CHILDREN'S UNDERSTANDING OF NUMBER PATTERNS

Anna Susanne Steinweg
University of Dortmund, Germany
susanne.steinweg@mathematik.uni-dortmund.de

This article deals with children's reactions when they are confronted with problems concerning number patterns. In German schools, problems concerning patterns and number patterns are rare, some curricula do not even mention them.
Therefore my documented tests are the first contact with these problems for most children. This is also a big opportunity for my research, because the aim is to show the genuine reactions of the children. The main topic of this article will be some results of a school-project with 9 to lO-yearolds.

1 Introduction

Like I already mentioned in the abstract, tasks about number patterns or tasks dealing with the inner mathematical structure are barely found in German Schools. Every Federal State in Germany is allowed to create its own curriculum for primary and secondary schools, and this law is used. The differences in German curricula are considerable, and the differences are not only in an exchange of contents, they are qualitative differences!

In North Rhine-Westphalia structure orientation is established as the principle for lessons. The curriculum gives the following definition:

"Structure orientation: Mathematics lessons have to make visible the rules, laws and formulas that stamp order and organisation onto an appearance. Within this view the learning of mathematics consists to a major part of finding and describing structures." (Kultusministerium NRW, 1985, p.25) 1. The acknowledgement of equality to application-oriented mathematics is a great opportunity in the NRW curriculum for mathematics. But the latest curricula lead in a completely different direction. Application-orientation and classroom-projects are considered to be revolutionary. A connection to the children's life is searched for in every task. If none is found, the tasks are happily left out, without recognising that this cuts off an important branch of mathematics.

It is often argued that the children should not be bored with pure 'calculation', that such tasks are out of the children's field of interest and therefore too much to handle. But already DEWEY (18591952) had found out (in BOYDSTON, 1985, p.353): "Teacher who have heard that they should avoid matters foreign to pupils' experience, are frequently surprised to find pupils wake up when something beyond their ken is introduced, while they remain apathetic in considering the familiar." For the children I worked with during my research the tasks about number patterns were in no way *familiar,* and I would like to present an excerpt of the analysis of reactions and answers.

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[1] translated by the author

2 Some Results of a School-Project

I will not and cannot give information about if number patterns pave the way for algebra, like the National Curriculum wants us to believe. The focal point of my research is not on finding a possible connection between algebra and the arithmetic of elementary lessons, but on the observation of the first contact between child and task. For that purpose app. 260 children took paper pencil tests as part of a pilot study, after that 60 interviews were held in all 4 grades of German Primary School (6 to 10-year-olds), and finally there has been a school-project which was realised during the period from April 20th 1998 to June 24th 1998 in two 4th grades (9 to lO-year-olds).
Of course, the pupils taking part in this project showed very different reactions to the tasks. Unfortunately it is not possible to describe the whole range of different types of pupils that were observed here. I therefore tried to choose two very different children (Lena and Andreas) as examples. Lena is regarded as a so called high-achiever and Andreas is marked as a low-achiever. In away, these children are the poles of a scale of reactions. Therefore it is possible to give you an overview about the opportunities that lessons about number patterns have offered for the children and how they have been received.
A main focus of the analysis lies on how far the children used the three steps for understanding number patterns

- Seeing the pattern
- Describing the pattern - oral and written
- Explaining the pattern - oral and written

and if it is possible to show preferences for specific types of number patterns. The task which were offered during the project can be categorised in three different types:

Type A	recognising and continuing the pattern (pattern is created by numbers)	e.g. sequences, numberwalls, number&partner-number
Type B	recognising and continuing (geom. objects creating a shape or dot pattern)	e.g. dot pattern, sequence of geom. objects
Type C	recognising and repairing the pattern, offering solutions	e.g. series of tasks

To show the different requirements of the types some examples oftasks and solutions are presented in the following. These examples can also show the variety of procedures which were used by the children.

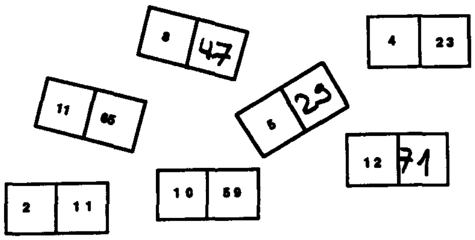

fig. 1

The number&partner-number task belongs to Type A. The underlying mathematical structure of this task is a function which assigns a y to every x. Both pupils are able to complete pairs of numbers which were given to them. They also formulate a rule that leads to the results. Lena comments on the following number&partner-numbers (fig. 1): "Always times 6 and then minus I". Andreas uses a shorter form by writing: "Always x 6 - I".

One important aspect in dealing with number pattern is the opportunity to create own patterns. Andreas did not create a new pattern while Lena presented the following (fig. 2) to her classmates.

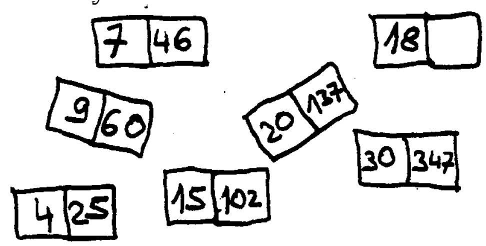

fig. 2

It is important to see that she does not go for an easy solution which can be found by describing a pure addition or a pure multiplication. She uses a combination of both which is far more sophisticated.

Dot pattern are a typical example for Type B tasks. Many children - like Andreas - had great difficulties to find the next dot pattern in a sequence despite the "obvious" structure of the dots. Often the arithmetical pattern and the number of dots were in the focus of interest. It was looked for the next element which fits in the arithmetical structure or which copies a detected increase from one element to another. The spatial arrangement was often neglected.

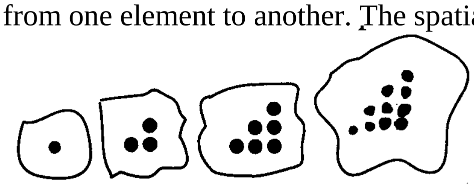

fig. 3

Lena was able to find the correct answers. The example shows her solution of the sequence of triangular numbers (fig. 3).

You can see in her explanation that she is aware of the structure behind the pattern. She knows that the increase of the sequence of the triangular numbers follows the sequence of the natural numbers. Lena understands the dot pattern while Andreas is not even able to see the geometrical structure of the dots.

I do not think that working with number patterns without the question 'why?' is in principle no mathematics, like Kerslake (1994) tries to show. I rather think that it is a form of mathematics. Nevertheless the aim should be to make the children ask why.

Tasks of Type A and B offer various possibilities to describe patterns. In Type C a further explanation of the patterns is stimulated (see, for instance, STEINWEG, 1997). Type C tasks are a series of tasks with always surprising results. One has to further examine these tasks to find out why these results occur.

Andreas had major difficulties to spot the patterns because he was often lacking the arithmetical skills to solve the tasks. Without a solution it is often difficult to see the pattern at all.

Lena often offers an explanation of the phenomena in addition to the arithmetical solution. For example, she uses the place value table for her explanation of those series in which the sum and the first addend are inverted numbers:

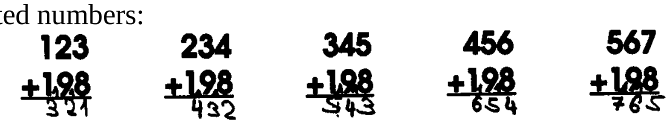

fig. 4a

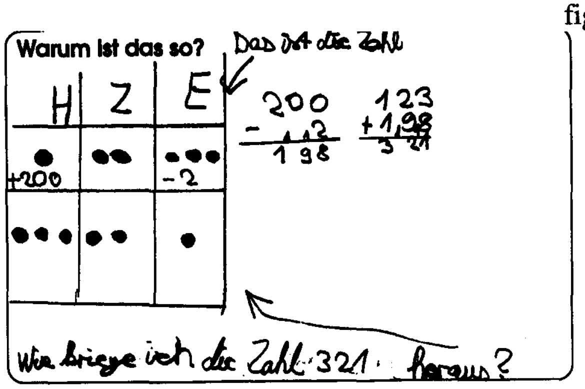

fig. 4b

Her diagram (fig. 4b) shows the process of finding the solution. In the first line of the place value table she writes down the given number ("That's the number.") At the bottom she asks "How do I get the number 321 ?". The solution is made clear by the counters and the calculations. In the classroom discussion she was able to show that she was aware that these process of transformation can be used for the other numbers, because the difference between the hundred digit and the unit digit is always 2.

3 Conclusion

Because these two children are only two possible examples out of 46, I would like to add a quick overview about a final test which has been taken by (almost) all children of the two classes and which again should show the preferences and probabilities for solution of the different types of tasks. If one looks at the children's results sorted by types, one gets the following picture:

	TYPE A	TYPE B	TYPE C
correct	66,59	63,96	75,58
aspects of the pattern have been considered	12,33	5,81	3,49
no recognisable connection with the pattern	5,97	9,30	8,14
not worked on	15,11	20,93	12,79
total	100	100	100

fig. 5

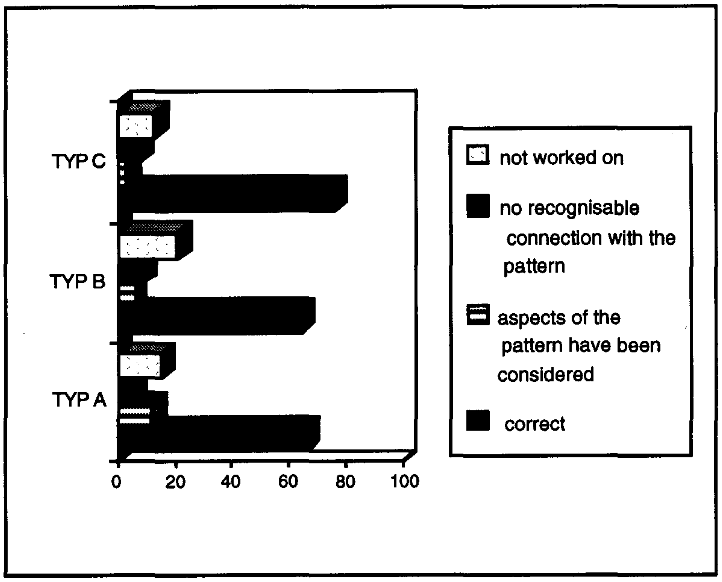

fig. 6

The children have no obvious preference for any type. It is remarkable that Type C tasks come in first with 75.58%. They had been considered as more difficult in the run-up, but obviously for the children they were not. Certainly the project lessons had their share in this results, because the children were familiar with this kind of tasks and therefore had not to search blindly for *some* pattern in the series. This aspect strikes me as remarkable, because in no time at all the pupils have developed a nose for number pattern tasks which were previously unknown to them. This could not result from corresponding instructions alone. Also such instructions were not given during the project lessons. The tasks were just presented, and the different solutions of the pupils were compared in a classroom talk.

Type A and B showed similar values. However, it can be observed that more tasks of Type B have not been worked on then of any other type. If one combines the solutions where aspects of the pattern had been considered and the correct solutions, Type A ranks before Type B. This is remarkable, because no knowledge about numbers is required for Type **B,** and therefore they should be "more easy" to solve than the tasks about sequences and numberwalls.

It can be shown that *every* type of pupil can get access to number patterns. The differentiation in dealing with them is made naturally the intensity and precision of the children's descriptions, not by a variation of the tasks in advance.

Tasks of Type B offer access to number patterns in two ways. First, the hidden arithmetic of this tasks is not that obvious, so low-achiever and average pupils as well try to work on the dot pattern numbers. Secondly these tasks deviate considerably from the mathematics tasks experienced to date, it is not supposed that the tasks are insoluble from the start, and thereby a new access to mathematics is made possible.

Good mathematical skills and linguistic competence could be of help for working on number patterns. The discoveries are written down easier and more distinct. This does not mean that children who have linguistic difficulties are not able to recognise number patterns, but they have more difficulties to communicate about them. Because number patterns are something new, open up a new field of mathematics, the children are demanded to develop a new language: "At every stage of mathematical education we find that new discoveries demand the development of a language that gives the neatest expression to the relations and operations that are being considered. The way is than open for further discoveries through the use of the ideas expressed in language." (Wll.LIAMS/SHURAD, 1982,9)

If a description or explanation of the patterns is demanded, a good general linguistic usage can be an optimal assistance to the mathematical text. To find a mathematical language that is more accurate and precise it is however necessary to introduce new conventions or furnish the children with certain means of description, for example graphics, place value tables etc. For example, the use of the word "always" to describe a regularity was especially fruitful. In the comments to number & partner-number (Type A) the term is adopted by all (types of) pupils. So it should be aimed for an agreement about a language which can be understood by all participants.

Some patterns only appear after solving a column of tasks. If there are miscalculations in the solutions, the pattern is often considerably disturbed and therefore can not be found. This means that a good skill in calculation assists the ability to deal with number patterns. But in no way it can be said that because of this number patterns are only suitable for lessons with pupils who are high achievers. Other examples definitely showed that especially pupils who where supposed to be lowachievers are able to see the general pattern of a task - maybe even better then the high-achievers, who try to integrate all phenomena they have detected into their description.

The reactions to number patterns presented show that there are no predictable "simple" or "difficult" tasks in primary lessons. The children show a high degree of individuality and a very differentiate dealing with mathematics and its' structures.

It would be a catastrophe if the development in German curricula would permanently curtail this branch of mathematics lessons. The interest that all types of pupils showed for this tasks contradicts the usual descriptions of what is within the children's field of interest. Apart from that, only a small part of mathematics would be presented in Primary School lessons, and a major part, its inner structure and many associations, would be neglected.

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