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Nonlinear expectation formation, endogenous business cycles and stylized facts

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Abstract

We modify Samuelson's multiplier-accelerator model to explore the influence of expectations on fluctuations in economic activity. Within our model, the agents use a nonlinear mix of extrapolative and regressive forecast rules to predict the output. Our model is able to mimic some generic features of business cycles. In particular, consumption is procyclical and fluctuates less than output while investment is procyclical and fluctuates more than output.

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1. Introduction

Modern industrial economies undergo significant short-run output variations. Alternating periods of expansion and contraction are, of course, welfare decreasing. Every recession in which workers become involuntary unemployed results in a loss of output that cannot be regained. The potential origins of business cycles therefore belong to the most challenging issues of macroeconomics. A good understanding of the causes of economic fluctuations, e.g., is essential for policy makers who seek to tame them. As demonstrated by Baumol (1961), the variability of national income may increase if stabilization measures are ill designed.

This paper investigates the role of expectations for business cycles. We extend the popular multiplier-accelerator model of Samuelson (1939) by allowing the agents to use a nonlinear mix of extrapolative and regressive expectation formation rules to predict their income. We find that economic activity endogenously depends on the mood of the agents. If they are optimistic (pessimistic), output is above (below) its long-run equilibrium value. Moreover, our nonlinear deterministic discrete-time model can account for several stylized facts of business cycles. First, business cycles do not exhibit any simple regular or cyclical pattern, e.g. output declines vary considerably in size and timing. Second, consumption and investment are procyclical to output. Third, fluctuations are distributed very unevenly among the components of output. Most importantly, consumption is less volatile than output while investment is more volatile than output.

Note that the core elements of Samuelson's model, extended and discussed by Hicks (1950), Duesenberry (1950), Goodwin (1951) and Baumol, W. (1961), still receive much academic attention (e.g. Gandolfo 1985, Puu 1989, Hommes 1995, Sushko, Puu and Gardini 2003, Puu, Gardini and Sushko 2004). Many more interesting contributions to business cycle modeling have been presented during recent decades, using more sophisticated mechanisms and involving, for instance, aspects such as the monetary sector, inventory adjustments or international trade relations (see, e.g., Medio 1992, Day and Chen 1993, Day 1999, Rosser 2000). To clarify the role of expectations, however, we focus on a stylized version of the economy. Already within our simple setting intricate business cycles may emerge. Adding more complications to the model would only increase the complexity of the dynamics.

The remainder of our paper is organized as follows. In section 2, we reconsider Samuelson's business cycle model. In section 3, we propose a few modifications. In section 4, we investigate the new model. In section 5, we seek to replicate the stylized facts of business cycles. The final section concludes the paper.

2. Samuelson's Approach

Let us briefly recall the seminal business cycle model of Samuelson (1939) which was developed to study the interplay between the multiplier analysis and the principle of acceleration: The Keynesian "multiplier" is a multiplicative factor that relates (autonomous) expenditures to national income. The accelerator principle states that induced investment is proportional to the time increase of consumption. An increase in (autonomous) investment therefore leads to an increase in national income and consumption (via the multiplier effect) which in turn raises investment (via the accelerator process). This feedback mechanism repeats itself and may generate an oscillatory behavior of output.

The model may be formalized as follows. Consumption in period t depends on national income in period t-1

$$C_t = aY_{t-1}, \tag{1}$$

where $0 < a < 1$ is the marginal propensity to consume. Investment may be expressed as

$$I_t = I^a + b(C_t - C_{t-1}). \tag{2}$$

Autonomous investment I^a is independent of the business cycle and induced investment is proportional to changes in consumption according to $b > 0$. National income for a closed economy results in

$$Y_t = C_t + I_t. \tag{3}$$

Combining (1), (2) and (3), we obtain the recurrence equation in the income variable

$$Y_t = I^a + a(1+b)Y_{t-1} - abY_{t-2}, \tag{4}$$

which is a second-order linear difference equation. According to (4), current national income depends on autonomous investment and on the output of the last two periods.

The fixed point of (4), i.e. the long-run equilibrium output, is determined as

$$\overline{Y} = \frac{1}{1-a} I^a, \tag{5}$$

implying furthermore $\overline{C} = a\overline{Y}$ and $\overline{I} = I^a$. Stability of the fixed point requires that

$$0 < a < \frac{1}{b} < 1 \tag{6}$$

and the condition for oscillations is

$$a < \frac{4b}{(1+b)^2}. \tag{7}$$

As is well known, dampened oscillations occur if (6) and (7) are true. Temporary business cycles then arise due to the interplay of the multiplier and the accelerator: Increased investment increases output and increased output induces increased investment.

A major criticism of Samuelson's model is that changes in economic activity either die out or explode (sustained swings only occur for a nongeneric boundary case). Hicks (1950) thus developed the first nonlinear business cycle model. He limited the evolution of an otherwise explosive output path by proposing upper and lower bounds for investment. As already mentioned, the models of Samuelson and Hicks are still valid. They are frequently used as workhorses to study new additional elements that may stimulate business cycles.

3. Some Modifications

As argued by Simon (1955), economic agents are boundedly rational in the sense that they lack knowledge and computational power to derive fully optimal actions. Instead, they tend to use simple heuristics which have proven to be useful in the past (Kahneman, Slovic and Tversky 1986). Survey studies reveal that agents typically use a mix of extrapolative and regressive expectation formation rules to forecast economic variables (Ito 1990, Takagi 1991). Similar results are observed in asset pricing experiments. For instance, Smith (1991) and Sonnemans et al. (2004) report that financial market participants typically extrapolate past price trends or expect a reversion of the price towards its long-run equilibrium value. Indeed, the dynamics of group expectations have successfully been modeled for financial markets. Contributions by Day and Huang (1990), Kirman (1993), de Grauwe et al. (1993), Brock and Hommes (1998) or Lux and Marchesi (2000) demonstrate that interactions between heterogeneous agents who rely on heuristic forecasting rules may cause complex financial market dynamics, as observed in actual markets.

Our goal is to investigate the importance of expectations for the variability of output. Our main modification of Samuelson's model is that the agents' consumption depends on their expected current income (and not on their past realized income).[1] Note that Flieth and Foster (2002) and Hohnisch et al. (2005) model socioeconomic interactions between heterogeneous agents to explain the evolution of business confidence indicators. Both papers are able to replicate typical patterns in the German business-climate index (the so-called Ifo index), yet refrain from establishing a link between expectations and economic activity. We believe, however, that mass psychology, expressed via expectations and visible in business confidence indicators, is a major factor that may cause swings in national income. For example, new era thinking may lead to optimistic self-fulfilling prophecies (e.g. the New Economy hype) while general pessimism may cause economic slumps (Shiller 2000).

Let us be more precise. We assume that the agents' consumption in period t is a constant fraction of their expected income for that period

$$C_t = a\,E[Y_t], \tag{8}$$

[1] In Westerhoff (2005), the investment function depends on boundedly rational expectations while the consumption function is as in Samuelson's original setup. In addition, that paper does not try to mimic the stylized facts of business cycles.

where a is again the marginal propensity to consume. Since the agents form their expectations at the beginning of period t, they possess information until the end of t-1. The agents use a weighted average of extrapolative and regressive expectations to predict their income

$$E[Y_t] = W_t E^e[Y_t] + (1 - W_t) E^r[Y_t]. \tag{9}$$

The relative impact of both expectation formation rules depends on $0 < W < 1$.

According to the extrapolative expectation formation rule, agents either optimistically believe in a boom or pessimistically expect a downturn. Such expectations are formalized as

$$E^e[Y_t] = Y_{t-1} + c(Y_{t-1} - \overline{Y}), \tag{10}$$

where c is a positive extrapolation parameter. If output is above its long-run equilibrium value $\overline{Y}$, people think that the economy is in a prosperous state and thus predict that their income will remain high. If output is below $\overline{Y}$, the agents are depressed and consequently expect a rather low income (such a heuristic has also been applied by Day and Huang 1990).

Regressive expectations are formed as

$$E^r[Y_t] = Y_{t-1} + d(\overline{Y} - Y_{t-1}). \tag{11}$$

The mean-reversion parameter $0 < d < 1$ captures the agents' expected adjustment speed of the output towards its long-run equilibrium value.

The more the economy deviates from $\overline{Y}$, the less weight the agents put on extrapolative expectations. Clearly, the agents believe that extreme economic conditions are not sustainable. The relative impact of the extrapolative rule may be written as

$$W_t = \frac{1}{1 + (Y_{t-1} - \overline{Y})^2}. \tag{12}$$

Since the relative importance of extrapolative and regressive expectations is time-varying, the agents' expectations are nonlinear.

Samuelson's accelerator principle takes into account that investment increases when consumption increases. If consumption is constant, investment goes back to an autonomous level. But this seems to be odd in the presence of multiple steady states. When current consumption remains for some time above (below) its long-run equilibrium value $\overline{C} = a\overline{Y}$, a situation in which output also tends to remain above (below) potential output, investment should be elevated (lower). We thus condition autonomous investment on the level of consumption and write

$$I_t = I^a(1 + e(C_t - \overline{C})) + b(C_t - C_{t-1}). \tag{13}$$

The parameter $e > 0$ indicates the variability of investment to out-of-equilibrium consumption.[2] Note that investment is subject to the agents'

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[2] Note that the (Kaldorian) investment function in Bischi et al. (2001) includes a constant autonomous component and a short-run, time-varying component which is conditioned on the current level of economic activity.

expectations via consumption.

4. Analysis of the Model

4.1 The dynamical system

Let us express income in terms of deviations from equilibrium income through the following change of variable

$$x_t \equiv Y_t - \overline{Y}, \tag{14}$$

i.e.

$$Y_t = x_t + \overline{Y}. \tag{15}$$

Note that W_t is a function h of x_{t-1}

$$W_t = h(x_{t-1}) \equiv \frac{1}{1+x_{t-1}^2} \tag{16}$$

and that

$$C_t = \overline{C} + ax_{t-1}(1+h(x_{t-1})(c+d)-d), \tag{17}$$

$$\begin{aligned} I_t = {} & I^a(1+eax_{t-1}(1+h(x_{t-1})(c+d)-d)) \\ & + bax_{t-1}(1+h(x_{t-1})(c+d)-d) \\ & - bax_{t-2}(1+h(x_{t-2})(c+d)-d) \end{aligned}. \tag{18}$$

Substituting (15)-(18) into (1) and rearranging terms yields the following autonomous nonlinear second-order difference equation

$$\begin{aligned} x_t = {} & a(1+b+eI^a)x_{t-1}(1+h(x_{t-1})(c+d)-d) \\ & - bax_{t-2}(1+h(x_{t-2})(c+d)-d) \end{aligned}, \tag{19}$$

which can be rewritten as a two-dimensional nonlinear dynamical system

$$\begin{cases} x_t = a(1+b+eI^a)x_{t-1}(1+h(x_{t-1})(c+d)-d) \\ \qquad - baz_{t-1}(1+h(z_{t-1})(c+d)-d) \\ z_t = x_{t-1} \end{cases}, \tag{20}$$

where in general

$$h(x) \equiv \frac{1}{1+x^2}. \tag{21}$$

Note that the equilibrium levels $\overline{Y}$ and $\overline{C}$ no longer appear in this dynamical system.

4.2 Steady states

Note that $(0,0)$ is a steady state of the model, which obviously corresponds to the "fundamental" steady state of the classical multiplier-accelerator model. In general, the steady states are the points $(\overline{x}, \overline{z})$ where $\overline{x}$ solves

$$\bar{x} = a(1+b+eI^a)\bar{x}(1+h(\bar{x})(c+d)-d)) - ba\bar{x}(1+h(\bar{x})(c+d)-d), \tag{22}$$

i.e.

$$1 = a(1+eI^a)(1+\frac{1}{1+\bar{x}^2}(c+d)-d), \tag{23}$$

which reduces to

$$\bar{x}^2 = \frac{a(1+eI^a)(c+1)-1}{1-(1-d)a(1+eI^a)}. \tag{24}$$

It follows that two further "non-fundamental" steady states exist provided that

$$\frac{1}{1+c} < a(1+eI^a) < \frac{1}{1-d}. \tag{25}$$

Note that the parameter space for which three steady states exist increases with c and d.

4.3 Local stability of the steady state $(0,0)$

Note that $h(0)=1$ and $h'(0)=0$. It follows that the Jacobian matrix evaluated at the steady state is given as

$$J = \begin{bmatrix} a(1+b+eI^a) & -ba(1+c) \\ 1 & 0 \end{bmatrix}. \tag{26}$$

Denoting by $Tr(J)$ and $Det(J)$ the trace and the determinant of J, sufficient conditions for local asymptotic stability are (i) $1+Tr(J)+Det(J)>0$, (ii) $1-Tr(J)+Det(J)>0$, and (iii) $1-Det(J)>0$ (see, e.g. Medio and Lines 2001). We thus obtain

$$1+a(1+b+eI^a)+ba(1+c)>0, \tag{27}$$

$$1-a(1+eI^a)(1+c)>0, \tag{28}$$

and

$$ba(1+c)<1. \tag{29}$$

Note that (27) is always satisfied for positive parameters. If we fix the parameters a, e, and I^a, we can represent the stability region and the bifurcation curves in the plane (b,c), with $b,c>0$. The stability region is defined by

$$\begin{cases} c < \dfrac{1}{a(1+eI^a)} - 1 \\ c < \dfrac{1}{ba} - 1 \end{cases} \tag{30}$$

and represented in figure 1.

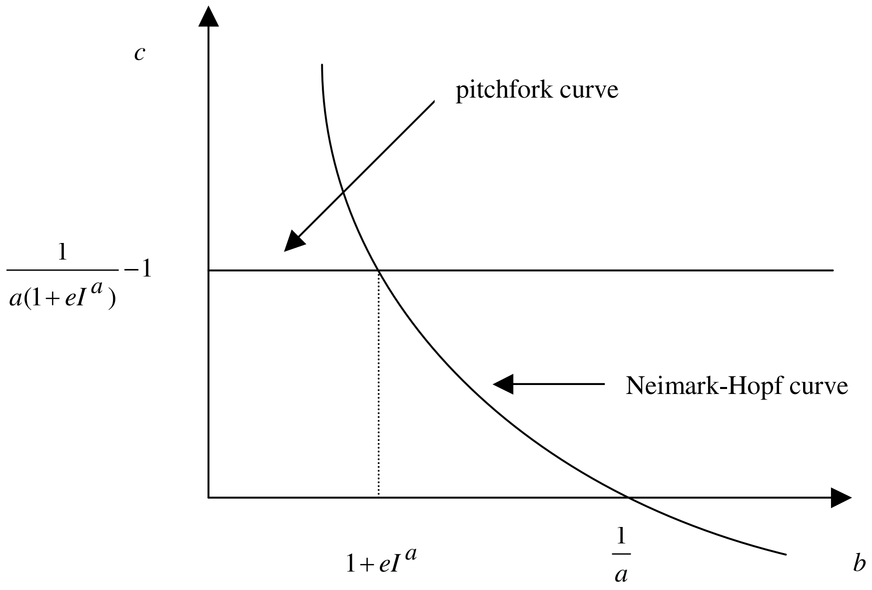

Figure 1: Stability region and bifurcation curves in the plane (b,c).

The case represented in figure 1 is the one where $a(1+eI^a)<1$. Note that the stability region is empty in the opposite case! Note also from figure 1 that stability may be lost via a pitchfork bifurcation (where a stable steady state becomes unstable and two new stable steady states are created) or via a supercritical Neimark-Hopf bifurcation (where the steady state changes from stable to unstable focus. The pitchfork bifurcation occurs when c in increased beyond the threshold $c_P \equiv 1/(a(1+eI^a))-1$, for $b<1+eI^a$. The Neimark-Hopf bifurcation occurs when c becomes larger than the threshold $c_N \equiv 1/ba-1$, for $1+eI^a<b<1/a$ (or when $c<1/(a(1+eI^a))-1$ and b becomes sufficiently high).

4.4 Numerical bifurcation scenarios

Let us further explore these bifurcation routes by assuming $a=0.5$, $b=0.5$, $c=0.5$, $d=1$, $e=0$ and $I^a=10$. Figure 2 first presents a bifurcation diagram for the extrapolation parameter *c*, which is increased in 250 discrete steps from 0 to 20. The upper two panels of figure 2 only differ with respect to their initial values, whereas the bottom panel shows them on top of each other. What are the results? If c becomes larger than 1, then the "fundamental" steady state $\overline{Y}=20$ becomes unstable and output converges to one of the two "non-fundamental" steady states. The output level then depends on the initial value of the system and permanently remains either above or below $\overline{Y}=20$. Moreover, for about $c\approx 7$, two period-doubling bifurcations emerge, one above and one below $\overline{Y} = 20$. In that case, national income fluctuates back and forth between two values. If c becomes larger, even more complicated (chaotic) dynamics may emerge. Overall, the amplitude of output fluctuations seems to increase with the extrapolation parameter. Figure 3 presents a bifurcation diagram in which the parameter b is increased in 250 discrete steps from 0 to 5. The simulation design is the same as in figure 2. As predicted by our analysis, output converges towards its "fundamental" steady state as long as b is below $b = 4/3$. Afterwards, (quasi-)periodic motion sets in. As is visible, additional complicated bifurcations follow.

5. Business Cycles and Stylized Facts

In this section, we seek to show that our model has the potential to replicate a few stylized facts of business cycles. For the simulation analysis, we fix the parameters of the model as follows: $a = 0.9$, $b = 1.5$, $c = 11$, $d = 0.85$, $e = 0.065$. Moreover, setting $I^a = 10$, long-run equilibrium output and consumption are given as $\overline{Y} = 100$ and $\overline{C} = 90$, respectively. Figure 4 depicts a typical simulation run with 300 observations. If one interprets one time step as one month, then 300 periods stand for a time span of 25 years. The first, second, third and fourth panels show the evolution of expected output, actual output, consumption and investment, respectively. All time series display quite intricate motion. For instance, there is almost no regularity in the timing and duration of booms and recessions. National income remains above its long-run equilibrium value $\overline{Y} = 100$ for some time but then, out of the blue, a downturn sets in. After some time of economic distress, output suddenly switches back to a higher value. It is important to note that actual business cycles are indeed relatively irregular, see, for instance, the empirical survey of Stock and Watson (1999).

As is further visible, output, consumption and investment vary procyclically. If output is high, then consumption and investment are also high. While the level of investment is smaller than the level of consumption, its relative amplitude, defined as the relative distance between the variable's extreme values and its long-run average, is larger. To be precise, the relative amplitude of national income is about 20 percent, whereas the relative amplitudes of consumption and investment are about 7 and 100 percent, respectively (these amplitudes may easily be up- or downscaled). Our simple model therefore has the potential to replicate – in a certain stylized way – some important regularities of business cycles.[3]

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[3] The multiplier-accelerator model is sometimes criticized since it implies frequent disinvestments. Within our setting, there are only very few occasions of actual disinvestment.

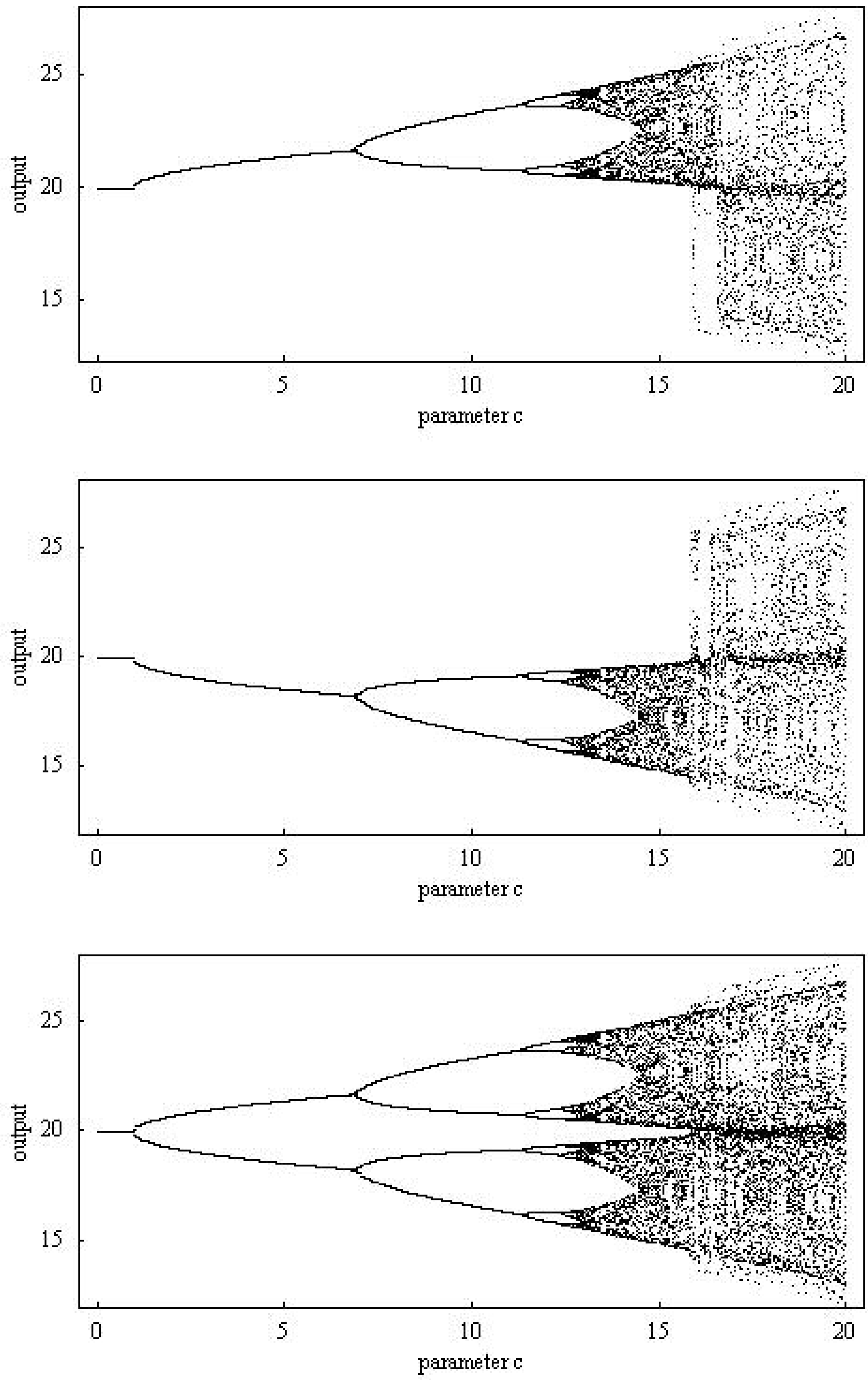

Figure 2: Bifurcation diagrams. Parameter *c* is increased as indicated on the axis, and the remaining parameters are as in section 4. The first two panels result from different initial values, whereas the bottom panel presents the first two panels on top of each other.

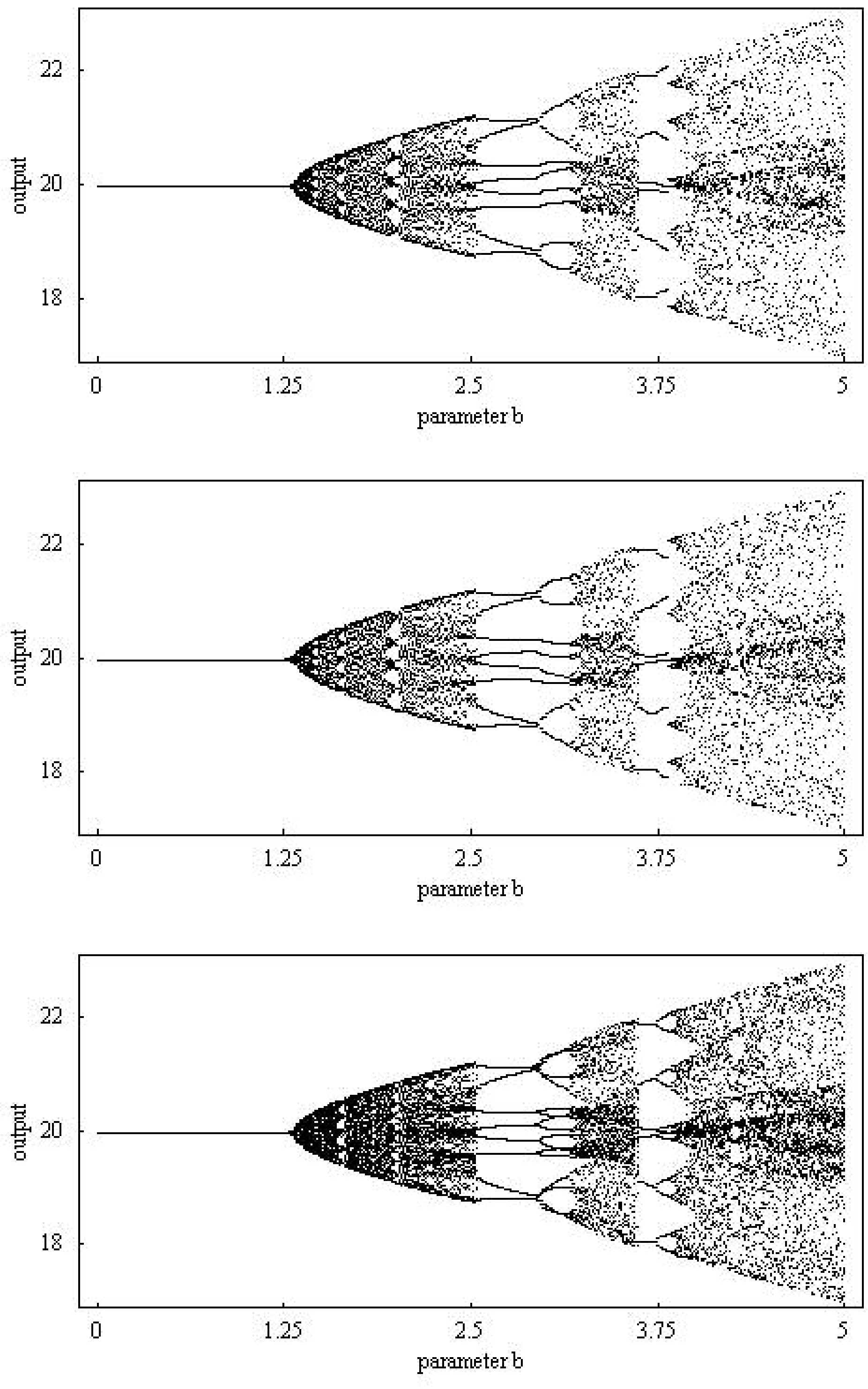

Figure 3: Bifurcation diagrams. Parameter *b* is increased as indicated on the axis, and the remaining parameters are as in section 4. The first two panels result from different initial values, whereas the bottom panel presents the first two panels on top of each other.

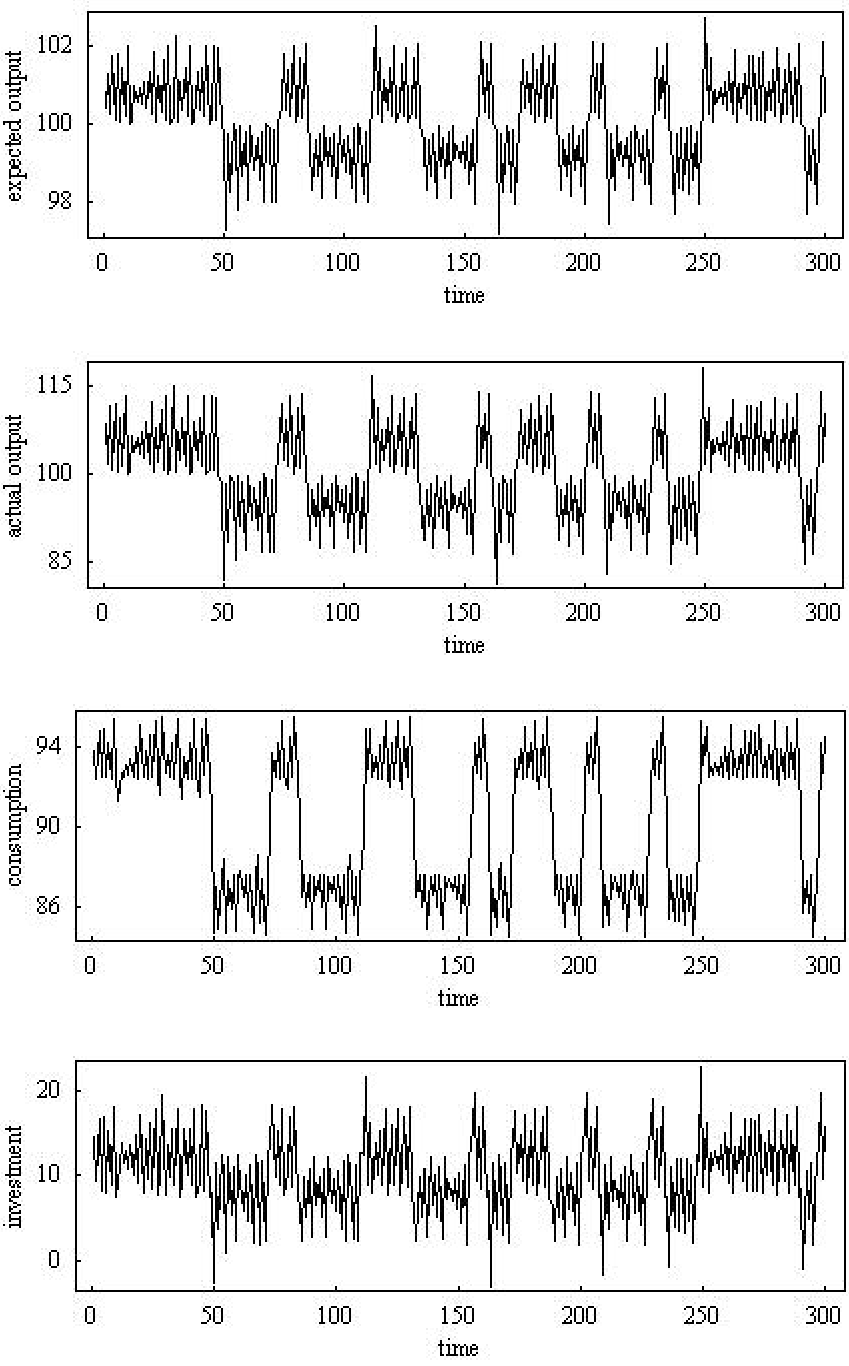

Figure 4: Endogenous business cycles. The first, second, third and fourth panels show the expected output, the actual output, consumption and investment for 300 observations. Parameter setting as in section 5.

From an economic point of view there is no reason why business cycles have to occur at all. In principle, an economy can operate at the full employment level. However, business cycles do in fact occur. Within out setup, a single disturbance suffices to trigger lasting endogenous swings in economic activity. What drives the dynamics is the agents' nonlinear expectations. As revealed by the top two panels, output tends to be high when the agents expect it to be high. In the presence of optimistic agents, the economy performs well. First, optimistic agents consume heavily, which raises output. Second, elevated consumption creates additional investment which, in turn, further boosts the economy. Conversely, if the agents become depressed, national income will decrease.[4] We would like to point out that the model's agents should not be called irrational; their expectations tend to be right concerning the general state of the economy. During a prosperous state, for instance, the agents correctly predict that actual output will be above potential output.

Summing up, our model obviously has the potential to generate various dynamic outcomes: unstable steady states, multiple stable steady states, cycles of various length and chaos. What is more important, however, is that our model has the potential to mimic certain stylized facts of actual business cycles. The ebb and flow of economic activity is driven by the mood of the agents via their nonlinear expectation formation process. Business cycles may thus have a strong endogenous component. This observation is important for policy makers. If they achieve to create an optimistic atmosphere, a recession may be shortened.

6. Conclusions

Economies experience significant swings in economic activity. A number of quite interesting mechanisms have been suggested which may explain business cycles (for surveys see, e.g., Medio 1992, Day and Chen 1993, Day 1999, Rosser 2000). The focus of this paper is on the role of expectations for output variability. Guided by empirical studies, we assume that agents use a combination of extrapolative and regressive expectation formation rules. Since the agents favor regressive expectations if the distance between national income and its long-run equilibrium value is large, the expectations of the agents are nonlinear.

We find that the mood of the agents may stimulate endogenous output fluctuations. If the agents are optimistic (pessimistic) output tends to be above (below) its long-run equilibrium value. Moreover, our model has the potential to mimic certain stylized facts of business cycles. Simulated business cycles display (1) irregular output movements, (2) output, consumption and

[4] A depression may emerge as follows. Suppose output is high but the agents expect it to return to its long-run equilibrium value. If consumption and (induced) investment decrease strongly, output may be pushed below its long-run equilibrium value. Should that be the case, it is likely that agents expect the economy to remain in recession.

investment are procyclical, and (3) consumption is less volatile than output while investment is more volatile than output.

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