bar{b} = \bar{b}DT \).

Finally we, assume that households value real bond holdings relative to consumption such that, in steady state, they make optimal decisions. It then follows from (9) and (A.7) that

\[ \bar{\Lambda} = \left( \frac{\bar{b}}{1 - \bar{g}} \right)^{\sigma} = \left( \frac{(\bar{\tau}^{\theta - 1} - \bar{g})}{(1 - \bar{g})(1 - \beta)} \right)^{\sigma}. \quad (A.11) \]

B Log-linear model

B.1 Optimal consumption decision

The log-linearized optimality conditions (including budget constraints) are given by

\[ \hat{C}^i_s = \hat{C}^i_{s+1} - \frac{1}{\sigma} (E^i_s \hat{\pi}_s - E^i_{s+1} \hat{\pi}_{s+1}), \quad s = t, t + 1, \ldots, t + T - 1, \quad (B.1) \]

\[ \tilde{b}^i_{t+T+1} = \bar{b} \hat{C}^i_{t+T} + \frac{\bar{b}}{\sigma} E^i_{t+T} \hat{\pi}_{t+T} + \frac{\bar{b}}{\sigma} \hat{\Lambda}^i_t, \quad (B.2) \]

\[ \eta \hat{H}^i_s = -\sigma \hat{C}^i_s - \frac{E^i_{t+T} \hat{\pi}_s}{1 - \bar{\tau}} + E^i_t \hat{\pi}_s, \quad s = t, t + 1, \ldots, t + T, \quad (B.3) \]

\[ \tilde{b}^i_{s+1} = \frac{\bar{w}}{\beta} ((1 - \bar{\tau})(E^i_t \hat{\pi}_s + \hat{H}^i_s) - E^i_{t+1} \hat{\pi}_s) + \frac{1}{\beta} \tilde{b}^i_s + \tilde{b}(i_s - \frac{1}{\beta} E^i_t \hat{\pi}_s) + \frac{\bar{w}}{Y} E^i_t \hat{\pi}_s - \frac{1 - \bar{g}}{\beta} \hat{C}^i_s, \quad (B.4) \]

\[ s = t, t + 1, \ldots, t + T, \]
where we used that $\bar{H} = \bar{Y}$ and $\frac{\bar{C}}{\bar{Y}} = 1 - \bar{g}$.

Iterating the log-linearized budget constraints from period $t+T$ backward and multiplying both sides by $\beta^{T+1}$ gives

$$
\beta^{T+1} \hat{b}_t^{i+1} = \hat{b}_t^{i+1} - (1 - \bar{g}) \sum_{s=0}^{T} \beta^s (\hat{C}_t^{i+s}) + \frac{\bar{\pi}}{\bar{Y}} \sum_{s=0}^{T} \beta^s (E_t^{i+s} \hat{\pi}_s^{i+s})
$$

$$
+ \frac{\bar{b}}{\beta} \sum_{s=0}^{T} \beta^s (\beta E_t^{i+s} \hat{\tau}_s^{i+s}) + \hat{w} \sum_{s=0}^{T} \beta^s ((1 - \bar{\tau})(E_t^{i+s} \hat{\pi}_t^{i+s} + \bar{H}_t^{i+1} - E_t^{i+s} \hat{\tau}_t^{i+s})).
$$

We can plug in $\hat{b}_t^{i+1}$ from (B.2) and labor from (B.3) to get

$$
\beta^{T+1} \hat{b}_t^{i+1} \hat{C}_t^{i+1} + \beta^{T+1} \frac{\bar{b}}{\sigma} E_t^{i+1} \bar{\pi}_t^{i+1} + \beta^{T+1} \frac{\bar{b}}{\sigma} \hat{\lambda}_t^{i+1} = \hat{b}_t^{i+1} - (1 - \bar{g}) \sum_{s=0}^{T} \beta^s (\hat{C}_t^{i+s}) + \frac{\bar{\pi}}{\bar{Y}} \sum_{s=0}^{T} \beta^s (E_t^{i+s} \hat{\pi}_s^{i+s})
$$

$$
+ \frac{\bar{b}}{\beta} \sum_{s=0}^{T} \beta^s (\beta E_t^{i+s} \hat{\tau}_s^{i+s}) + \hat{w} \sum_{s=0}^{T} \beta^s ((1 - \bar{\tau})(E_t^{i+s} \hat{\pi}_t^{i+s} + \bar{H}_t^{i+1} - E_t^{i+s} \hat{\tau}_t^{i+s})).
$$

Next we use the Euler equation to substitute for future consumption. Iterating the Euler equation gives

$$
\hat{C}_t^{i+s} = \hat{C}_t^{i} + \sum_{j=0}^{s-1} \frac{1}{\sigma} (E_t^{i+j} - E_t^{i+j+1}), \quad T - s \geq 1.
$$

Rearranging (B.5) and substituting for future consumption gives

$$
\beta^{T+1} \frac{\bar{b}}{\sigma} \hat{C}_t^{i+1} + \sum_{j=0}^{T-1} \frac{1}{\sigma} (E_t^{i+j} - E_t^{i+j+1}) + \beta^{T+1} \frac{\bar{b}}{\sigma} E_t^{i+1} \bar{\pi}_t^{i+1} + \beta^{T+1} \frac{\bar{b}}{\sigma} \hat{\lambda}_t^{i+1} = \hat{b}_t^{i+1} - (1 - \bar{g}) \sum_{s=0}^{T} \beta^s (\hat{C}_t^{i+s})
$$

$$
+ \frac{\bar{b}}{\beta} \sum_{s=0}^{T} \beta^s (\beta E_t^{i+s} \hat{\tau}_s^{i+s}) + \frac{\bar{\pi}}{\bar{Y}} \sum_{s=0}^{T} \beta^s (E_t^{i+s} \hat{\pi}_s^{i+s}) + \hat{w} \sum_{s=0}^{T} \beta^s ((1 - \bar{\tau})(E_t^{i+s} \hat{\pi}_t^{i+s} - E_t^{i+s} \hat{\tau}_t^{i+s})).
$$
Taking contemporaneous consumption to one side of the equation gives the current decision of consumer $i$

$$
\left( \beta^{T+1} \hat{b}_i + \left( \frac{\sigma}{\eta} \tilde{w}(1 - \tilde{\tau}) + (1 - \tilde{g}) \right) \frac{1 - \beta^{T+1}}{1 - \beta} \right) \hat{C}_i = \quad (\text{B.8})
$$

$$
- \beta^{T+1} \frac{\bar{\Lambda}_i + \hat{b}_i}{\sigma} + \tilde{w} \sum_{s=0}^{T} \beta^s ((1 + \frac{1}{\eta})((1 - \tilde{\tau})E_t^i \tilde{w}_{t+s} - E_t^i \tilde{\tau}_{t+s}) + \frac{\bar{\bar{\gamma}}}{\bar{\bar{Y}}} \sum_{s=0}^{T} \beta^s (E_t^i \tilde{\tau}_{t+s})
$$

$$
= \left( \frac{\sigma}{\eta} \tilde{w}(1 - \tilde{\tau}) + (1 - \tilde{g}) \right) \sum_{s=1}^{T} \beta^s \sum_{j=0}^{s-1} \frac{1}{\sigma} (E_t^i \tilde{\tau}_{t+j} - E_t^i \tilde{\tau}_{t+j+1})
$$

$$
+ \bar{b} \sum_{s=0}^{T} \beta^s (\beta E_t^i \tilde{c}_{t+s} - E_t^i \tilde{\tau}_{t+s}) - \beta^{T+1} \frac{\bar{b}}{\sigma} \sum_{j=0}^{T-1} \frac{1}{\sigma} (E_t^i \tilde{\tau}_{t+j} - E_t^i \tilde{\tau}_{t+j+1}) - \beta^{T+1} \frac{\bar{b}}{\sigma} E_t^i \tilde{\tau}_{t+T}.
$$

Aggregating this equation over all households yields an expression for aggregate consumption as a function of aggregate expectations about aggregate variables, only.

$$
\left( \beta^{T+1} \bar{b} + \left( \frac{\sigma}{\eta} \bar{w}(1 - \bar{\tau}) + (1 - \bar{g}) \right) \frac{1 - \beta^{T+1}}{1 - \beta} \right) \bar{C}_t = \quad (\text{B.9})
$$

$$
- \beta^{T+1} \frac{\bar{\bar{\Lambda}}}{\sigma} + \bar{\bar{b}} + \bar{\bar{w}} \sum_{s=0}^{T} \beta^s ((1 + \frac{1}{\eta})((1 - \bar{\tau})E_t \tilde{w}_{t+s} - E_t \tilde{\tau}_{t+s}) + \frac{\bar{\bar{\gamma}}}{\bar{\bar{Y}}} \sum_{s=0}^{T} \beta^s (E_t \tilde{\tau}_{t+s})
$$

$$
= \left( \frac{\sigma}{\eta} \tilde{w}(1 - \tilde{\tau}) + (1 - \tilde{g}) \right) \sum_{s=1}^{T} \beta^s \sum_{j=0}^{s-1} \frac{1}{\sigma} (E_t \tilde{\tau}_{t+j} - E_t \tilde{\tau}_{t+j+1})
$$

$$
+ \bar{b} \sum_{s=0}^{T} \beta^s (\beta E_t \tilde{c}_{t+s} - E_t \tilde{\tau}_{t+s}) - \beta^{T+1} \frac{\bar{b}}{\sigma} \sum_{j=0}^{T-1} \frac{1}{\sigma} (E_t \tilde{\tau}_{t+j} - E_t \tilde{\tau}_{t+j+1}) - \beta^{T+1} \frac{\bar{b}}{\sigma} E_t \tilde{\tau}_{t+T}.
$$

B.2 Optimal pricing decision

Log linearizing (16) gives

$$
\hat{p}_t^i (j) - \hat{p}_t = \frac{1 - \omega \beta}{1 - \omega^{T+1} \beta^{T+1}} \left[ \hat{m}_t + \bar{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \left( \bar{m}_{c_{t+s}} + \bar{\bar{\tau}}_{t+s} \right) \right]
$$

which can be written in terms of inflation expectations as

$$
\hat{p}_t^i (j) - \hat{p}_t = \frac{1 - \omega \beta}{1 - \omega^{T+1} \beta^{T+1}} \left[ \hat{m}_t + \bar{E}_t^j \sum_{s=1}^{T} \omega^s \beta^s \left( \bar{m}_{c_{t+s}} + \sum_{\tau=1}^{s} \tau_{t+\tau} \right) \right].
$$
Next, (17) can be log-linearized to

\[ \hat{p}_t = \omega \hat{p}_{t-1} + (1 - \omega) \int_0^1 \hat{p}_t^*(j) dj, \]

from which it follows that

\[ \pi_t = \frac{1 - \omega}{\omega} \left( \int_0^1 \hat{p}_t^*(j) dj - \hat{p}_t \right). \]  \hspace{1cm} (B.11)

Aggregating (B.10) and plugging this in in the above expression gives

\[ \pi_t = \frac{(1 - \omega)(1 - \omega \beta)}{\omega(1 - \omega T + \beta T + 1)} \left( \hat{m}c_t + \sum_{s=1}^T \omega^s \beta^s \hat{E}_t \hat{m}c_{t+s} + \sum_{s=1}^T \omega^s \beta^s \sum_{\tau=1}^s \hat{E}_t \pi_{t+s} \right). \]  \hspace{1cm} (B.12)

### B.3 Final model

To complete the model we first log-linearize the government budget constraint, (19), to

\[ \tilde{b}_{t+1} = \frac{1}{\beta} \tilde{g}_t - \bar{\omega} (\tilde{w}_t + \tilde{H}_t) + \bar{\tau}_t + \frac{1}{\beta} \tilde{b}_t + \bar{b}(\tilde{i}_t - \frac{1}{\beta} \tilde{\pi}_t). \]  \hspace{1cm} (B.13)

Next, we log-linearize the market clearing condition, (20)

\[ \hat{Y}_t = (1 - \bar{g}) \hat{C}_t + \hat{g}_t, \]  \hspace{1cm} (B.14)

and write wages and marginal costs as

\[ \hat{m}c_t = \hat{w}_t = \eta \tilde{H}_t + \sigma \hat{C}_t + \frac{\tilde{\tau}_t}{1 - \bar{\tau}} = (\eta + \frac{\sigma}{1 - \bar{g}}) \hat{Y}_t - \sigma \frac{\tilde{g}_t}{1 - \bar{g}} + \frac{\tilde{\tau}_t}{1 - \bar{\tau}}. \]  \hspace{1cm} (B.15)

Finally, log-linearizing profits of firm \( j \) to

\[ \hat{\Xi}_t(j) = \frac{1}{1 - \hat{m}c(j)} (\hat{p}_t(j) - \hat{p}_t) + \hat{Y}_t(j) - \frac{\hat{m}c}{1 - \hat{m}c} \hat{m}c_t. \]  \hspace{1cm} (B.16)

Aggregate profits can be written as

\[ \hat{\Xi}_t = \hat{Y}_t - (\theta - 1) \hat{m}c_t, \]  \hspace{1cm} (B.17)

where we used that \( \hat{m}c = \frac{\theta - 1}{\sigma} \).
Using (B.14) in (B.9) results an expression for aggregate output.

\[
\hat{Y}_t = \frac{1}{\rho} \hat{b}_t + g_t + \delta \sum_{s=0}^{T} \beta^s (1 - \hat{\tau}) \hat{E}_t \hat{\pi}_{t+s} - \hat{E}_t \hat{\pi}_{t+s} + \frac{\bar{\xi}}{\rho} \sum_{s=0}^{T} \beta^s (\hat{E}_t \hat{\pi}_{t+s}) \\
- \frac{\bar{b}}{\rho} \sum_{s=0}^{T} \beta^s (\hat{E}_t \hat{\pi}_{t+s} - \hat{E}_t \hat{\pi}_{t+s}) \\
- \beta^{T+1} \frac{\bar{b}}{\sigma \rho} \sum_{j=0}^{T-1} (\hat{E}_t \hat{\pi}_{t+j} - \hat{E}_t \hat{\pi}_{t+j+1}) - \beta^{T+1} \frac{\bar{b}}{\sigma \rho} \hat{E}_t \hat{\pi}_{t+T} - \beta^{T+1} \frac{\bar{b}}{\sigma \rho} \hat{\lambda}_t,
\]

\[
\delta = \frac{\bar{\omega} \eta + 1}{\rho},
\]

\[
\mu = \frac{1}{\rho} \left( \frac{\bar{\omega}}{\eta} (1 - \hat{\tau}) + \frac{1 - \bar{g}}{\sigma} \right),
\]

\[
\rho = \frac{1}{1 - \bar{g}} \left[ \beta^{T+1} \tilde{b} + \left( \frac{\sigma}{\eta} \bar{\omega} (1 - \hat{\tau}) + (1 - \bar{g}) \right) \frac{1 - \beta^{T+1}}{1 - \beta} \right].
\]

We now assume that agents know, or have learned about the above relations between aggregate variables (which hold in every period). Therefore, expectations about wages and profits can be substituted for, using (B.15) and (B.17). This gives the following system of 3 equations that, together with a specification of monetary and fiscal policy, completely describe our model

\[
(1 - \nu_y) \hat{Y}_t = \frac{1}{\rho} \hat{b}_t + g_t + \nu_T + \nu_r \sum_{s=0}^{T} \beta^s (\hat{E}_t \hat{\pi}_{t+s}) + \nu_g \sum_{s=0}^{T} \beta^s (\hat{E}_t \hat{y}_{t+s}) + \nu_y \sum_{s=0}^{T} \beta^s (\hat{E}_t \hat{y}_{t+s}) \\
- \mu \sum_{s=1}^{T} \beta^s \sum_{j=1}^{s} (\hat{E}_t \hat{\pi}_{t+j-1} - \hat{E}_t \hat{\pi}_{t+j}) + \frac{\bar{b}}{\rho} \sum_{s=0}^{T} \beta^s (\hat{E}_t \hat{\pi}_{t+s} - \hat{E}_t \hat{\pi}_{t+s}) \\
- \beta^{T+1} \frac{\bar{b}}{\sigma \rho} \sum_{j=0}^{T-1} (\hat{E}_t \hat{\pi}_{t+j} - \hat{E}_t \hat{\pi}_{t+j+1}) - \beta^{T+1} \frac{\bar{b}}{\sigma \rho} \hat{E}_t \hat{\pi}_{t+T} - \beta^{T+1} \frac{\bar{b}}{\sigma \rho} \hat{\lambda}_t,
\]

\[
\pi_t = \tilde{\kappa} (\eta + \frac{\sigma}{1 - \bar{g}}) \sum_{s=0}^{T} \omega^s \beta^s \hat{E}_t \hat{y}_{t+s} - \frac{\tilde{\kappa} \sigma}{1 - \bar{g}} \sum_{s=0}^{T} \omega^s \beta^s \hat{E}_t \hat{y}_{t+s} \\
+ \frac{\tilde{\kappa}}{1 - \hat{\tau}} \sum_{s=0}^{T} \omega^s \beta^s \hat{E}_t \hat{\pi}_{t+s} + \kappa \sum_{s=1}^{T} \omega^s \beta^s \sum_{\tau=1}^{s} \hat{E}_t \hat{\pi}_{t+\tau},
\]
\[ \tilde{b}_{t+1} = \frac{1}{\beta} \tilde{g}_t - \frac{\tilde{w}}{\beta} \left[ \tilde{\tau} \left( (1 + \eta + \frac{\sigma}{1 - \tilde{g}}) \tilde{Y}_t - \sigma \tilde{g}_t + \tilde{\tau}_t \right) + \tilde{\tau} \right] + \frac{1}{\beta} \tilde{b}_t + \tilde{b}(\tilde{t}_t - \frac{1}{\beta} \tilde{\tau}_t), \quad (B.24) \]

with

\[ \nu_g = \frac{1}{\theta \rho} + \left( \delta (1 - \tau) - \frac{\theta - 1}{\theta \rho} \right) (\eta + \frac{\sigma}{1 - \tilde{g}}), \quad (B.25) \]

\[ \nu_g = \left( \frac{\theta - 1}{\theta \rho} - \delta (1 - \tau) \right) \frac{\sigma}{1 - \tilde{g}}, \quad (B.26) \]

\[ \nu_\tau = - \frac{\theta - 1}{\theta \rho (1 - \tau)}, \quad (B.27) \]

\[ \tilde{\kappa} = \frac{(1 - \omega) (1 - \omega \beta)}{\omega (1 - \omega T + 1 \beta T + 1)}. \quad (B.28) \]

**C Model under infinitely forward looking rational expectations**

When we let the planning horizon, T, go to infinity (B.18) reduces to

\[ \tilde{Y}_t = \frac{1}{\rho} b_t + g_t + \delta \sum_{s=0}^{\infty} \beta^s \left( (1 - \tilde{\tau}) \tilde{E}_t \tilde{w}_{t+s} - \tilde{E}_t \tilde{\tau}_{t+s} \right) + \frac{\tilde{\kappa}}{\theta \rho} \sum_{s=0}^{\infty} \beta^s (\tilde{E}_t \tilde{\Xi}_{t+s}) \]

\[ - \mu \sum_{s=1}^{\infty} \beta^s \sum_{j=0}^{s-1} (\tilde{E}_t \tilde{i}_{t+j} - \tilde{E}_t \pi_{t+j+1}) + \frac{\tilde{b}}{\rho} \sum_{s=0}^{\infty} \beta^s (\beta \tilde{E}_t \tilde{i}_{t+s} - \tilde{E}_t \tilde{\pi}_{t+s}), \]

which can be written as

\[ \tilde{Y}_t = \frac{1}{\rho} b_t + \tilde{g}_t + \delta \sum_{s=0}^{\infty} \beta^s \left( (1 - \tau) \tilde{E}_t \tilde{w}_{t+s} - \tilde{E}_t \tilde{\tau}_{t+s} \right) + \frac{1}{\theta \rho} \sum_{s=0}^{\infty} \beta^s (\tilde{E}_t \tilde{\Xi}_{t+s}) \]

\[ - \frac{\mu \beta}{1 - \beta} \sum_{j=0}^{\infty} \beta^j (\tilde{E}_t \tilde{i}_{t+j} - \tilde{E}_t \pi_{t+j+1}) + \frac{\tilde{b}}{\rho} \sum_{s=0}^{\infty} \beta^s (\beta \tilde{E}_t \tilde{i}_{t+s} - \tilde{E}_t \tilde{\pi}_{t+s}). \]

Leading this equation 1 period and taking expectations gives

\[ \tilde{E}_t \tilde{Y}_{t+1} = \frac{1}{\rho} \tilde{E}_t \tilde{b}_{t+1} + \tilde{E}_t \tilde{g}_{t+1} + \delta \sum_{s=1}^{\infty} \beta^{s-1} \left( (1 - \tilde{\tau}) \tilde{E}_t \tilde{w}_{t+s} - \tilde{E}_t \tilde{\tau}_{t+s} \right) \quad (C.1) \]

\[ + \frac{1}{\theta \rho} \sum_{s=1}^{\infty} \beta^{s-1} (\tilde{E}_t \tilde{\Xi}_{t+s}) - \frac{\mu \beta}{1 - \beta} \sum_{j=1}^{\infty} \beta^{j-1} (\tilde{E}_t \tilde{i}_{t+j} - \tilde{E}_t \pi_{t+j+1}) \]

\[ + \frac{\tilde{b}}{\rho} \sum_{s=1}^{\infty} \beta^{s-1} (\beta \tilde{E}_t \tilde{i}_{t+s} - \tilde{E}_t \tilde{\pi}_{t+s}). \]
We can therefore rewrite output recursively as

\[
\hat{Y}_t = \beta \bar{E}_t \hat{Y}_{t+1} + \frac{1}{\rho} (\bar{b}_t - \beta \bar{E}_t \bar{b}_{t+1}) + (\bar{g}_t - \beta \bar{g}_{t+1}) + \delta (1 - \tau) \hat{w}_t - \delta \bar{\tau}_t + \frac{1}{\theta \rho} \bar{e}_t \\
+ \left( \beta \frac{\bar{b}}{\rho} - \frac{\mu \beta}{1 - \beta} \right) i_t - \frac{b}{\rho} \hat{\pi}_t + \frac{\mu \beta}{1 - \beta} \bar{E}_t \bar{\pi}_{t+1}.
\]

Plugging in (B.15) and (B.17) this reduces to

\[
(1 - \nu_y) \hat{Y}_t = \beta \bar{E}_t \hat{Y}_{t+1} + \frac{1}{\rho} (\bar{b}_t - \beta \bar{E}_t \bar{b}_{t+1}) + (1 + \nu_g) \bar{g}_t - \beta \bar{g}_{t+1} + \nu \bar{\tau}_t \\
+ \left( \beta \frac{\bar{b}}{\rho} - \frac{\mu \beta}{1 - \beta} \right) i_t - \frac{b}{\rho} \hat{\pi}_t + \frac{\mu \beta}{1 - \beta} \bar{E}_t \bar{\pi}_{t+1}.
\]

The same kind of derivation on the production side results in the standard New Keynesian Phillips curve. For infinite horizons we get

\[
\pi_t = \frac{(1 - \omega)(1 - \omega \beta)}{\omega} \left( \sum_{s=0}^{\infty} \omega^s \beta^s \bar{E}_t \bar{mc}_{t+s} + \frac{\omega \beta}{1 - \omega \beta} \sum_{s=0}^{\infty} \omega^s \beta^s \bar{E}_t \bar{\pi}_{t+j+1} \right),
\]

Leading 1 period and taking expectations gives

\[
\bar{E}_t \bar{\pi}_{t+1} = \frac{(1 - \omega)(1 - \omega \beta)}{\omega} \left( \sum_{s=1}^{\infty} (\omega \beta)^{s-1} \bar{E}_t \bar{mc}_{t+s} + \frac{\omega \beta}{1 - \omega \beta} \sum_{s=1}^{\infty} (\omega \beta)^{s-1} \bar{E}_t \bar{\pi}_{t+j+1} \right).
\]

So that inflation writes recursively as

\[
\pi_t = \omega \beta \bar{E}_t \pi_{t+1} + (1 - \omega) \beta \bar{E}_t \pi_{t+1} + \hat{\kappa} \bar{mc}_t = \beta \bar{E}_t \pi_{t+1} + \hat{\kappa} \bar{mc}_t,
\]

\[
\hat{\kappa} = \frac{(1 - \omega)(1 - \omega \beta)}{\omega}.
\]

Using (B.15) then gives

\[
\pi_t = \beta \bar{E}_t \pi_{t+1} + \hat{\kappa} (\eta + \frac{\sigma}{1 - \bar{g}}) \hat{Y}_t - \hat{\kappa} \sigma \frac{\bar{g}_t - \hat{g}_t}{1 - \bar{g}} + \hat{\kappa} \frac{\bar{\tau}_t}{1 - \tau}.
\]
D Steady states log-linear model for given $\Lambda$

Below we derive steady state relations that hold for a given value of the valuation of end of horizon wealth: $\hat{\Lambda}_t = \Lambda$. This is the steady state that agents expect the model to have reached after their horizon.\textsuperscript{18}

In steady state, Equations (21) through (29) reduce to

\begin{align*}
i &= \phi_1 \pi + \phi_2 Y, \quad \text{(D.1)} \\
g &= -\mathds{1}_{\text{cons}} \alpha \gamma_{\text{cons}} (b - DT), \quad \text{(D.2)} \\
\tau &= \tau^{DT} + (\gamma_0 + \mathds{1}_{\text{cons}} (1 - \alpha) \gamma_{\text{cons}}) (b - DT). \quad \text{(D.3)}
\end{align*}

Plugging in these steady states values for monetary and fiscal variables in the steady state versions of Equations (24) through (26), we can rearrange these three equations for output, inflation and debt to

\begin{align*}
Y &= \frac{1}{\bar{\rho}} b + (\phi_1 \psi_i / \epsilon - \psi_\pi / \epsilon) \pi - \gamma_{\text{DT}} (b - DT) - \gamma_\tau \tau^{DT} - \beta^{T+1} \bar{b} \bar{\sigma} \Lambda, \quad \text{(D.4)} \\
\pi &= \chi (\eta + \sigma / (1 - \bar{g})) Y + \chi \sigma_1 (b - DT) + \chi \frac{\tau^{DT}}{1 - \bar{\tau}}, \quad \text{(D.5)} \\
\beta b &= b + \left( \phi_2 \beta \bar{b} - \bar{w} \bar{\sigma} (1 + \eta + \sigma / (1 - \bar{g})) \right) Y \\
&\quad + (\beta \phi_1 - 1) \bar{b} \pi - \delta_{\text{DT}} (b - DT) - \delta_\tau \tau^{DT}, \quad \text{(D.6)}
\end{align*}

With

\begin{align*}
\epsilon &= 1 - \frac{1 - \beta^{T+1}}{1 - \beta} \nu_{\gamma} - \phi_2 \psi_i, \quad \text{(D.7)} \\
\bar{\rho} &= \rho \epsilon, \quad \text{(D.8)} \\
\psi_i &= \left( -\beta^{T+1} \frac{\bar{b} \sigma}{\rho (T + 1)} + \frac{\beta \bar{b} (1 - \beta^{T+1})}{\rho (1 - \beta)} - \mu \frac{(\beta T - T - 1) \beta^{T+1} + \beta}{(1 - \beta)^2} \right), \quad \text{(D.9)} \\
\psi_\pi &= \left( -\beta^{T+1} \frac{\bar{b} \sigma}{\rho T} + \frac{\bar{b} (1 - \beta^{T+1})}{\rho (1 - \beta)} - \mu \frac{(\beta T - T - 1) \beta^{T+1} + \beta}{(1 - \beta)^2} \right), \quad \text{(D.10)}
\end{align*}

\textsuperscript{18}The steady state that the model will reach once $\Lambda_t$ has converged can then be obtained by plugging the value of $\Lambda$ that solves $\Lambda = (1 - \gamma) \Lambda + \gamma \left( \frac{\sigma}{\bar{g}} b + \frac{\sigma}{\bar{g}} Y + \frac{\sigma}{\bar{g}} \bar{g} - i \right)$. \hfill 47
\[
\gamma_{DT} = \frac{1}{\epsilon} \left[ I_{\text{cons}} \alpha \gamma_{g} + \frac{1 - \beta^{T+1}}{1 - \beta} \left( -\nu_{\tau} (\gamma^{0}_{\tau} + I_{\text{cons}} (1 - \alpha) \gamma_{\tau}^{\text{cons}}) + \nu_{g} I_{\text{cons}} \alpha \gamma_{g} \right) \right], \tag{D.11}
\]

\[
\gamma_{\tau} = -\frac{1}{\epsilon} \frac{1 - \beta^{T+1}}{1 - \beta}, \tag{D.12}
\]

\[
\chi = \frac{\tilde{K} 1 - \omega^{T+1} \beta^{T+1}}{1 - \omega \beta}, \tag{D.13}
\]

\[
\sigma_{1} = \sigma \left[ I_{\text{cons}} \alpha \gamma_{g} \frac{\gamma^{0}_{\tau} + I_{\text{cons}} (1 - \alpha) \gamma_{\tau}^{\text{cons}}}{1 - \tilde{\tau}} \right], \tag{D.14}
\]

\[
\delta_{DT} = \left[ I_{\text{cons}} \alpha \gamma_{g} \gamma_{\tau}^{\text{cons}} + \bar{w} \left( \gamma^{0}_{\tau} + I_{\text{cons}} (1 - \alpha) \gamma_{\tau}^{\text{cons}} + \tilde{\tau} \sigma_{1} \right) \right], \tag{D.15}
\]

\[
\delta_{\tau} = \bar{w} \left( 1 + \frac{\tilde{\tau}}{1 - \tilde{\tau}} \right). \tag{D.16}
\]

Now, plugging in (D.5) in (D.4) gives

\[
Y = \frac{1}{\hat{\rho} v} \left\{ b - \beta^{T+1} \frac{\bar{b}}{\sigma \hat{\rho} v} \Lambda + \frac{\sigma}{\bar{\tau} \epsilon} \left[ \chi \sigma_{1} (\phi_{1} \psi_{i} - \psi_{\pi} \epsilon) - \gamma_{DT} \right] (b - DT) \right\} + \frac{1}{\nu} \left[ \frac{\sigma}{1 - \tilde{\tau}} \left( \phi_{1} \psi_{i} - \psi_{\pi} \epsilon \right) - \gamma_{\tau} \right] \tilde{\tau}_{DT}, \tag{D.17}
\]

\[
v = 1 - (\phi_{1} \psi_{i} - \psi_{\pi} \epsilon) \chi (\eta + \frac{\sigma}{1 - \tilde{\tau}}). \tag{D.18}
\]

Plugging in (D.5) in (D.6) gives

\[
\beta b = b + \sigma_{2} Y + \left[ -\delta_{DT} + (\beta \phi_{1} - 1) \bar{b} \chi \sigma_{1} \right] (b - DT) + \left[ -\delta_{\tau} + (\beta \phi_{1} - 1) \bar{b} \chi \frac{1}{1 - \tilde{\tau}} \right] \tilde{\tau}_{DT}, \tag{D.19}
\]

\[
\sigma_{2} = \left( \phi_{2} \beta \bar{b} - \bar{w} \tilde{\tau} (1 + \frac{\sigma}{1 - \tilde{\tau}}) + (\beta \phi_{1} - 1) \bar{b} \chi (\eta + \frac{\sigma}{1 - \tilde{\tau}}) \right). \tag{D.20}
\]

Finally plugging in (D.17) in (D.19) and rearranging gives

\[
b = \frac{1}{\sigma_{3}} \left( \omega_{DT} DT - \omega_{\tau} \tilde{\tau}_{DT} - \beta^{T} \frac{\bar{b}}{\sigma \hat{\rho} v} \Lambda \sigma_{2} \right), \tag{D.21}
\]

with

\[
\sigma_{3} = 1 - \frac{1}{\beta} \left[ 1 + \frac{1}{\hat{\rho} v} \sigma_{2} \right] + \omega_{DT}, \tag{D.22}
\]

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\[
\omega_{DT} = \frac{\delta_{DT}}{\beta} - \frac{1}{\beta} (\beta \phi_1 - 1) \bar{b} \chi \sigma_1 - \frac{1}{\beta} \sigma_2 \frac{1}{\nu} \left[ \chi \sigma_1 \left( \frac{\psi_1}{\epsilon} - \frac{\psi_2}{\epsilon} \right) \right] - \gamma_{DT}, \tag{D.23}
\]

\[
\omega_\tau = \frac{\delta_\tau}{\beta} - \frac{1}{\beta} (\beta \phi_1 - 1) \bar{b} \chi \frac{1}{1 - \bar{\tau}} - \frac{1}{\beta} \sigma_2 \frac{1}{\nu} \left[ \chi \frac{1}{1 - \bar{\tau}} \left( \frac{\psi_1}{\epsilon} - \frac{\psi_2}{\epsilon} \right) \right] - \gamma_\tau. \tag{D.24}
\]

Steady state output and inflation now can by plugging in (D.21) in (D.17) and plugging in the resulting steady state output, as well as (D.21), in (D.5).

E Robustness

E.1 Uncertainty about type of consolidation

In the paper, we have assumed that when the debt target drops, agents know that consolidations will be implemented next period and also know their composition (i.e. spending- or tax-based), with certainty. In this appendix, we assume that agents initially think consolidations are a mix of 50% spending-based and 50% tax-based. This reflects the case where the government does not credibly announce which type of consolidations it will implement. When consolidations are implemented for the first time, agents observe the type of consolidation, and update their beliefs accordingly.

This case of initial uncertainty about the type of consolidations is plotted in Figure 9, where we assume \( T = 8, \phi_1 = 1.3 \) and \( \gamma_{cons}^g = 0.3 \), as in Figure 5. We see that now consumption and hence output do not react much in the period where the drop in the debt target is announced (period 2). This is because agents do not expect a big change in their future consumption, since spending-based and tax-based consolidations have opposite effects on that variable. As a result, debt does not change much either in the period where the debt target is dropped. Comparing this with Figure 5, under spending-based consolidations (green) the induced recession upon implementation is now deeper, while debt drops with a slower pace in the subsequent quarters. The opposite holds for tax-based consolidations (black).

In the first period where consolidations are implemented (period 3), agents still do not know what the composition will be. Hence, output is again lower under spending-based consolidations than in Figure 5, while it is higher in case of a tax-based consolidation. This
Figure 9: Impulse responses of spending-based (green) and tax-based (black) consolidations for $T = 8$, in the case where agents do initially not have perfect knowledge about the type of consolidation.
results in a relative advantage of tax-based consolidations in reducing debt, already in the short run. In the following periods agents have updated their beliefs about the type of consolidations, and dynamics are very similar to those in Figure 5. Slight differences arise because of slightly different debt levels, and hence different values of taxes and government spending.

The other figures of Section 5 change in a similar fashion under uncertainty about the type of consolidation. That is, the initial two periods favor tax-based consolidations and hurt spending-based consolidations compared to the case of fully anticipated consolidations, while medium to long run dynamics are hardly affected.

E.2 Gain parameter for updating of $\Lambda^i_t$

As discussed in Section 5, agents with shorter horizons respond in a boundedly rational way to fiscal consolidations caused by a drop in the debt target. More precisely, when they anticipate a change in their wealth (bond holdings) they are not immediately able to infer how this should change their valuation of future wealth. Instead, when the debt threshold is dropped they start updating their relative valuation of future wealth $\Lambda^i_t$, and only after several periods they have learned how to value future wealth in their new environment.

In this section we investigate how sensitive results of Section 5 are to the speed with which they update $\Lambda^i_t$, which is determined by the gain parameter $\gamma$. A slower updating process (lower $\gamma$) would make the boundedly rational wealth effects for agents with shorter horizons more pronounced. Faster updating, with a higher gain parameter, might make agents respond more rationally to a drop in the debt target than what was found in Figures 3 and 4. In Figures 10 and 11, we plot the most extreme case, where the gain parameter is set equal to $\gamma = 1$ instead of $\gamma = 0.25$.

In the figures it can be seen that the blue curves of $T = 3$ and the green curves of $T = 8$ converge faster to the steady state than in Figures 3 and 4. Initial responses are however very similar, and qualitative dynamics, especially for the case of $T = 8$, remain the same. Considering present value multipliers, we find that multipliers for $T = 8$ and $T = 100$ change only marginally when we increase the gain parameter. For $T = 3$ the gain parameter has however a significant effect on present value multipliers.

Table 3 presents the multipliers of Table 1, for the case where the gain parameter is $\gamma = 1$
Figure 10: Transitions to new steady state following a drop in the debt target from 75% of GDP ($\Delta T_t = 0.05$) to 70% of GDP ($\Delta T_t = 0$) when the gain parameter is increased to $\gamma = 1$ in case of spending-based consolidations, for different horizons. In particular, $T = 3$ (blue), $T = 8$ (green), $T = 100$ (yellow) and infinite horizon rational expectations (thin black). The last 2 curves overlap.
Figure 11: Transitions to new steady state following a drop in the debt target from 75% of GDP ($\tilde{DT}_t = 0$) to 70% of GDP ($\tilde{DT}_t = 0.05$) when the gain parameter is increased to $\gamma = 1$ in case of tax-based consolidations, for different horizons. In particular, $T = 3$ (blue), $T = 8$ (green), $T = 100$ (yellow) and infinite horizon rational expectations (thin black). The last 2 curves overlap.
rather than $\gamma = 0.25$. Comparing the two tables, it can be seen that the impact multiplier does not change. This is not surprising since the faster updating of $\Lambda_i$ only starts to take effect when the consolidations have started. The present value multipliers after 4 and 8 quarters are however less negative under the larger gain parameter. This reflects the less persistent recession that can also be observed in Figures 10 and 11. Multipliers for $T = 3$ remain however considerably more negative than those of $T = 8$ and $T = 100$ and change in a similar way as before when we make monetary policy more aggressive or increase the strength of consolidations. We can therefore conclude that the qualitative results of this paper are robust to the calibration of $\gamma$.

Panel a: $\phi_1 = 1.3, \gamma_{\text{cons}}^{\phi_1} = 0.3$

<table>
<thead>
<tr>
<th>$\frac{\Delta Y}{\Delta g}$</th>
<th>1qr</th>
<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
<th>$\frac{\Delta Y}{\Delta \tau}$</th>
<th>1qr</th>
<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
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</thead>
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<tr>
<td>$T = 3$</td>
<td>-0.82</td>
<td>-0.81</td>
<td>-0.75</td>
<td>-0.68</td>
<td>$T = 3$</td>
<td>-0.53</td>
<td>-0.65</td>
<td>-0.66</td>
<td>-0.62</td>
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</table>

Panel b: $\phi_1 = 1.3, \gamma_{\text{cons}}^{\phi_1} = 0.6$

<table>
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<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
<th>$\frac{\Delta Y}{\Delta \tau}$</th>
<th>1qr</th>
<th>4qr</th>
<th>8qr</th>
<th>12qr</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T = 3$</td>
<td>-0.83</td>
<td>-0.81</td>
<td>-0.73</td>
<td>-0.65</td>
<td>$T = 3$</td>
<td>-0.35</td>
<td>-0.41</td>
<td>-0.34</td>
<td>-0.24</td>
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Panel c: $\phi_1 = 1.8, \gamma_{\text{cons}}^{\phi_1} = 0.3$

<table>
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<th>$\frac{\Delta Y}{\Delta \tau}$</th>
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</thead>
<tbody>
<tr>
<td>$T = 3$</td>
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<td>-0.47</td>
<td>-0.49</td>
<td>-0.44</td>
<td>-0.36</td>
</tr>
</tbody>
</table>

**Table 3:** Output Multipliers

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