

Axiomatizing an Algebra of Step Reactions for Synchronous Languages

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Abstract. This paper introduces a novel algebra for reasoning about step reactions in synchronous languages, such as macro steps in Harel, Pnueli and Shalev's Statecharts and instantaneous reactions in Berry's Esterel. The algebra describes step reactions in terms of configurations which can both be read in a standard operational as well as in a model-theoretic fashion. The latter arises by viewing configurations as propositional formulas, interpreted intuitionistically over finite linear Kripke structures. Previous work by the authors showed the adequacy of this approach by establishing compositionality and full-abstraction results for Statecharts and Esterel. The present paper generalizes this work in an algebraic setting and, as its main result, provides a sound and complete equational axiomatization of step reactions. This yields, for the first time in the literature, a complete axiomatization of Statecharts macro steps, which can also be applied, modulo encoding, to Esterel reactions.

1 Introduction

Synchronous languages provide a popular framework for designing and programming *event-based reactive systems*. Prominent examples of such languages include Harel's *Statecharts* [5], which is a graphical language that extends finite-state machines by concepts of state hierarchy, concurrency, and event priority, and Berry's *Esterel* [2, 3], which is a textual language having similar features to Statecharts. Today, both languages are supported by commercial tools, including *Statemate* [7] and *Esterel Studio* [4], which mainly focus on generating running code. The development of semantic-based verification tools is still in its infancy, partly due to the lack of sufficiently simple compositional semantics.

The semantics of Statecharts, as conceived by Pnueli and Shalev [20], and of Esterel are based on the idea of *cycle-based reaction*, where first the input events, as defined by a system's environment, are sampled at the beginning of each cycle, then the system's reaction in form of the emission of further events is determined, and finally the generated events are output to the environment. Statecharts and Esterel differ in the details of what exactly constitutes a cycle, which is also called a *macro step* in Statecharts and an *instantaneous reaction* in Esterel. Moreover, Esterel refers to events as signals. Both languages have in common that they obey the semantic principles of *synchrony* and *causality*. The

synchrony requirement reflects the mechanism behind cycle-based reaction and is mathematically modeled via the *synchrony hypothesis*. This hypothesis ensures that reactions and the propagations of events are instantaneous, which is an idealized system behavior, practically justified by the observation that reactive systems usually perform much faster than their environments. Causality refers to the requirement that the reason for an event to be generated in a system reaction must be traced back to the input events provided by the environment. Esterel differs from Statecharts in that it further adopts the principles of *reactivity* and *determinism*. Reactivity implies that, in each cycle, a system response in the form of generated events can be constructed, for any inputs an environment may provide. Determinism requires for this response to be unique.

This brief discussion highlights the variety of possible choices when defining a semantics for step reactions, with different choices implying subtly different semantics. Recent research by the authors, aiming at a unifying semantic framework for synchronous languages, has concentrated on employing ideas from *intuitionistic logic* for describing step reactions [12–15]. Intuitionistic logic, in contrast to classical logic, is constructive and thus truly reflects the operational character of step reactions in the light of causality: it rejects the classical principle of the excluded middle, i.e., events are either always present or always absent *throughout* a reaction, which cannot be maintained for a compositional semantics that allows the system environment to inject events *during* a step reaction. Indeed, our intuitionistic setting has led to compositional and fully-abstract characterizations of Statecharts macro steps and Esterel reactions [12, 14].

This paper introduces a simple yet expressive algebra for describing and reasoning about step reactions in terms of so-called *configurations* and presents an equational axiomatization for it. In particular, this gives for the first time in the literature a sound and complete axiomatization for Statecharts macro steps, which can also be applied, modulo encoding, to Esterel reactions. The step algebra's semantics is inspired by the authors' previous work and reads configurations as propositional formulas, interpreted intuitionistically over finite linear Kripke structures, to which we refer as *sequence structures* (cf. Sec. 2). Our axiomatization is then built on top of this algebra (cf. Sec. 3), and its proof of completeness combines techniques used in process algebras [1] and logics (cf. Sec. 4); it employs a process-algebraic notion of *normal form* that in turn is defined by model-theoretic means. Our axioms have an appealing operational intuition that shades light on the semantics of step reactions. They also provide groundwork for an axiomatic comparison of popular synchronous languages.

2 Step Algebra

This section introduces our step algebra for reasoning about those step reactions that may be specified within *event-based synchronous languages*. Usually, synchronous languages, such as *Statecharts* [5, 20] or *Esterel* [2, 3] (with its graphical front-end *SyncCharts*), enrich the notation of finite state machines by mechanisms for expressing hierarchy, concurrency, and priority. This allows one to

refine a single state by a state machine, to run several state machines in parallel which may coordinate each other by broadcasting events, and to give some transitions precedence over others, respectively.

In event-based synchronous languages, each transition t is labeled by two sets of events, which are referred to as *trigger* and *action*. The trigger is a set of positive events P and negative events \bar{N} , taken from a countable universe of events Ev and their negated counterparts in $\overline{Ev} =_{\text{df}} \{\bar{e} : e \in Ev\}$, respectively. For convenience, we define $\bar{\bar{e}} =_{\text{df}} e$. Intuitively, t is enabled and forced to fire if the transition's environment signals all events in P but none in N . The effect of firing t is the generation of all events in the transition's action $A \subseteq Ev$. These events might in turn trigger transitions in different parallel components, thus enabling a *causal* chain reaction whose length is bounded by the number of parallel components within the program under consideration. A *step reaction* is then the set of all events that are already valid at the beginning of the step or generated during the step. When constructing steps in the suggested operational manner, it is possible to experience inconsistencies, namely when a firing transition generates some event e , whose absence, i.e., its negation \bar{e} , was assumed when firing a previous transition in the step. Since an event cannot be both present and absent within the same step reaction, due to the principle of *global consistency* [20], the choice sequence leading to the inconsistency is rejected, and a different sequence must then be chosen. If no consistent sequence is possible, the step construction fails altogether. Alternatively, one could also say that the step construction remains unfinished: it waits for the environment to provide additional events to disable the transitions that produced the inconsistency.

The semantic subtlety of step reactions arises precisely from the capability of defining transitions whose enabledness disables other transitions, as well as from the interpretation of negated trigger events. Thus, the difficulties in defining a clean semantic account of a synchronous language lie not in the semantics of sequences of step reactions but in the semantics of single step reactions, which are thus the focus of study in this paper. In the light of the above discussion, the key operators for combining transitions in synchronous languages are *parallel composition* and *event negation*. State hierarchy is merely a notationally convenient rather than a semantically relevant operator. Observe that parallel composition and event negation also allow one to express *nondeterministic choice* [13]. For example, a choice between two transitions $P_1, \bar{N}_1/A_1$ and $P_2, \bar{N}_2/A_2$ might be written as the parallel composition $P_1, \bar{N}_1, \bar{e}_2/A_1, e_1 \parallel P_2, \bar{N}_2, \bar{e}_1/A_2, e_2$, where e_1, e_2 are distinguished events not occurring in the triggers or actions of the two original transitions and where the comma notation stands for union, i.e., $X, Y =_{\text{df}} X \cup Y$ and $X, x =_{\text{df}} X \cup \{x\}$.

Syntax. For the purposes of this paper, it is convenient to work with a quite general syntax for step reactions, which allows us to encode several dialects of synchronous languages, including Statecharts and Esterel. The terms describing step reactions, to which we also refer as *configurations*, are defined inductively as follows:

$$C ::= 0 \mid A \mid I/C \mid C \parallel C,$$

where $A \subseteq Ev$ and $I \subseteq Ev \cup \overline{Ev}$. Intuitively, 0 stands for the configuration with the empty behavior, $A \subseteq Ev$ denotes the signaling of all events in A , configuration I/C encodes that configuration C is triggered by the presence of the positive events in I and the absence of the negative events in I , and $C_1 \parallel C_2$ describes the parallel composition of configurations C_1 and C_2 . Observe that the semantics of configuration 0 coincides with the semantics of $A = \emptyset$; nevertheless, it seems natural to include 0 . For notational convenience, we let the transition slash $/$ to have higher binding power than parallel composition \parallel and interpret a nesting of transition slashes in a right-associative manner. Note that our syntax does not contain a special failure configuration for expressing global inconsistency. This could easily be included but can also be encoded as \overline{N}/N , if $N \neq \emptyset$ is the set of events causing the inconsistency to occur. Finally, for notational convenience, we often write x for the singleton set $\{x\}$.

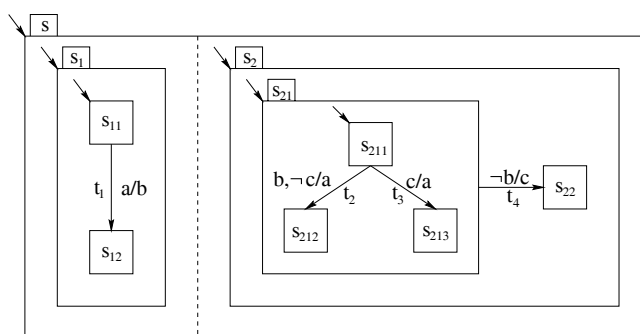


Fig. 1. Example Statechart

We illustrate our syntax by means of an example. Consider the Statechart depicted in Fig. 1 and assume that all components are in their initial states marked by small unlabeled arrows. Then the first Statechart step determining the initial Statechart reaction, may be encoded in our syntax as the configuration

$$C_{ex} =_{df} a/b \parallel b, \overline{c}, \overline{e_3}, \overline{e_4}/a, e_2 \parallel c, \overline{e_2}, \overline{e_4}/a, e_3 \parallel \overline{b}, \overline{e_2}, \overline{e_3}/c, e_4 .$$

Although the main body of this paper focuses on Statecharts, it is worth mentioning here that reactions of Esterel programs can be encoded in our syntax as well. The key idea is to let events encode signal statuses, i.e., to define $Ev =_{df} \{s = 1, s = 0 : s \text{ is a signal}\}$, where $s = 1$ stands for signal s is present ‘high’ and $s = 0$ for s is present ‘low’. The adaptation of the techniques and results of this paper to Esterel will be discussed in Sec. 5.

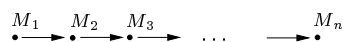
Semantics. In order for a semantics on configurations to be useful for the purposes of this paper, it must meet several requirements. First, it must be *compositional* to be axiomatizable, i.e., it must give rise to a semantic equivalence on configurations that is a congruence. Second, it should be compatible with existing semantics of the synchronous languages of interest, in particular with Statecharts and Esterel. Unfortunately, many semantics for synchronous languages, including the one of Statecharts as originally conceived by Harel, Pnueli

and Shalev [5, 20], are not compositional. Indeed, Huizing and Gerth proved more than a decade ago that synchrony, causality, and compositionality cannot be combined in a simple mathematical framework that models reactions as input–output–functions over event sets [9]. Research by the authors in this field over the past few years has revealed an appealing model–theoretic framework for studying Statecharts and Esterel semantics, which is based on reading configurations as simple propositional formulas that are *intuitionistically* interpreted over finite linear Kripke structures [12–15]. Our model–theoretic approach allows not only for a compositional semantics but also for establishing full–abstraction results. This paper generalizes this work in an algebraic setting.

The key idea is to consider a step reaction not as an arbitrary computation but as a *stabilization* process, where the synchronous environment is only concerned with the final response, while the system takes care of the actual sequence of events that leads to the stationary state. The main feature that distinguishes a stabilization process from an arbitrary computation is that it is a *monotonically increasing approximation* of the final step reaction. This means that once an event from the final step reaction has become present or asserted by the firing of a transition generating it, the event will remain present until the stationary state is reached. However, a *compositional* semantics of synchronous reactions must also cater for any potential interaction with the environment *during* the stabilization. Hence, for event–based languages, a suitable model is given by monotonically increasing sequences of sets of events M_i , for $1 \leq i \leq n \in \mathbb{N}$,



in which external input (solid arrows) alternates with internal reactions (dashed arrows). In each “external step” $M_i \subsetneq M_i^*$, the environment injects new events into the system, making it unstable. In the ensuing “internal steps” $M_i^* \subseteq M_{i+1}$, the system responds to the external stimulus along a number of intermediate stabilization steps (shaded areas) until it reaches the next stationary state M_{i+1} . The synchrony hypothesis abstracts all internal steps into a single *instantaneous* step reaction. Thus, when we specify a synchronous system from the external point of view, we only specify the sequence



along stationary states. Each global state M_i then records not only the input from the environment but *at the same time* also all causal reactions of the system within the same step reaction.

We are now able to formally equip configurations with this intuitive semantics, which can naturally be presented in a model–theoretic fashion. As illustrated above, we interpret configurations over finite, nonempty, strictly increasing sequences $M = (M_1, M_2, \dots, M_n)$, for $n \in \mathbb{N}$ and $M_i \subseteq Ev$, called *sequence structures*. ‘Strictly increasing’ means $M_i \subsetneq M_{i+1}$, for all $1 \leq i < n$. We say that M satisfies C , or that M is a *sequence model* of C , in signs $M \models C$, if $M_i \models C$

for all $1 \leq i \leq n$, where

$$\begin{aligned} M_i &\models 0 && \text{always} \\ M_i &\models A && \text{if } A \subseteq M_i \\ M_i &\models I/C && \text{if } (I \cap Ev \subseteq M_i \text{ and } \overline{(I \cap \overline{Ev})} \cap M_n = \emptyset) \text{ implies } M_i \models C \\ M_i &\models C_1 \parallel C_2 && \text{if } M_i \models C_1 \text{ and } M_i \models C_2 \end{aligned}$$

This definition is a shaved version of the standard semantics obtained when reading a configuration as a formula in propositional intuitionistic logic [22], i.e., when taking events to be atomic propositions and replacing \bar{e} by the negation $\neg e$, concatenation of events in sets and ‘ \parallel ’ by conjunction ‘ \wedge ’, and the transition slash ‘/’ by implication ‘ \supset ’. An empty trigger and the configuration 0 are identified with *true*. Then we have $M \models C$ if and only if C is valid in the intuitionistic Kripke structure M . Note that, for sequence structures $M = (M_1)$ of length one, the notions of sequence model and classical model coincide; hence, intuitionistic logic is a refinement of classical logic and we simply write M_1 for (M_1) . The utility of intuitionistic logic comes into play when ensuring global consistency within step reactions. This is because intuitionistic logic interprets negation globally for a sequence M , and not locally for single states M_i . In particular, $M_i \models I$ in intuitionistic logic if $I \cap Ev \subseteq M_i$ and $I \cap \overline{Ev} \cap M_n = \emptyset$, i.e., the final state M_n in M determines the absence of events.

Our semantics suggests the following equivalence on configurations. Configurations C_1, C_2 are *step congruent*, in signs $C_1 \simeq C_2$, if $M \models C_1 \Leftrightarrow M \models C_2$ holds for all sequence structures M . From the definition above we can immediately derive the following proposition.

Proposition 1 (Congruence). *The equivalence \simeq is indeed a congruence, i.e., $C_1 \simeq C_2$ implies $C_1 \parallel D \simeq C_2 \parallel D$ and $I/C_1 \simeq I/C_2$, for all configurations C_1, C_2, D and for all triggers $I \subseteq Ev \cup \overline{Ev}$.*

It was proved in [14] that the step congruence \simeq is *compositional* and *fully-abstract* with respect to Statecharts macro-step semantics. In particular, the macro steps for a configuration C correspond to those classical models N , for which no refinement $N' \subsetneq N$ satisfying $(N', N) \models C$ exists. Moreover, it is sufficient to consider sequence structures of length one and two only. If $2SM(C)$ denotes the sequence models of configuration C of length at most two, then $C_1 \simeq C_2$ if and only if $2SM(C_1) = 2SM(C_2)$ [14]. Finally, our compositional semantics can be carried over to Esterel, as was shown in [15].

Returning to our example and taking $Ev = \{a, b, c, e_2, e_3, e_4\}$, we obtain $(\{c, e_4\}), (\{a, b, e_2\}), (\{\}, \{a, b, e_2\}) \in 2SM(C_{ex})$. The first sequence model of length one corresponds to the valid Statecharts macro step in which only transition t_4 fires. This is witnessed by the fact that $(M', \{c, e_4\}) \notin 2SM(C_{ex})$, for any proper subset $M' \subsetneq \{c, e_4\}$. However, the second sequence model $\{a, b, e_2\}$ corresponds to a step reaction where both transitions t_1 and t_2 fire. This is not a valid Statecharts macro steps since it violates causality; observe that a/b and b/a “bite in each others tails.” Our semantic framework witnesses this situation by the fact that $(\{\}, \{a, b, e_2\})$ is a sequence model of length two that refines the one-sequence model, or classical model, $\{a, b, e_2\}$.

3 Axiomatization

The remainder of this paper presents an axiomatic characterization of our step congruence. This section first presents our axioms and shows them to be correct. The next section then proves the axiom set complete.

(A1)	$\emptyset = 0$	(A2)	$A \parallel B = A \cup B$
(A3)	$\emptyset/C = C$	(A4)	$I/0 = 0$
(A5)	$I_1, I_2/C = I_1/(I_2/C)$	(A6)	$I/(C_1 \parallel C_2) = I/C_1 \parallel I/C_2$
(B1)	$C_1 \parallel C_2 = C_2 \parallel C_1$	(B2)	$(C_1 \parallel C_2) \parallel C_3 = C_1 \parallel (C_2 \parallel C_3)$
(B3)	$C \parallel C = C$	(B4)	$C \parallel 0 = C$
(C1)	$P, I/P = 0$		
(C2)	$C = C \parallel I/C$		
(C3)	$A \parallel A, I/C = A \parallel I/C$		
(C4)	$P, \overline{N}/C = 0$		if $P \cap N \neq \emptyset$
(D1)	$P, \overline{N}/A = P, \overline{N}/A, B$		if $N \cap A \neq \emptyset$
(D2)	$P, \overline{N}/A = P, e, \overline{N}/A \parallel P, \overline{N}, \overline{e}/A$		if $N \cap A \neq \emptyset$
(D3)	$\overline{N}/C \parallel P, \overline{N}/A = \parallel \{ \overline{N}, \overline{e}/C : e \in P \} \parallel P, \overline{N}/A$		if $N \cap A \neq \emptyset$ and $P \neq \emptyset$

Table 1. Axioms for the step congruence

Our axioms system is displayed in Table 1, where $A, B, N, P \subseteq Ev$, $I, I_1, I_2 \subseteq Ev \cup \overline{Ev}$, and $e \in Ev$, and where C, C_1, C_2, C_3 are configurations. We write $\vdash C_1 = C_2$ to state that $C_1 = C_2$ can be derived from the axioms by standard equational reasoning. Axioms (A1)–(A6) and (B1)–(B4) are fairly natural and do not need much operational justification. When taking them together, it is easy to see that every configuration is equivalent to a flat parallel composition of transitions, without nested triggers and where ordering and duplication is immaterial. Note that Axioms (B3) and (B4) can actually be deduced from Axioms (A1)–(A6), (B1), and (B2), by induction on the structure of C . Because of their fundamental nature, however, we have included them as first-class axioms.

We concentrate on explaining the remaining, more interesting axioms. Axiom (C1) describes that, if the firing of a transition merely reproduces in its action some of the events required by its trigger, then we might just as well not fire the transition at all. Hence, it is equivalent to configuration 0. Axiom (C2) states that by adding in parallel to a configuration C a guarded version I/C of it, the behavior remains unchanged. This is intuitively clear since the extra I/C component, when and if it fires at all, only produces the behavior encoded by C , which is already present anyway. Logically speaking, guarding is a *weakening* operation. Axiom (C3) is perhaps the most important equation, as it emulates the firing of transitions. The left-hand side $A \parallel A, I/C$ represents a situation in which some events A have become present while at the same time there is a pending transition $A, I/C$ that is waiting, among other preconditions I , for the events in A . Hence, it is safe to cancel out all dependencies of C on A and to replace $A, I/C$ by I/C . Logically, Axiom (C3) is a version of the *cut* rule. Axiom (C4) deals with inconsistencies in triggers. If a configuration C is guarded

by a trigger P, \overline{N} in which some event is required to be both present and absent, i.e., $P \cap \overline{N} \neq \emptyset$, then this guarded configuration will never become active. In this case, $P, \overline{N}/C$ is equivalent to the inactive configuration 0.

The remaining Axioms (D1)–(D3) are concerned with conflicts between the trigger and action parts of a transition. They axiomatize the effect of transitions that produce a failure under certain trigger conditions. More precisely, these axioms involve a transition $P, \overline{N}/A$ with $N \cap A \neq \emptyset$, whose firing leads to a global inconsistency. Such a transition rejects the completion of all macro steps in which its trigger P, \overline{N} is true. Thus, since $P, \overline{N}/A$ can never fire in a consistent way, the step construction cannot terminate in a situation in which trigger P, \overline{N} is true. In other words, whenever all P have become present, the step construction must continue until at least one event in N is present, in order to inactivate the transition. If this does not happen, the step construction fails. Axioms (D1)–(D3) formalize three different consequences of this. Axiom (D1) reflects the fact that, since $P, \overline{N}/A$ can never contribute to a completed step, if $N \cap A \neq \emptyset$, we may add arbitrary other events B to its action, without changing its behavior. Logically, this axiom corresponds to the laws $e \wedge \neg e \equiv \text{false}$ and $\text{false} \supset B$, for any B . Axiom (D2) offers a second way of reading the inconsistency between triggers and actions. Since at completion time any event e is either present or absent, the same rejection that $P, \overline{N}/A$ produces can be achieved by $P, \overline{N}, e/A \parallel P, \overline{N}, \overline{e}/A$. This is because if e is present at completion time, then $P, \overline{N}, e/A$ raises the failure; if e is absent, then $P, \overline{N}, \overline{e}/A$ does the job. This is the law $\neg\neg(x \vee \neg x)$ in intuitionistic logic. Finally, consider Axiom (D3) that encodes the following intuition. Instead of saying that $P, \overline{N}/A$ generates a failure, if all events in P are present and all events in N are absent, we might say that, if all events in N are absent, then at least one of the events in P must be absent, provided the step under consideration is to be completed without failure. But then any parallel component of the form \overline{N}/C , which becomes active on the absence of all events in N , can be replaced by the parallel composition $\parallel \{\overline{N}, \overline{e}/C : e \in P\}$. The reason is that, if \overline{N}/C fires at all in the presence of transition $P, \overline{N}/A$, then at least one of the weaker transitions $\overline{N}, \overline{e}/C$ will be able to fire at some point, depending on which of the events in P it is that will be absent to avoid failure. Again there is a logic equivalent for this, namely the intuitionistic law $\neg(p_1 \wedge p_2) \equiv \neg p_1 \vee \neg p_2$ that holds for *linear* Kripke models.

Last, but not least, it is important to note that configuration $P, \overline{N}/A$, for $N \cap A \neq \emptyset$, behaves not the same as configuration 0, since the former inevitably produces a failure if its trigger is true, while 0 does not respond at all, not even by failure. As expected, all of our axioms can be proved correct.

Theorem 1 (Correctness). *Let C_1, C_2 be configurations such that $\vdash C_1 = C_2$. Then, $C_1 \simeq C_2$.*

Proof. The correctness of each of our axioms can be established directly along our notion of sequence models. However, since this is exactly the standard interpretation of propositional intuitionistic formulas over finite linear Kripke structures, one may simply employ the wealth of knowledge on intuitionistic logic for the proof [22].

Here we only present the proof of Axiom (D2) as an example. First observe that Axiom (D2) is interderivable with the equivalence $\overline{N}/A = e, \overline{N}/A \parallel \overline{N}, \overline{e}/A$ by Axioms (A5) and (A6), whence it suffices to prove the latter. Let $M = (M_1, M_2, \dots, M_n)$ be a sequence structure and assume $M_i \models \overline{N}/A$, for all $1 \leq i \leq n$. It is trivial to show that, for any choice of $e \in Ev$, we have $M_i \models e, \overline{N}/A$ and $M_i \models \overline{N}, \overline{e}/A$. We thus concentrate on proving the other direction. Suppose we know that $M_i \models e, \overline{N}/A$ and that $M_i \models \overline{N}, \overline{e}/A$, for all $1 \leq i \leq n$ and $N \cap A \neq \emptyset$. We claim that $M_n \cap N \neq \emptyset$, from which $M_i \models \overline{N}/A$ follows trivially. Assume otherwise, i.e., $M_n \cap N = \emptyset$. Then, since in the final world M_n either $e \in M_n$ or $e \notin M_n$, the assumptions $M_n \models e, \overline{N}/A$ and $M_n \models \overline{N}, \overline{e}/A$ would imply $A \subseteq M_n$. Because of $N \cap A \neq \emptyset$, this contradicts $M_n \cap N = \emptyset$. \square

4 Completeness

This section proves our axiomatization to be complete with respect to the step congruence \simeq , i.e., whenever $C_1 \simeq C_2$ we can transform C_1 into C_2 by equational reasoning based on our axioms. The proof of completeness employs a notion of *normal form*: we first show that every configuration can be rewritten into one in normal form using our axioms and then establish the desired completeness result for configurations in normal form. Our notion of normal form is inspired by our previous studies of the sequence–model semantics of Statecharts and, in particular, by a characterization of this semantics in terms of semi–lattice structures [14]. The intuition behind normal forms, however, can also be understood in isolation.

The purpose of the normal form is to lay out explicitly, in a canonical syntactic form, the behavior offered by a configuration relative to a fixed and finite set of relevant events. Typically, these are all the events that occur in the configurations we wish to normalize. For simplicity, let us take Ev to be this finite set; the complement $A^c = Ev \setminus A$ of any set $A \subseteq Ev$ is then also finite. A normal form relative to Ev is a parallel composition of simple transitions

$$(\parallel_{i \in I} P_i, \overline{N}_i/A_i) \parallel (\parallel_{j \in J} E_j, \overline{E}_j^c/Ev).$$

The transitions are grouped into two categories, indexed by I and J , respectively. The former category encodes individual, partial or complete, step reactions, whereas the latter category records the conditions under which the step construction fails (to complete). A transition $P_i, \overline{N}_i/A_i$ of the first kind specifies that, if the events in P_i are known to be present and those in N_i are absent, then A_i is one possible reaction of the configuration to P_i . A transition $E_j, \overline{E}_j^c/Ev$ of the second kind enforces that the step construction cannot consistently complete with just the events in E_j present. In order to complete the reaction, at least one event outside E_j must become available as well. In the light of this discussion, a normal form may thus be seen as a “response table,” where, given a set of environment events, one may look up the associated partial or complete step reaction or learn about immanent failure. This response–table interpretation is

reflected in a number of structural properties on normal-form configurations, which are made precise in the following definition.

Definition 1 (Normal form). *A configuration C is in normal form, if it has the shape*

$$(\|_{i \in I} P_i, \overline{N_i/A_i}) \parallel (\|_{j \in J} E_j, \overline{E_j^c/Ev}),$$

where I, J are disjoint finite index sets, $E_j \subsetneq Ev$, for all $j \in J$; and if it satisfies the following conditions:

1. $P_i \subseteq A_i$ and $P_i \cap N_i = \emptyset$, for all $i \in I$;
2. $B \models C$ iff $\forall j \in J. B \neq E_j$;
3. $B \models C$ iff $\exists i \in I. B = N_i^c$;
4. $B \models C$ and $P \subseteq B$ implies $\exists i \in I. P_i = P$ and $B = N_i^c$;
5. $(P_i, N_i^c)^* = A_i$, for all $i \in I$ with $N_i \cap A_i = \emptyset$; and
6. $N_i \cap A_i = \emptyset$, for all $i \in I$.

where $(P, N)^* =_{df} \bigcap \{E : (E, N) \models C, P \subseteq E \subseteq N\}$ and $B \subseteq Ev$ arbitrary.

Conds. (2)–(5) encode structural properties that refer to our model-theoretic semantics. It is through these that the normal form obtains its essential semantic nature. The other conditions, Conds. (1) and (6), are simple local consistency requirements. Note that the side condition $N_i \cap A_i = \emptyset$ of Cond. (5) is redundant due to Cond. (6); however, its presence will simplify matters later.

Proposition 2 (Normalization). *For every configuration C there exists a configuration C' in normal form such that $\vdash C = C'$.*

Proof. Let C be an arbitrary configuration. Because of Axioms (A3)–(A6) we may assume without loss of generality that C is given as a flat parallel composition of simple transitions $P, \overline{N^c/A}$. We will rewrite C using our axioms in six steps, obtaining configurations C_i , for $1 \leq i \leq 6$, such that C_i satisfies normal-form Conds. (1) through (i). We say that C_i is in i -normal form, or i -nf for short. At each stage we define the J -part of C_i to be the collection of all transitions of the form $B, \overline{B^c/Ev}$, where $B \subsetneq Ev$. All other transitions make up the I -part. In this way each C_i naturally splits into the form $(\|_{i \in I} P_i, \overline{N_i/A_i}) \parallel (\|_{j \in J} E_j, \overline{E_j^c/Ev})$ such that $E_j \subsetneq Ev$, for all $j \in J$.

We will employ the associativity and commutativity Axioms (B1) and (B2) for parallel composition without explicit mentioning, whenever convenient. Furthermore, observe that, since all C_i have the same semantics, it does not matter whether we read validity \models in Conds. (2)–(5) relative to C or C_i .

1. Assume Cond. (1) is violated by a transition $P, \overline{N/A}$ in C , i.e., $P \not\subseteq A$ or $P \cap N \neq \emptyset$. In the second case we can simply drop the transition because of Axioms (C4) and (B4). In the former case, we can transform $P, \overline{N/A}$ so that Cond. (1) is satisfied:

$$\begin{aligned} \vdash P, \overline{N/A} &= P, \overline{N/A} \parallel 0 && \text{(B4)} \\ &= P, \overline{N/A} \parallel P, \overline{N/P} && \text{(C1)} \\ &= P, \overline{N/(A \parallel P)} && \text{(A6)} \\ &= P, \overline{N/A}, P && \text{(A2)} \end{aligned}$$

Making these first adjustments yields C_1 , with $\vdash C = C_1$, where C_1 is in 1-nf. All successive transformations to C_1 either introduce new transitions that satisfy Cond. (1) or, if not, we can repeat this step to clean out or transform the transitions such that Cond. (1) does hold.

2. Next we consider Cond. (2), starting off with direction (\implies). Let $B \models C_1$, i.e., B is a classical model, and further $B = E_j$ for some $j \in J$. Then $B \subsetneq Ev$ and also $Ev \subseteq B$, since $B \models B, \overline{B^c}/Ev$. This is an obvious contradiction, whence this direction of Cond. (2) is automatically fulfilled for C_1 .

For the other direction (\impliedby) we show that, for any $B \not\models C_1$, the equivalence $\vdash C_1 = C_1 \parallel B, \overline{B^c}/Ev$ is derivable. If we apply this for every such B we get our desired 2-nf C_2 , subsuming the new transitions $B, \overline{B^c}/Ev$ in the J -part of the 2-nf. Note that always $Ev \models C$, which means $B \subsetneq Ev$ in such a case. The transformation $\vdash C_1 = C_1 \parallel B, \overline{B^c}/Ev$ is obtained in the following fashion. Since by assumption $B \not\models C_1$, there must be some transition $P, \overline{N}/A$ in C_1 such that $B \not\models P, \overline{N}/A$. Hence, $\vdash C_1 = C'_1 \parallel P, \overline{N}/A$, where C'_1 stands for C_1 without the transition $P, \overline{N}/A$. Now observe that $B \not\models P, \overline{N}/A$ implies $P \subseteq B$ and $N \cap B = \emptyset$, but $A \not\subseteq B$. We then reason as follows, abbreviating $P, \overline{N}/A \parallel B, \overline{B^c}/B$ by D .

$$\begin{aligned}
 & \vdash P, \overline{N}/A \\
 & = P, \overline{N}/A \parallel B, \overline{B^c}/B && \text{(B4,C1)} \\
 & = D \parallel B, P, \overline{B^c}, \overline{N}/A \parallel B, P, \overline{B^c}, \overline{N}/B && \text{(C2, A5, twice)} \\
 & = D \parallel B, \overline{B^c}/A \parallel B, \overline{B^c}/B && \text{(} P \subseteq B, N \cap B = \emptyset, \text{ i.e., } N \subseteq B^c \text{)} \\
 & = D \parallel B, \overline{B^c}/(A \parallel B) && \text{(A6)} \\
 & = D \parallel B, \overline{B^c}/A, B && \text{(A2)} \\
 & = D \parallel B, \overline{B^c}/Ev && \text{(D1, } A \not\subseteq B, \text{ i.e., } A \cap B^c \neq \emptyset \text{)} \\
 & = P, \overline{N}/A \parallel B, \overline{B^c}/B \parallel B, \overline{B^c}/Ev \\
 & = P, \overline{N}/A \parallel B, \overline{B^c}/Ev && \text{(A6, A2)}
 \end{aligned}$$

This shows $\vdash C_1 = C'_1 \parallel P, \overline{N}/A = C'_1 \parallel P, \overline{N}/A \parallel B, \overline{B^c}/Ev = C_1 \parallel B, \overline{B^c}/Ev$.

3. The direction (\implies) of Cond. (3) can be trivially satisfied by inserting parallel transitions $B, \overline{B^c}/B$ for those $B \subsetneq Ev$ that satisfy $B \models C_2$, via Axioms (B4) and (C1). This preserves Conds. (1) and (2). Note that we accommodate $B, \overline{B^c}/B$ in the I -part.

Suppose direction (\impliedby) is violated by $B \not\models C_2$, for which there exists a transition $P_i, \overline{N}_i/A_i$ with $B = N_i^c$ in the I -part of C_2 . We must have $N_i = B^c \neq \emptyset$, for otherwise $B = \overline{N}_i^c = Ev$, contradicting $B \not\models C_2$. By Cond. (2) there exists a transition $B, \overline{B^c}/Ev$ in the J -part of C_2 . Hence,

$$\vdash C_2 = C'_2 \parallel P_i, \overline{B^c}/A_i \parallel B, \overline{B^c}/Ev.$$

We distinguish several cases. If $A_i = \emptyset$, then $P_i, \overline{B^c}/A_i$ is the same as $P_i, \overline{B^c}/\emptyset$ which can be eliminated from C_2 right away by Axioms (A1), (A4), and (B4). If $P_i \cap B^c \neq \emptyset$ we can drop $P_i, \overline{B^c}/A_i$ by way of Axioms (B4) and (C4). Hence, assume that $A_i \neq \emptyset$ and $P_i \cap B^c = \emptyset$. Now, if $B = \emptyset$, then $B^c = Ev$ and $P_i = \emptyset$. Thus, we use Axioms (A2) and (A6) to derive $\vdash P_i, \overline{B^c}/A_i \parallel B, \overline{B^c}/Ev =$

$\overline{Ev}/A_i \parallel \overline{Ev}/Ev = \overline{Ev}/Ev = B, \overline{B^c}/Ev$, which gets rid of the culprit $P_i, \overline{B^c}/A_i$. It remains to tackle the situation in which $B \neq \emptyset$. But then, since also $B^c \neq \emptyset$, we can use the equational rewriting

$$\begin{aligned} & \vdash P_i, \overline{B^c}/A_i \parallel B, \overline{B^c}/Ev \\ & = \overline{B^c}/(P_i/A_i) \parallel B, \overline{B^c}/Ev & (A5) \\ & = \parallel \{ \overline{B^c}, \overline{e}/(P_i/A_i) : e \in B \} \parallel B, \overline{B^c}/Ev & (D3) \\ & = \parallel \{ P_i, \overline{B^c}, \overline{e}/A_i : e \in B \} \parallel B, \overline{B^c}/Ev & (A5) \end{aligned}$$

to replace in C_2 , effectively, the offending $P_i, \overline{B^c}/A_i$ by the parallel composition of transitions $P_i, \overline{B^c}, \overline{e}/A_i$, for $e \in B$, each of which has a negative trigger strictly larger than the one in $P_i, \overline{B^c}/A_i$ we started off with.

By iterating these transformations over all B 's and i 's such that $B \not\sqsubseteq C_2$ and $B = N_i^c$, Cond. (3) (\Leftarrow) can be achieved. The normalization must terminate since the sets B to consider become smaller and smaller in the process. Note that the resulting configuration C_3 also satisfies Conds. (1) and (2), whence it is in 3-nf.

4. Cond. (4) may be achieved by inserting into C_3 the transitions $P, \overline{B^c}/P$, for all P, B such that $P \subseteq B \models C_3$. The insertions may be done via Axioms (B4) and (C1). Note that the resulting configuration C_4 still satisfies Conds. (1)–(3), since $P \subseteq B$ is equivalent to $P \cap \overline{B^c} = \emptyset$; whence it is in 4-nf.
5. Consider an arbitrary transition $P_i, \overline{N_i}/A_i$, satisfying $N_i \cap A_i = \emptyset$, in the I -part of C_4 . We will show how to enforce Cond. (5) for this transition. Under the assumptions, we know $P_i \subseteq A_i \subseteq N_i^c$ and $N_i^c \models C_4$ by Conds. (1) and (3). In order to show $(P_i, N_i^c)^* = A_i$, it is sufficient to establish the following two properties:

- (a) $(A_i, N_i^c) \models C_4$; and
- (b) $(X, N_i^c) \models C_4$ and $P_i \subseteq X \subseteq N_i^c$ implies $A_i \subseteq X$, for any $X \subseteq Ev$.

Assume that Property (5a) is not yet satisfied, i.e., $(A_i, N_i^c) \not\models C_4$. Then, there must be a transition $P, \overline{N}/A$ in C_4 such that $(A_i, N_i^c) \not\models P, \overline{N}/A$. This transition could be of the form $P_k, \overline{N_k}/A_k$, for some $k \in I$, or of the form $E_j, \overline{E_j^c}/Ev$, for some $j \in J$.

Because of $N_i^c \models C_4$, we have $N_i^c \models P, \overline{N}/A$. But then, $(A_i, N_i^c) \not\models P, \overline{N}/A$ means that $(A_i, N_i^c) \models P, \overline{N}$ and $(A_i, N_i^c) \not\models A$. Hence, in particular, $A_i \supseteq P$, $A_i \not\supseteq A$, and $N_i^c \cap N = \emptyset$, i.e., $N \subseteq N_i$.

We now show that $P_i, \overline{N_i}/A_i \parallel P, \overline{N}/A = P_i, \overline{N_i}/A_i, A \parallel P, \overline{N}/A$ by the following calculations, where $N_1 =_{\text{df}} N_i \setminus N$ and $A_1 =_{\text{df}} A_i \setminus P$:

$$\begin{aligned} & \vdash P_i, \overline{N_i}/A_i \parallel P, \overline{N}/A \\ & = P_i, \overline{N_1}, \overline{N}/A_1, P \parallel P, \overline{N}/A & (A2, A6) \\ & = P_i, \overline{N_1}, \overline{N}/A_1 \parallel P_i, \overline{N_1}, \overline{N}/P \parallel P, \overline{N}/A & (A5, A6) \\ & = P_i, \overline{N_1}, \overline{N}/A_1 \parallel \overline{N}/(P_i, \overline{N_1}/P \parallel P/A) & (C2) \\ & = P_i, \overline{N_1}, \overline{N}/A_1 \parallel \overline{N}/(P_i, \overline{N_1}/P \parallel P_i, \overline{N_1}/(P/A) \parallel P/A) & (A6) \\ & = P_i, \overline{N_1}, \overline{N}/A_1 \parallel \overline{N}/(P_i, \overline{N_1}/(P \parallel P/A) \parallel P/A) & (C3) \\ & = P_i, \overline{N_1}, \overline{N}/A_1, P, A \parallel P, \overline{N}/A & (A2, A5, A6) \\ & = P_i, \overline{N_i}/A_i, A \parallel P, \overline{N}/A \end{aligned}$$

This allows us to replace transition $P_i, \overline{N_i}/A_i$ for which $(A_i, N_i^c) \not\models C_4$ by the transition $P_i, \overline{N_i}/A_i, A$. If now $N_i \cap (A_i \cup A) = \emptyset$, i.e., $A_i \cup A \subseteq N_i^c$, then we find $(A_i \cup A, N_i^c) \models P, \overline{N}/A$ and $A_i \cup A \supseteq A_i$ since $A_i \not\supseteq A$. Thus, using this technique, one can saturate the A_i until, for all transitions $P, \overline{N}/A$, there exists no $i \in I$ such that $N_i \cap A_i = \emptyset$ and $(A_i, N_i^c) \not\models P, \overline{N}/A$. This will ensure Property (5a), for all $i \in I$.

It remains to establish Property (5b). Let $(X, N_i^c) \models C_4$ for some $X \subseteq Ev$ such that $P_i \subseteq X \subseteq N_i^c$. Hence, $(X, N_i^c) \models P_i, \overline{N_i}/A_i$. Since $(X, N_i^c) \models P_i, \overline{N_i}$ we consequently know that $(X, N_i^c) \models A_i$, i.e., $A_i \subseteq X$ as desired. Let C_5 denote the 5-nf configuration resulting from this normalization step.

6. Let us assume that some transition $P, \overline{N}/A$ in C_5 violates Cond. (6). Then, using Axiom (D1) we rewrite $\vdash P, \overline{N}/A = P, \overline{N}/Ev$ first, and then by repeated applications of Axiom (D2) we obtain $\vdash P, \overline{N}/Ev = \|\{E, \overline{E^c}/Ev : P \subseteq E \subseteq N^c\}$. In this way, the offending original transition $P, \overline{N}/A$ in C_5 can be eliminated completely in terms of transitions indexed by J . This establishes Cond. (6) and does not destroy any of the conditions previously established. The result is a 6-nf C_6 with $\vdash C = C_6$.

Configuration C_6 is now the desired normal form of C . \square

Having Prop. 2 in hand, the completeness proof for our axiomatization is now a simple exercise.

Theorem 2 (Completeness). *Let C_1 and C_2 be arbitrary configurations such that $C_1 \simeq C_2$. Then, $\vdash C_1 = C_2$.*

Proof. Let C_1, C_2 be given such that $C_1 \simeq C_2$. We may assume by Prop. 2 that C_1, C_2 are both in normal form. It suffices to show that every parallel component, i.e., transition, of C_1 also occurs in C_2 ; by symmetry also the reverse will hold. Then the completeness statement follows by suitably employing Axioms (B1)–(B4) for parallel composition.

Consider a transition of the form $P_i, \overline{N_i}/A_i$ occurring in C_1 . Since C_1 is in normal form we know by Cond. (3) that $N_i^c \models C_1$. Hence, by premise $C_1 \simeq C_2$, we have $N_i^c \models C_2$. We may now apply Cond. (4), since $P_i \subseteq N_i^c$ by Cond. (1), to obtain some index $i' \in I$ such that $P_{i'}, \overline{N_{i'}}/A_{i'}$ is a transition in C_2 with $N_{i'} = N_i$ and $P_{i'} = P_i$. By Cond. (5) of normal forms $A_{i'} = (P_{i'}, N_{i'}^c)^* = (P_i, N_i^c)^* = A_i$, as desired. Note that the definitions of $(P_{i'}, N_{i'}^c)^*$ in C_2 and $(P_i, N_i^c)^*$ in C_1 coincide, because of $C_1 \simeq C_2$.

Consider a transition of the form $E_j, \overline{E_j^c}/Ev$ in C_1 . Since C_1 is in normal form we know by Cond. (2) that $E_j \not\models C_1$. Hence, $E_j \not\models C_2$ by the premise $C_1 \simeq C_2$. Further, by Cond. (2) applied to normal form C_2 we conclude the existence of some $j' \in J$ such that $E_{j'}, \overline{E_{j'}^c}/Ev$ is a transition in C_2 with $E_{j'} = E_j$. \square

5 Discussion and Related Work

There exists a wealth of related work on the semantics of synchronous languages, especially Statecharts [23]. Our paper focused on the most popular original se-

mantics of Harel's Statecharts, as defined by Pnueli and Shalev in their seminal paper [20]. Since this semantics combines the synchrony hypothesis and the causality principle, it cannot be compositional if step reactions are modeled by input–output–functions on event sets, according to a result by Huizing and Gerth [9]. Within the traditional style of labeled transition systems, researchers have then concentrated on providing compositionality for Pnueli and Shalev's semantics either by taking transition labels to be partial orders encoding causality [11, 17, 21] or by explicitly including micro–step transitions [16]. Our step algebra is related to the former kind of semantics, where causality is encoded via intuitionistically interpreted sequence structures. However, in contrast to the other mentioned work, our logical approach lends itself to establishing full–abstraction results [12, 14] and the equational axiomatization of Statecharts macro steps presented here.

A different approach to axiomatizing Statecharts was suggested by de Roever et al. for an early and later on rejected Statecharts semantics that does not obey global consistency [6]. In their setting, it is admissible for a firing transition to generate an event, whose absence was assumed earlier in the construction of the macro step under consideration. This leads to a very different semantics than the one of Pnueli and Shalev [20], for which Huizing, Gerth, and de Roever gave a denotational account in [10]. This denotational semantics provided the groundwork for an axiomatization by Hooman, Ramesh, and de Roever [8]. However, in contrast to our work which *equationally* axiomatized the step congruence underlying Pnueli and Shalev's semantics, Hooman et al. supplied a Hoare–style axiomatization for both liveness and safety properties of Statecharts, which was proved to be sound and complete with respect to the denotational semantics of Huizing et al. [10]. A similar approach was taken by Levi regarding a process–algebraic variant of Pnueli and Shalev's Statecharts and a real–time temporal logic [11]. It should be noted that the settings of de Rover et al. and of Levi deal with sequences of macro steps and not just single macro steps, as our step algebra does. However, extending the step algebra and its axiomatization to sequences of macro steps should not be difficult. In such a more general development, the step algebra introduced here would play the role of a synchronization algebra [24], around which a macro–step process language is built.

The results of this paper are not restricted to Statecharts but can also be applied to other languages, in particular to Berry's Esterel [2, 3]. The authors have shown in [15], using the same model–theoretic framework of intuitionistic sequence structures as for Statecharts, how the instantaneous core of Esterel can be faithfully and compositionally encoded in terms of propositional formulas. This is done in such a way that the operational execution of the encoding produces the same responses as the execution of the original program under the semantics of Esterel [2]. It is not difficult to see that the propositional formulas corresponding to Esterel configurations build a sublanguage in our step algebra, when taking $Ev =_{\text{df}} \{s=1, s=0 : s \text{ is a signal}\}$, where $s=1$ stands for signal s is present 'high' and $s=0$ for s is present 'low'. This sublanguage, however, requires the full syntax of our step algebra, which allows for nested transition triggers.

For example, the instantaneous Esterel program

```
present  $a$  then present  $b$  else emit  $c$  end end
```

would be translated into the configuration $a=1/(b=0/c=1)$; see [15] for details. Because of the existence of this encoding of Esterel reactions into our step algebra, which preserves Esterel’s semantics, the axiomatization presented here can directly be used to reason about Esterel reactions. For the sake of completeness, it needs to be mentioned that some initial work on axiomatizing Esterel has been carried out within an encoding of Esterel programs in a variant of the duration calculus [19]. However, this work aims at an axiomatic semantics for Esterel rather than an equational axiomatization of the underlying step congruence.

The step algebra presented in this paper focused on the most essential operators in synchronous languages. In the future we would like to enrich our algebra and its axiomatization to accommodate an operator for *event scoping*, or signal hiding, which is used in Esterel [2] and Argos [18]. Moreover, instead of encoding the external-choice operator $+$ (as found in the hierarchy operator of Statecharts) via parallel composition and negated events, it is possible to include $+$ as a primitive operator in our step algebra. To do so, one only needs to add a silent, non-synchronizing event in the action of every transition; see [14]. We might mention here that, when axiomatizing external choice, one key axiom will be the “tie-break axiom” $a/b \parallel b/a = a \parallel b + b \parallel a$.

6 Conclusions and Future Work

This paper presented a uniform algebra, to which we referred to as step algebra, for reasoning about step reactions in synchronous languages, such as those originating from Statecharts and Esterel. The algebra’s semantics was inspired by previous work of the authors, which adapted ideas from intuitionistic logics for defining a compositional semantics for these languages. Our main result is a sound and complete axiomatization of the resulting step congruence in our step algebra, whose completeness proof mixes techniques from process algebra and logic. This yields, for the first time in the literature, a complete axiomatization of Statecharts macro steps, in the sense of Pnueli and Shalev. Modulo a simple syntactic translation, this axiomatization can be adapted to instantaneous reactions in Esterel as well. We believe that our approach provides important groundwork for comparing popular synchronous languages by means of axioms, an approach which already proved successful in process algebra, and also for developing compositional verification methods highly needed in this area.

Regarding future work, we plan to extend our step algebra by other operators employed in some synchronous languages, in particular by operators for event scoping. While this will probably lead to slight changes in our semantics, we expect to carry over the style of axiomatization and the techniques for proving the resulting axiom set complete.

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