

# **Business cycle synchronization in a simple Keynesian macro model with socially-transmitted economic sentiment and international sentiment spill-over \***

Martin Hohnisch<sup>a</sup> and Frank Westerhoff<sup>b\*\*</sup>

<sup>a</sup> University of Bonn, Department of Economics, Adenauerallee 24-42, D-53113 Bonn, Germany

<sup>b</sup> University of Bamberg, Department of Economics, Feldkirchenstrasse 21, D-96045 Bamberg, Germany

## **Abstract**

We propose a simple Keynesian business cycle model in which national income expectations of heterogeneous interacting investors affect their investment decisions. The investors' expectation formation is influenced by their sentiment: investors who hold optimistic views about the future state of the economy expect a higher aggregate demand in the following period and thus invest more than pessimistic investors. The investors' sentiment is, in turn, subject to socio-economic interactions. Simulations show that our model has the potential to generate complex business cycle dynamics. Based on that framework, we provide a three-country model of business cycle synchronization in which spill-over effects on the level of sentiment synchronize national cycles, provided that investors believe that the economies are indeed coupled.

## **Keywords**

Business cycles; expectation formation; socio-economic interactions; optimism and pessimism; synchronization of business cycles.

## **JEL Classification**

D11; E11; E32.

---

\* We thank two anonymous referees and the managing editor (Harald Hagemann) for their many constructive and helpful comments.

\*\* Corresponding author. Email: frank.westerhoff@uni-bamberg.de

## 1 Introduction

Recently, there has been a great deal of interest among economists in the role of interactions-driven socio-economic processes which may lead to important phenomena, such as herding in financial markets (Kirman 1993, Lux 1995) or the formation of economic sentiment in macroeconomic models (Franke 2007a, Westerhoff and Hohnisch 2007). Some of the basic principles of these models have already been proposed by Weidlich and Haag (1983). Since they are not reliant on strong rationality concepts, such models are well-suited for formalizing notions such as “animal spirits” (Keynes 1936).

Following this general approach, the present paper defines and analyzes a modified Keynesian multiplier-accelerator model<sup>1</sup> which is intertwined with an interactions-driven process of individual economic sentiment. In our model, economic sentiment affects investment expenditures via investors’ national income expectations which, in turn, are affected by social interactions and national income movements. To be more precise, we replace the Samuelsonian accelerator term  $i(Y_t - Y_{t-1})$  with an expectation-based term  $i(E[Y_t] - Y_t)$ , i.e. investment expenditures do not depend on past changes in national income but on the current expected change in national income. We then assume that each investor expects an increase in national income in the following period if he is optimistic and a decrease if he is pessimistic. Investors are more likely to become optimistic if national income is increasing, and are more likely to become pessimistic if national income is decreasing, thereby providing a macroscopic feedback from national income movements to expectation formation. Simulations reveal that our simple model is able to produce intricate oscillations in national income.

---

<sup>1</sup> We use a simple goods market setup to be able to pin down some of the causalities acting inside our model. For recent contributions that also rely on the multiplier-accelerator model or related Keynesian approaches, see, e.g. Chiarella, Flaschel and Franke (2005) or Puu and Sushko (2006).

We then use the model to propose a sentiment-based explanation of international business cycle synchronization in a three-country model. The economies become coupled and their cycles synchronized through “sentiment-spillover effects” across the economies if investors believe that the economies are indeed coupled. This effect occurs in our model even without trade between the economies, pointing to the potential role of economic-sentiment-based effects in business cycle synchronization. Our work complements real-sector (trade-based) arguments explaining business cycle synchronization, as put forward by Frankel and Rose (1998), among others.

The structure of the paper is as follows: Section 2 specifies the basic model and analyzes its dynamics. Section 3 extends the basic model into a three-country model in which the emergence of business cycle synchronization due to business-sentiment spill-over between countries is demonstrated. Section 4 concludes the paper with a number of additional comments.

## **2 The basic model**

Our model rests on the multiplier-accelerator model of Samuelson (1939). Accordingly, national income at time step  $t + 1$  is given by

$$Y_{t+1} = C_{t+1} + I_{t+1}, \quad (1)$$

where  $C$  and  $I$  denote aggregate consumption and aggregate investment expenditures, respectively.

Consumption depends on the last period’s national income

$$C_{t+1} = c Y_t. \quad (2)$$

The marginal propensity to consume is denoted by  $0 < c < 1$ .

As in Westerhoff (2006), we do not assume that induced investments are proportional to the observed change in national income between period  $t$  and period  $t-1$  (as in Samuelson 1939), but are proportional to the expected change in national income between period  $t$  and period  $t+1$ . Thus, we specify that

$$I_{t+1} = \bar{I} + i(E[Y_{t+1}] - Y_t), \quad (3)$$

where the first component stands for autonomous investments and the second component reflects the accelerator principle, which now entails expectations.

Expectation formation is sentiment-driven: if an investor is optimistic (pessimistic), he expects national income to increase (decrease) by an exogenous amount  $e > 0$ . The average value of expected national income may thus be written as

$$E[Y_{t+1}] = W_t(Y_t + e) + (1 - W_t)(Y_t - e), \quad (4)$$

where  $W_t$  and  $(1 - W_t)$  are the fractions of optimistic and pessimistic investors, respectively.

Inserting (2)-(4) in (1) yields

$$Y_{t+1} = cY_t + \bar{I} + i(eW_t - e(1 - W_t)). \quad (5)$$

Note that for fixed fractions of optimistic and pessimistic agents, (5) turns into a first-order linear difference equation. Since  $0 < c < 1$ , the model then always generates a monotonic convergence towards the unique fixed point  $\bar{Y} = (\bar{I} + i e(2\bar{W} - 1))/(1 - c)$ . This observation will become important at a later stage.

Let us next give an informal description of how the fraction of optimistic (pessimistic) investors evolves over time. We assume that there are  $N$  investors in total, with each investor being in either an optimistic or a pessimistic state of mind. We follow Kirman (1993) in specifying that in each time period two investors meet at random and the first will adopt the state of mind of the other with a probability of  $1 - \delta$ .

In addition, there is a small probability  $\varepsilon$  that an investor will change his attitude independently. Unlike in Kirman's (1993) model, the probability that one agent may convince another agent is asymmetric in that an optimist is more likely to convert a pessimist when national income is increasing than a pessimist converting an optimist. Similarly, when national income decreases, the chances are higher that a pessimist will convert an optimist than the other way around.

Formally, let  $K_t = W_t N$  denote the number of optimistic agents at time step  $t$ .

The transition probability for the birth-and-death process of  $K$  is specified as follows

$$K_t = \begin{cases} K_{t-1} + 1 & \text{with probability } p_{t-1}^+ = \frac{N - K_{t-1}}{N} (\varepsilon + (1 - \delta_{t-1}^{P \rightarrow O}) \frac{K_{t-1}}{N-1}) \\ K_{t-1} - 1 & \text{with probability } p_{t-1}^- = \frac{K_{t-1}}{N} (\varepsilon + (1 - \delta_{t-1}^{O \rightarrow P}) \frac{N - K_{t-1}}{N-1}), \\ K_{t-1} & \text{with probability } 1 - p_{t-1}^+ - p_{t-1}^- \end{cases}, \quad (6)$$

where the probability that a pessimist converts into an optimist is

$$(1 - \delta_{t-1}^{P \rightarrow O}) = \begin{cases} 0.5 - \gamma & \text{for } Y_{t-1} - Y_{t-2} < 0 \\ 0.5 + \gamma & \text{otherwise} \end{cases} \quad (7)$$

and the probability that an optimist converts into a pessimist is

$$(1 - \delta_{t-1}^{O \rightarrow P}) = \begin{cases} 0.5 - \gamma & \text{for } Y_{t-1} - Y_{t-2} > 0 \\ 0.5 + \gamma & \text{otherwise} \end{cases}, \quad (8)$$

respectively.

We now turn to the dynamics of our model. The simulation run displayed in figure 1 was produced with the following parameter setting:

$$c = 0.9, i = 3.5, \bar{I} = 10, e = 0.5, N = 100, \gamma = 0.45, \varepsilon = 0.05.$$

The top panel of figure 1 shows the evolution of national income, while the bottom panel depicts the number of optimistic agents. The figure demonstrates that our model is

able to generate complex swings in economic activity. Both the duration and amplitude of the business cycles vary significantly over time.<sup>2</sup>

----- **Figure 1 goes about here** -----

Let us now discuss the origin of these business cycles in our model. For fixed fractions of optimistic and pessimistic agents, the evolution of national income is due to the iteration of a first-order linear difference equation. In this case, the level of autonomous expenditures is determined by the relation between optimistic and pessimistic investors. If all agents are optimistic, then total autonomous expenditures are  $\bar{I} + i e$ , so that equilibrium income is  $(\bar{I} + i e)/(1 - c)$ . Should there be as many optimistic as pessimistic agents, then autonomous expenditures amount to  $\bar{I}$  and equilibrium income is  $\bar{I}/(1 - c)$ . Finally, if all agents are pessimistic, autonomous expenditures decrease to  $\bar{I} - i e$ , and national income is equal to  $(\bar{I} - i e)/(1 - c)$ . Since the marginal propensity to consume is below one, our model implies that national income always monotonically approaches equilibrium income and thus may not leave the lower and upper boundaries  $(\bar{I} + i e)/(1 - c)$  and  $(\bar{I} - i e)/(1 - c)$ .

However, the equilibrium income itself undergoes a rather cyclical movement within these upper and lower boundaries. This is illustrated in figure 2 in which we have plotted national income in period  $t+1$  against national income in period  $t$ . The line segments  $K^{\min}$  and  $K^{\max}$  stand for the maps in which all agents are either optimistic or pessimistic. The corresponding equilibrium incomes are given by the intersections of these maps with the 45-degree line. Suppose now that we are in period  $\tau$  and that the fraction of optimistic agents results in map  $K_{\tau}$ . Obviously, national income converges

---

<sup>2</sup> For  $c=0.9$  and  $i=3.5$ , the multiplier-accelerator model of Samuelson (1939), which is a second-order linear difference equation, generates unstable (exploding) oscillations. However, our setup may cope with such parameter values, i.e. it generates enduring bounded business cycles.

within two time steps from  $Y_\tau$  to  $Y_{\tau+\Delta t}$ . Note that if the fraction of optimistic agents increases, map  $K_\tau$  shifts upwards, for instance to  $K_{\tau+\Delta t}$ . Again, we have drawn the evolution of national income for two additional time steps. Once national income increases (decreases), the bias of the social interactions and the income equilibrium movements jointly manage to sustain that movement. A turnaround may set in either randomly or when the upper (lower) boundary of national income is eventually reached.

----- **Figure 2 goes about here** -----

To further clarify the working of the model, a high time-resolution excerpt of the dynamics is presented in figure 3. The number of optimistic investors is depicted on the left-hand side. This number does not often change from one time step to the next. The evolution of national income for the corresponding time slot is presented on the right-hand side. National income changes in each time step according to (5). In addition, the dashed line represents the equilibrium income. Note that around time step 4880, the (relatively high) number of optimistic investors (randomly) decreases. As a result, equilibrium income also decreases and drops below the current national income. Now the herding process reverses. As more investors become pessimistic, a recession is about to set in.

----- **Figure 3 goes about here** -----

In the model of Kirman (1993), the relative values of  $\varepsilon$  and  $1 - \delta$  determine the distribution of the number of optimistic agents. He shows that when the probability of self-conversion is relatively high and the probability of being converted by another agent is relatively low, the distribution of the number of optimistic agents is unimodal and centered around  $K = 50$ . Conversely, when the probability of self-conversion is relatively low and the probability of being converted by another agent is relatively high, the distribution of optimistic agents is bimodal. In this scenario, nearly all agents hold

the same view for some time, but suddenly the whole crowd may switch to the other view. In between these two cases, there is a particular combination of  $\varepsilon$  and  $1 - \delta$  for which the number of optimistic agents is uniformly distributed. Recall that for  $\gamma = 0$ , the opinion formation part of our model is identical to that developed by Kirman.

Figure 4 illustrates how Kirman's results change if  $\gamma$  is positive. The panels show distributions of optimistic agents for different values of  $\gamma$  and  $\varepsilon$ . From left to right,  $\gamma$  takes the values 0.4, 0.45 and 0.5. From top to bottom,  $\varepsilon$  takes the values 0.01, 0.025, 0.05 and 0.1. Each distribution is computed out of 1 million observations. As we can see, other distribution types may emerge. In particular, our bimodal distributions resemble more an M than a U (as in Kirman's model). Although there are major swings in opinion, the herding process usually stops before all agents have turned either optimistic or pessimistic (which may be regarded as more realistic compared to the scenario where all agents in an economy hold the same view). This is even more the case in our benchmark parameter setting to which the central panel in the third line corresponds. We observe strong herding behavior, and the distribution of the number of optimistic agents is rather flat-topped, yet with a low probability mass in the tails of the distribution.

----- **Figure 4 goes about here** -----

Next we seek to explore how the dynamics of our model changes when the number of investors and the number of interactions between two consecutive updates of the Keynesian dynamics increase. In figure 5, we now assume that there are 500 investors (top set of panels) and 1000 investors (bottom set of panels), respectively.<sup>3</sup>

---

<sup>3</sup> If these investors reflect the largest 500 or 1000 firms of an economy, our simple model indeed captures a larger part of the total investment activity. The remaining investment expenditures are then summarized within the autonomous investment component.



We assume that there are 200 instances of social interaction for every macro time step. Hence, for 100000 social interactions we obtain 500 macro time steps. As can be seen in the figure, regular business cycles and sentiment swings emerge in the same way as in the introductory case (figure 1). It turns out that the paths of macro-variables appear more regular in both cases<sup>4</sup> and that the mean length of a business cycle increases (the mean length of business cycles depends on the ratio of interactions per macro time step to the number of investors). Note that again not all agents become simultaneously optimistic or pessimistic.

----- **Figure 5 goes about here** -----

Figure 6 extends the previous simulation experiment in the sense that we now also consider exogenous demand shocks. This is modeled in the following way. In each macro time step, we add a standard normal distributed random shock to the national income. The key insight of this exercise is that these shocks cause us to observe irregular business cycles with varying amplitude and frequency for both  $N=500$  and  $N=1000$ . Observe furthermore that also the evolution of the number of optimistic agents becomes more erratic.

----- **Figure 6 goes about here** -----

Two additional comments are in order. Suppose that we interpret one macro time step as one month. 500 months then correspond to a time span of about 40 years. For  $N=500$  investors we may identify roughly seven to eight business cycles. Hence the average duration of a business cycle is about five to six years for this particular parameter setting. We also remark that figures 5 and 6 are based on the same parameter

---

<sup>4</sup> For an interesting discussion of the impact of population size on the extent of macroscopic uncertainty in the presently discussed type of models, see Alfarano et al. (2008).

setting as figure 1, i.e. we did not need to fine-tune the model to generate interesting dynamics.

### 3 A multi-country model with business cycle synchronization

In this section, we consider three national economies, indexed by D, F and I. To demonstrate that a mechanism based on the international coupling of economic sentiment may in itself be sufficient to produce business cycle synchronization, we abstain in our three-country model from goods transfers between the countries.

The equations determining the national economy remain the same as in the basic model, i.e. we consider again (1) to (8), apart from being indexed by the respective national economy. For instance, for D we have

$$Y_{t+1}^D = C_{t+1}^D + I_{t+1}^D, \quad (9)$$

$$C_{t+1}^D = c Y_t^D, \quad (10)$$

$$I_{t+1}^D = \bar{I} + i(E[Y_{t+1}^D] - Y_t^D), \quad (11)$$

$$E[Y_{t+1}^D] = W_t^D (Y_t^D + e) + (1 - W_t^D)(Y_t^D - e), \quad (12)$$

and

$$K_t^D = W_t^D N, \quad (13)$$

respectively.

We introduce a transnational coupling of economic sentiment in the following way: for the transition probabilities, we now specify that the bias towards optimism (pessimism) in times when national income is rising (falling) also includes spill-over effects from the other two national economies (with  $p$  denoting a parameter representing

the degree of coupling between the economy of D with the two other countries, as perceived by investors in D). Therefore, we obtain

$$K_t^D = \begin{cases} K_{t-1}^D + 1 & \text{with probability } p_{t-1}^{D,+} = \frac{N - K_{t-1}^D}{N} (\varepsilon + (1 - \delta_{t-1}^{D,P \rightarrow O}) \frac{K_{t-1}^D}{N-1}) \\ K_{t-1}^D - 1 & \text{with probability } p_{t-1}^{D,-} = \frac{K_{t-1}^D}{N} (\varepsilon + (1 - \delta_{t-1}^{D,O \rightarrow P}) \frac{N - K_{t-1}^D}{N-1}), \\ K_{t-1}^D & \text{with probability } 1 - p_{t-1}^{D,+} - p_{t-1}^{D,-} \end{cases}, \quad (14)$$

$$(1 - \delta_{t-1}^{D,P \rightarrow O}) = \begin{cases} 0.5 - \gamma & \text{for } Y_{t-1}^D - Y_{t-2}^D + p(Y_{t-1}^F - Y_{t-2}^F + Y_{t-1}^I - Y_{t-2}^I) < 0 \\ 0.5 + \gamma & \text{otherwise} \end{cases} \quad (15)$$

and

$$(1 - \delta_{t-1}^{D,O \rightarrow P}) = \begin{cases} 0.5 - \gamma & \text{for } Y_{t-1}^D - Y_{t-2}^D + p(Y_{t-1}^F - Y_{t-2}^F + Y_{t-1}^I - Y_{t-2}^I) > 0 \\ 0.5 + \gamma & \text{otherwise} \end{cases}. \quad (16)$$

Analogous equations hold for the other countries.

Note that the coupling of the three national economies is a self-fulfilling prophecy in our model: if investors believe that the countries are coupled (i.e. that the rise or decline of national income in the other countries is relevant to the prospects in one's own country), our simulations discussed below show that business cycles resulting from Samuelson's multiplier-accelerator principle will indeed be synchronized (to an amount dependent on parameter  $p$ ). However, if the investors do not believe in such a coupling, then no mechanism in our model can account for business cycle synchronization.

Let us now present some simulation results of the dynamics of our three-country model. Figure 7 shows the time-parallel evolution of national income and the number of optimistic agents in each of the three countries for  $p=0$  (top set of panels) and  $p=0.5$

(bottom set of panels), respectively. Although investors are randomly matched in our model, local phenomena may nevertheless occur in individual countries. A casual investigation of the diagrams reveals, however, that the business cycles resulting from our model as well as the corresponding development of economic sentiment become synchronized for  $p=0.5$ . We see, for instance, that both the amplitudes and durations of business cycles are quite similar in all three countries if agents believe in a coupling. Moreover, fluctuations in economic activity and swings in economic sentiment then appear more regular.

----- **Figure 7 goes about here** -----

So far, the three countries in our model have been assumed to be symmetric, i.e. we have used the same parameter setting for all countries. One may wonder whether the synchronization of the business cycles just detected survives if the countries become asymmetric. To test this issue, we assume that the parameter setting for countries F and I remains as before. However, for country D we now assume a marginal propensity to consume of 0.8 rather than 0.9, thereby lowering the multiplier from 10 to 5. In addition, we increase the level of autonomous investments in country D from 10 to 20. Hence, if the agents are split equally between optimists and pessimists, the equilibrium incomes in all three countries are 100. What are the results of this exercise? The first set of panels again represents the time-parallel evolution of national income and the number of optimists in each of the three countries for  $p=0$ . Due to the change in parameter settings, fluctuations in economic activity in country D are now quite different to those observed in countries F and I. However, if agents believe in comovements between the countries, there will indeed be comovements between them. In the bottom set of panels we assume again that  $p=0.5$ . Note that there is a strong synchronization in business cycles and economic sentiment. While the durations of business cycles in all three

countries are quite comparable, the amplitude of the cycles in country D is much lower than in countries F and I.

----- **Figure 8 goes about here** -----

Let us finally try to quantify the degree of synchronization with respect to the coupling parameter  $p$ . This may be done by computing the correlation coefficients for the national income levels of countries D, F and I for increasing values of  $p$ . The solid, dashed and dotted lines in the two panels of figure 9 depict correlation coefficients for  $Y^F$  and  $Y^I$ ,  $Y^F$  and  $Y^D$  and  $Y^I$  and  $Y^D$ , respectively. In the left panel, we assume that the countries are symmetric (parameter setting as in figure 7), and in the right panel we assume that they are asymmetric (parameter setting as in figure 8). The correlation coefficients are estimated as averages over 50 simulation runs, each containing 10000 observations. The coupling parameter  $p$  is increased in 20 discrete steps from 0 to 0.5. As can be seen, there is a nonlinear relation between the correlation coefficients and the coupling parameter. Even “low” values of  $p$  may generate strong synchronization between the countries. Surprisingly, the degree of synchronization is only slightly weaker if the countries are asymmetric.

----- **Figures 9 goes about here** -----

#### **4 Discussion**

The aim of our paper is twofold. First, we show how social interactions at the level of individual economic sentiment may lead to enduring business cycles in Samuelson’s (1939) multiplier-accelerator model. The mechanism which engenders such cycles in the present paper is complementary to that suggested and analyzed by Westerhoff and Hohnisch (2007), who apply a pure multiplier model. In their paper, consumer sentiment affects individual consumption propensities, while in our paper economic

sentiment affects individual expectations which, in turn, impact upon investment. The present paper is also related to Westerhoff (2006) in which investors switch between extrapolative and regressive predictors with respect to market circumstances. In his model, nonlinear deterministic interactions between investors may lead to chaotic business cycles. In the present model, we abstain from such expectation formation rules to highlight which ingredients are *sufficient* to produce endogenous business cycles in our framework.<sup>5</sup> In that sense, our model is minimalist.

Second, with this basic model producing endogenous business cycles we illustrate in a three-country framework how business cycle synchronization may arise if an investor believes that information concerning the other national economies is relevant for the prospects of his own national economy. Again, the model is minimalist in the sense that we abstain from goods transfers between the countries which are presumably of importance for the phenomenon of business cycle synchronization.

It would be tempting to test the extent to which economic sentiment is synchronized over various countries. This empirical issue would presumably add to results on business cycle synchronization itself. However, it appears to be difficult to answer a basic question implied by the present paper: is the synchronization of business sentiment causal for the emergence of business cycle synchronization, or does the causality go in a different direction – if business cycle synchronization arises for different reasons then synchronization of business sentiment – if indeed it is evident from the data – might be a by-product of the former. Of course, it could well be possible

---

<sup>5</sup> It should be noted that “extrapolative” expectations at the aggregate level endogenously emerge in our model in the following sense: suppose that national income is increasing. In such a situation, the transition probabilities are such that more investors are likely to become optimistic and thus the mean income expectation rises (the analogous situation arises when income falls). For the sake of realism it would still be interesting to model investors’ individual expectation formation behavior in more detail. Useful empirical evidence is provided, for instance, by Nardo (2003).

that both phenomena are endogenously intertwined and that no clear direction of causality prevails. However, recent progress in estimating models with heterogeneous interacting agents has been achieved by Alfarano, Lux, and Wagner (2005), Lux (2007) and Franke (2007b).

## References

- Alfarano, S., Lux, T. and Wagner, F. (2005): Estimation of agent-based models: The case of an asymmetric herding model. *Computational Economics*, 26, 19-49.
- Alfarano, S., Lux, T. and Wagner, F. (2008): Time variation of higher moments in a financial market with heterogeneous agents: An analytical approach. *Journal of Economic Dynamics and Control*, 32, 101-136.
- Chiarella, C., Flaschel, P. and Franke R. (2005): Foundations for a disequilibrium theory of business cycles. Cambridge University Press, Cambridge.
- Franke, R. (2007a): Microfounded animal spirits, central bank credibility, and multiple equilibria. Working Paper, University of Kiel.
- Franke, R. (2007b): Estimation of a microfounded herding model on German survey expectations. Working Paper, University of Kiel.
- Frankel, J. and Rose A. (1998): The endogeneity of the optimum currency area criteria. *Economic Journal*, 108, 1009-1025.
- Keynes, J. M. (1936): *The general theory of employment, interest and money*. Harcourt Brace, New York.
- Kirman, A. (1993): Ants, recruitment and rationality. *Quarterly Journal of Economics*, 108, 137-156
- Kirman, A. (1991): Epidemics of opinion and speculative bubbles in financial markets. In: M. Taylor (Ed.): *Money and financial markets*. Blackwell, Oxford, 354-368.
- Lux T. (1995): Herd behaviour, bubbles and crashes. *Economic Journal*, 105, 881-896.
- Lux T. (2007): Rational forecasts or social opinion dynamics? Identification of interaction effects in a business climate survey. Working Paper, University of Kiel.



- Nardo, M. (2003): The quantification of qualitative survey data: A critical assessment. *Journal of Economics Surveys*, 17, 645-668.
- Puu, T. and Sushko, I. (2006): *Business cycle dynamics: Models and tools*. Springer, Berlin.
- Samuelson P. (1939): Interactions between the multiplier analysis and the principle of acceleration. *Review of Economic Statistics*, 21, 75-78.
- Weidlich W. and Haag G. (1983): *Concepts and models of a quantitative sociology*. Springer, Berlin.
- Westerhoff, F. (2006): Samuelson's multiplier-accelerator model revisited. *Applied Economics Letters*, 13, 89-92.
- Westerhoff, F. and Hohnisch, M. (2007): A note on interactions-driven business cycles. *Journal of Economic Interaction and Coordination*, 2, 85-91.

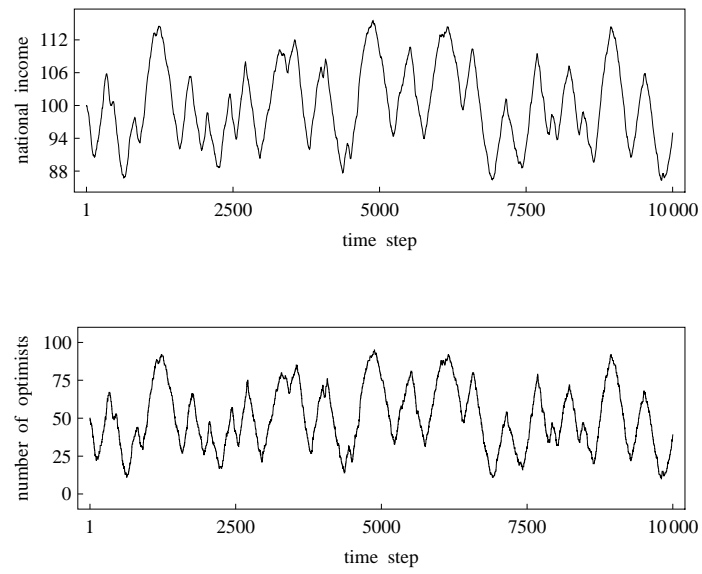


Figure 1: A snapshot of the dynamics of the basic model. The panels present the evolution of national income (top) and the number of optimistic investors (bottom) in the time domain with 10000 observations. Parameters as in section 2.

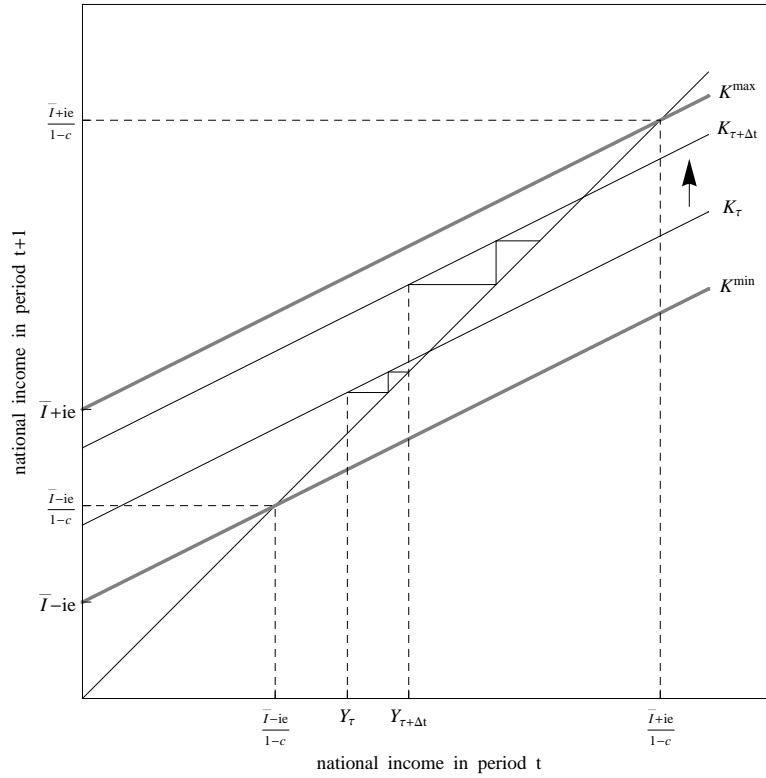


Figure 2: Illustration of the dynamics of the basic model for a fixed number of optimistic investors. National income in period  $t+1$  is plotted against national income in period  $t$ .

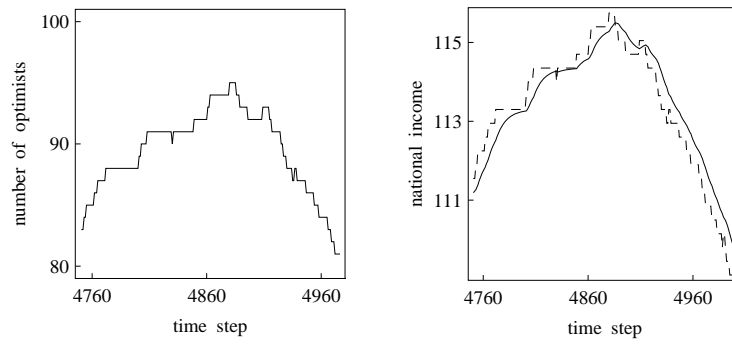


Figure 3: A high time-resolution excerpt of the dynamics. The number of optimistic investors (left) and national income (right) for a short time interval. The additional dashed line in the right panel depicts the equilibrium income. Parameters as in figure 1.

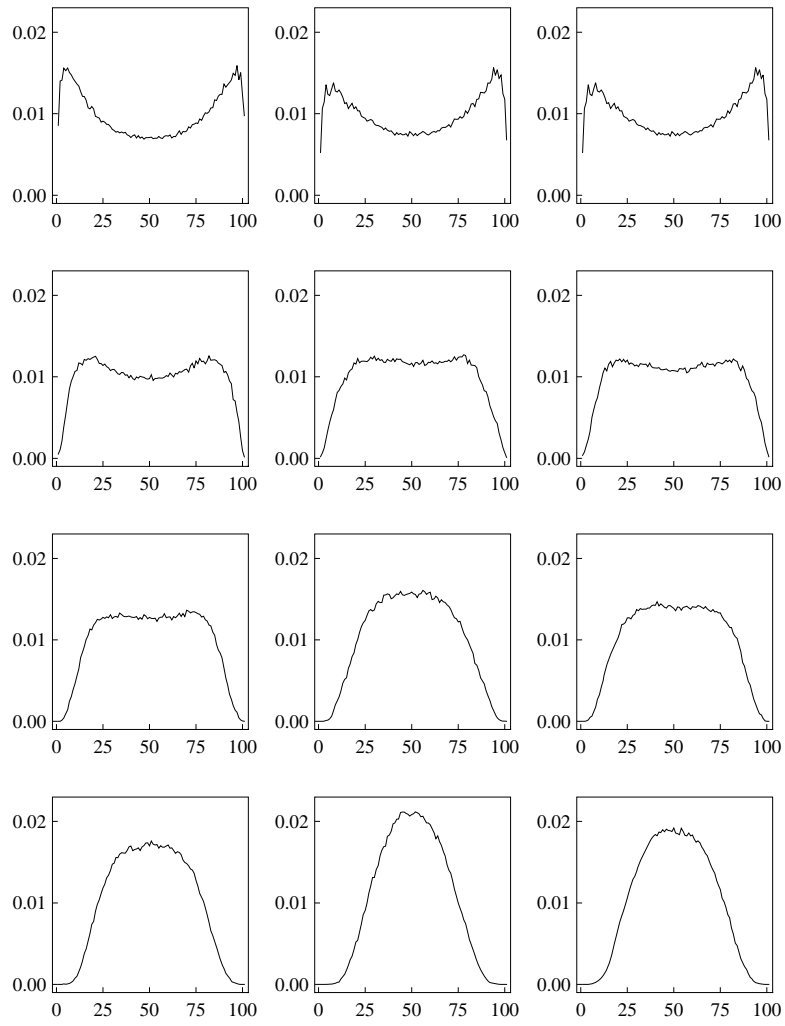


Figure 4: The distribution of the number of optimistic agents for different values of  $\gamma$  and  $\varepsilon$ . From left to right,  $\gamma$  takes the values 0.4, 0.45 and 0.5. From top to bottom,  $\varepsilon$  takes the values 0.01, 0.025, 0.05 and 0.1. Each distribution is computed taking 1 million observations into account. Other parameters as in figure 1.

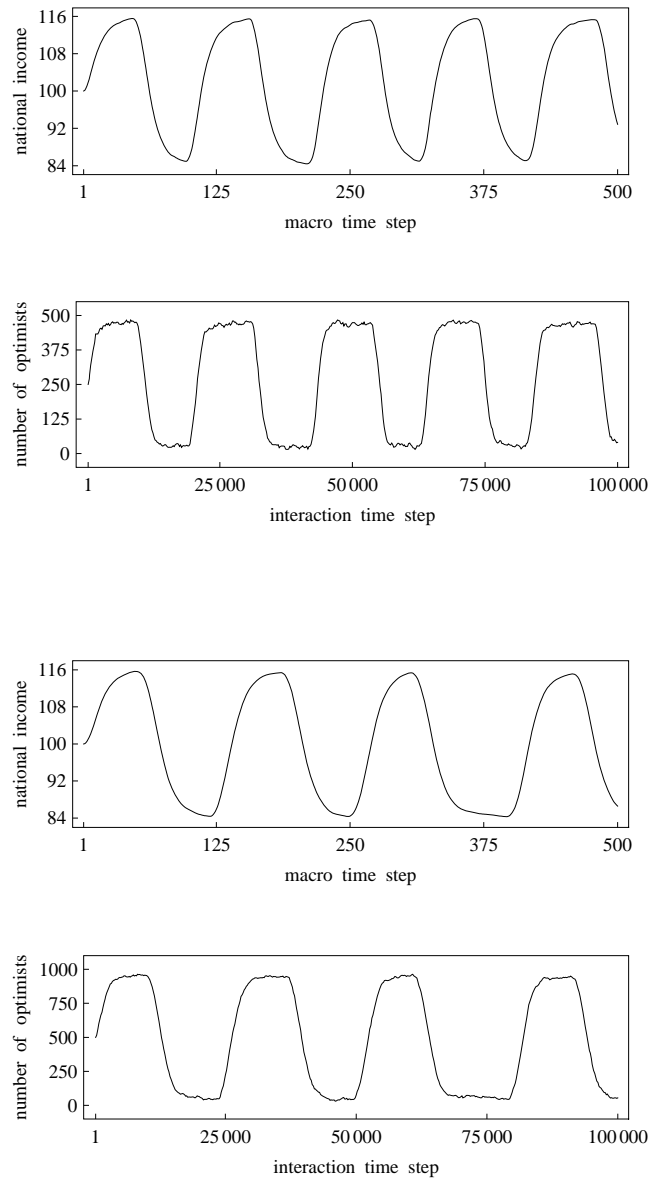


Figure 5: The dynamics of the model when the number of investors and social interactions per macro time step increases. The top set of panels shows the evolution of national income and the number of optimistic investors when there are 500 investors and 200 social interactions per macro time step. The bottom set of panels shows the same but now for 1000 investors. Other parameters as in figure 1.

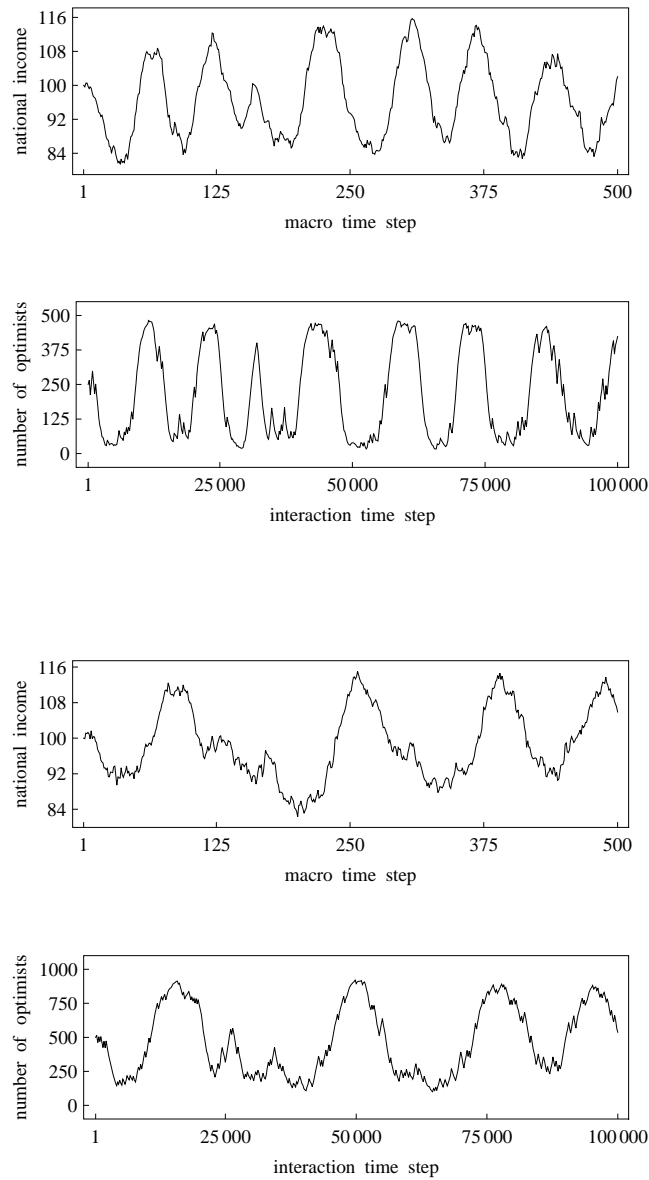


Figure 6: The dynamics of the extended model buffeted with exogenous shocks. The same simulation design as in figure 5 but now we add a standard normal distributed random variable to the income equation.

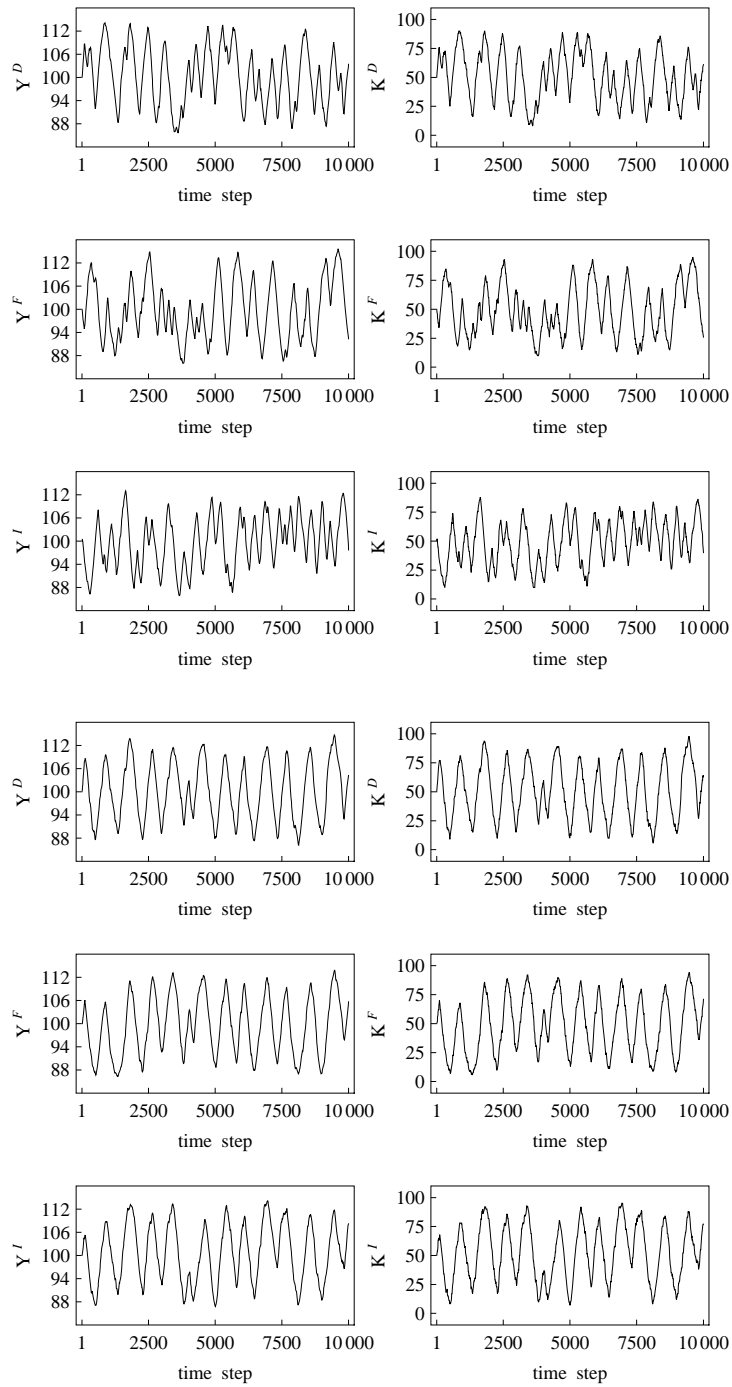


Figure 7: A snapshot of the dynamics of the three-country model of section 3. The upper set of panels shows the evolution of national income (left) and the number of optimistic investors (right). The countries are D (top), F (central) and I (bottom). The sentiment-spillover parameter is set to  $p = 0$  and the other parameters are as in section 2. The bottom set of panels shows the same but now for  $p = 0.5$ .



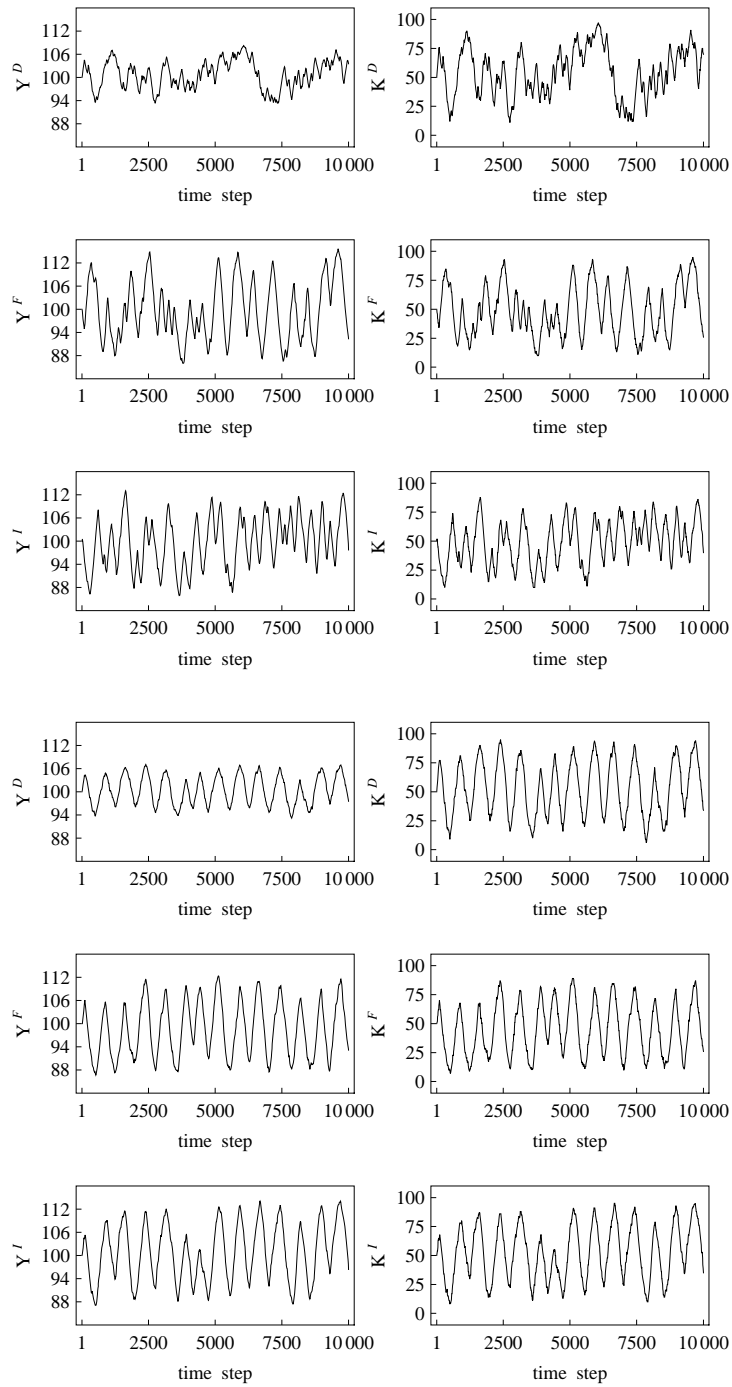


Figure 8: A snapshot of the dynamics of the three-country model of section 3 when the countries are asymmetric. The same simulation design as in figure 7 but for country D we now assume  $c = 0.8$  and  $\bar{I} = 20$ .

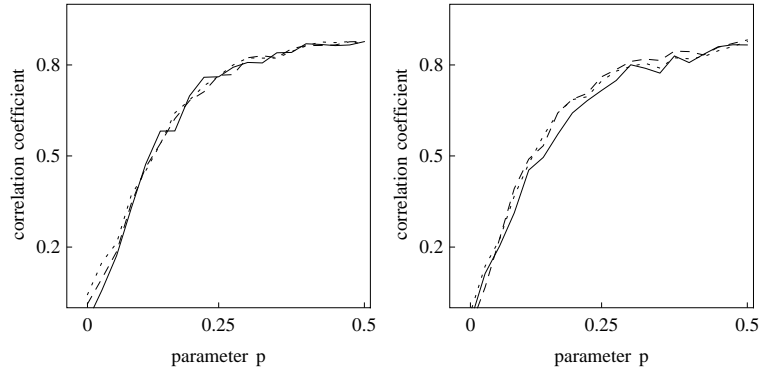


Figure 9: The synchronization of business cycles for an increasing sentiment-spillover. In the left panel the economies are symmetric (parameters as in figure 7) and in the right panel they are asymmetric (parameters as in figure 8). The solid, dashed and dotted lines represent correlation coefficients for  $Y^F$  and  $Y^I$ ,  $Y^F$  and  $Y^D$  and  $Y^I$  and  $Y^D$ , respectively. The correlation coefficients are estimated as averages over 50 simulation runs, each containing 10000 observations. Parameter  $p$  is increased in 20 discrete steps from 0 to 0.5.