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Expectations driven distortions in the foreign exchange market

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Abstract

This paper explores the phenomenon of lasting deviations of the exchange rate from its fundamental value in the foreign exchange market. Motivated by empirical observations a chartists–fundamentalists model is developed in which boundedly rational agents repeatedly choose between technical and fundamental trading rules to determine their speculative investment positions. Crucial for the dynamics is how the traders perceive the fundamental exchange rate. This perception process is based on psychological evidence. Simulations give rise to bubbles but simultaneously display quite realistic exchange rate dynamics (unit roots in the exchange rates, fat tails for returns, and volatility clustering).

JEL classification: F31; G14

Keywords: Exchange rate theory; Technical and fundamental trading rules; Expectations and learning; Market efficiency

1. Introduction

This paper aims at explaining the phenomenon of distortions in the foreign exchange market. Distortions in the sense of (lasting) deviations of the exchange rate from its fundamental value are a sign of market inefficiency. One well-known example for this is the bubble path of the US dollar in the 1980s. In January 1980, the mark-dollar exchange rate was around DM 1.70. In the next 5 years, the exchange rate increased over 100 percent. Its height of DM 3.46 was reached in February 1985. Afterwards, the exchange rate depreciated sharply by over 50 percent. At the end of 1987, the exchange rate dropped below DM 1.60.

More specifically, we want to develop a model which gives rise to bubble processes but yields realistic exchange rate movements at the same time, namely unit roots in exchange

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rates, fat tails for returns, and volatility clustering. Further aims include modeling the perception of the fundamental exchange rate more explicitly than usual on the grounds of psychological evidence and finally separating some of the forces responsible for the distortions.

Related behavioral finance research has already produced several papers which replicate some of the stylized facts of financial markets (Huang and Day, 1993; Lux, 1997; Brock and Hommes, 1997, 1998; Farmer, 1998). Moreover, the approach of Kirman (1991) is able to produce short-lived bubbles, whereas Frankel and Froot (1986) find an explanation for lasting bubbles. In their model, the bubble process is fed by a smoothly and slowly declining influence of the fundamentalists. The bubble path is at its turning point when the impact of the fundamentalists is at its minimum. In addition to the absence of typical time series properties, such a weighting scheme does not always hold.

The starting point for our investigation is a simple chartists–fundamentalists model. Motivated by empirical observations, a model is developed where boundedly rational market participants choose between a technical and a fundamental trading rule to determine their speculative investment positions. This decision is repeated at the beginning of each new trading period. If one subscribes to the strong assumption that the agents are able to determine the true fundamental value of the exchange rate, then the exchange rate fluctuates in a complex way around its fundamental value. The computed time series mimic the behavior of major currencies quite closely. Besides some excess volatility, the foreign exchange market seems to be efficient.

However, in this paper we try to go one step further and model the perception of the fundamental exchange rate more realistically. While the agents follow the news arrival process closely, mistakes in information processing occur. These mistakes are propagated over time since the agents tend to stick to their previously perceived fundamental value (anchor heuristic). If the agents believe that the exchange rate itself contains relevant information, they incorporate it into their "anchor" so that the exchange rate becomes even more disconnected from its true fundamental. Nevertheless, in the long-run the agents react to macroeconomic imbalances and adjust their perception. Through this learning procedure, the distortion eventually is corrected and some long-term mean-reversion sets in. Note that the results are not the outcome of strange exchange rate movements. On the contrary, the generated time series share some basic stylized facts with the empirical data.

The paper is organized as follows. Section 2 presents a simple chartists–fundamentalists framework and discusses its time series properties. Adopting heuristics from the psychological literature, Section 3 modifies the perception of the fundamental exchange rate. This extension gives rise to bubble processes. Section 4 offers some conclusions.

2. A simple chartists–fundamentalists model

First of all, the agents considered in this paper are not fully rational. Therefore, their behavior is not derived out of a well-defined utility maximization problem. Instead, we provide an empirical micro-foundation for the trading activity of the agents by using, for instance, observations from the market microstructure or psychological evidence. Our point of reference is that the agents are boundedly rational. According to Simon (1955), this term recognizes the cognitive limitations of a decision-maker with respect to both knowledge (including the relevant theory and necessary information) and computational capacity.

Psychologists like Tversky and Kahneman (1974) have produced a huge amount of experimental material which impressively demonstrates that people tend to rely on a limited number of heuristic principles. The complex tasks of assessing probabilities or predicting values are reduced to simpler judgmental operations hereby. It should be clear that heuristics are a kind of constraint. They prevent the agents from making an infinite number of useless inferences, but they also prevent the agents from making (a much smaller number of) useful inferences (Holyoak and Nisbett, 1988). In general, heuristics are quite useful, but sometimes they lead to severe and systematic errors.

Noteworthy examples for such failures are given in survey studies on expectation formation (Ito, 1990). Most importantly, market participants are heterogeneous, e.g. there exist individual peculiarities like wishful thinking which clearly violate the rational expectation hypothesis. Furthermore, the agents appear to adhere to destabilizing bandwagon expectations in the short-run, but display a stabilizing mean-reversion in the long-run.

Two observations are crucial for our model. In recent years, the daily foreign exchange turn-over has increased sharply. More and more, the trading volume reflects very short-term transactions, indicating a highly speculative component (BIS, 1999). Surprisingly, when the speculators determine their investment positions, rather simple trading rules are applied. Survey studies such as Taylor and Allen (1992) and Menkhoff (1997) unanimously confirm that most of the professional foreign exchange traders rely on both technical and fundamental trading rules.

Thus, our model lives from the facts that the foreign exchange market is dominated by speculative activity and that the traders are familiar with both technical and fundamental trading methods. The crucial idea of the model may be summarized as follows. At the beginning of every trading period, the traders choose a specific trading rule to determine their speculative investment position. The traders have the choice between technical and fundamental trading rules. Their selection depends on expected future performance possibilities.

2.1. Setup of the model

Technical analysis is a trading method that attempts to identify trends and reversals of trends by inferring future price movements from those of the recent past (see Murphy (1999) for a popular tutorial of technical analysis). Trading signals are spotted by applying both graphical (charts) and statistical tools. To cover these trading methods, the technical demand of the agents for period t is divided into two components

$$d_t^{\mathrm{C}}=\alpha^{\mathrm{C}}\{\alpha^{\mathrm{C},1}(0.6(\mathrm{Log}\, S_{t-1}-\mathrm{Log}\, S_{t-2})+0.4(\mathrm{Log}\, S_{t-2}-\mathrm{Log}\, S_{t-3}))+\alpha^{\mathrm{C},2}\delta_{t-1}\}. \tag{1}$$

The first one reflects the typical behavior of technical traders, where the trading signals are triggered by a simple moving average rule. In general, the demand of chartists is positive (negative) if the exchange rate S rises (declines). The second one represents additional random demand to allow for more complicated behavior. The stochastic variable δ is assumed to be normally distributed with mean zero and constant variance. The positive reaction coefficients α^{C}, $\alpha^{C,1}$, and $\alpha^{C,2}$ calibrate the total demand of (1) and the relation between the systematic and unsystematic component. Finally, note that in (1) a market order for period t is generated in response to past price changes. Such a lag structure is typical for technical trading rules since only past movements of the exchange rates are exploitable by these rules.

In contrast to technical trading rules, fundamental trading rules aim to gain from differences between the exchange rate and its fundamental value. Fundamentalists believe that the exchange rate will converge towards its equilibrium value in the future. The demand of the fundamentalists is formalized as

$$d_t^{F} = \alpha^{F} \left\{ \frac{E_t[S_{t+1}] - S_t}{S_t} \right\}, \tag{2}$$

where α^{F} is a positive reaction coefficient. The fundamental trading rule takes a long (short) position, if the expected future exchange rate is above (below) the spot rate. The amount of the demand depends on the relative distance between the expected rate and the spot rate.[1]

These expectations are modeled in a classical regressive manner. If the exchange rate deviates from its perceived fundamental value S^{FP}, a turn-back is expected. Thus

$$E_t[S_{t+1}] = \gamma_{t-1} S_{t-1}^{FP} + (1 - \gamma_{t-1}) S_{t-1}, \tag{3}$$

where γ stands for the expected adjustment speed of the exchange rate towards its fundamental. For instance, if γ is 0.25, the agents expect an adjustment of 25 percent. However, γ is not constant but drawn from a Uniform distribution. Since the expectation formation process for the trading period t has to be made in advance, the last available data is from period $t - 1$.

Decisive to this paper is the way in which the agents form their perception of the true fundamental exchange rate. This will be discussed in the next section. Here, we will simply assume that the agents are able to determine on average the true fundamental exchange rate S^{F}, although they do make some mistakes in every period

$$\mathrm{Log}\, S_t^{FP} = \mathrm{Log}\, S_t^{F} + \lambda_t, \tag{4}$$

where the mistake λ is normally distributed with mean zero and constant variance.

The development of the fundamental exchange rate is due to the news arrival process. The fundamental value follows a jump process. Its logarithm is given as

$$\mathrm{Log}\, S_t^{F} = \mathrm{Log}\, S_{t-1}^{F} + \eta_t, \tag{5}$$

where the news η is identically and independently distributed according to a Normal distribution with mean zero and constant variance. However, news does not occur in every trading period. In such a case η is zero.

The decision for a trading rule has to be made before the trading starts. The selection depends on expected future performance possibilities. Fundamentalism, compared to chartism,

[1] Due to the time structure of the model, the fundamentalists function as market makers. Technical traders derive their orders from past price movements; the market clearing is established by the fundamentalists. They adjust the price in order to absorb the excess demand.

becomes more popular the wider the spot rate deviates from its expected future rate. The weight of the chartists is defined as

$$m_t = \left(1 + \beta^1 + \beta^2_{t-1}\sqrt{\frac{|E_t[S_{t+1}] - S_{t-1}|}{S_{t-1}}}\right)^{-1}, \tag{6}$$

and the weight of the fundamentalists as $(1 - m_t)$. The coefficient β^1 represents the basic influence of the fundamentalists, e.g. $(1-1/(1+\beta^1))$ is the minimum fraction of agents who are always fundamentalists. Nevertheless, most of the traders adjust their trading strategy with respect to the relevant market conditions. As indicated by (6), the weight of the fundamentalists increases, though at a declining rate, as the relative distance between $E_t[S_{t+1}]$ and S_{t-1} rises. In such a situation, more and more of the speculators come to the conclusion that the exchange rate is mispriced so that fundamental analysis is preferable to technical analysis. The time dependent coefficient β^2_t reflects the popularity of fundamental analysis and stems from a Uniform distribution.

Demand from the international trade and the risk management of the firms play, compared to speculative transactions, a minor but still significant role. The liquidity needs of the firms for period t is given as

$$d_t^{\mathrm{IT}} = \alpha^{\mathrm{IT}}\left\{\alpha^{\mathrm{IT},1}\chi_{t-1} - \alpha^{\mathrm{IT},2}\frac{S_{t-1} - S^{\mathrm{F}}_{t-1}}{S^{\mathrm{F}}_{t-1}}\right\}, \tag{7}$$

where α^{IT}, $\alpha^{\mathrm{IT},1}$, and $\alpha^{\mathrm{IT},2}$ are reaction coefficients. The first source of the demand χ is normally distributed (with mean zero and constant variance), whereas the second source reflects the usual current account relationship. For example, if the currency is overvalued (i.e. $S > S^{\mathrm{F}}$), then imports exceed exports. Since the focus is on rather short time periods, say daily time intervals, the sign of the total demand of the firms is not a priori clear. Even a medium current account imbalance may be overcompensated by the random component.

The market clearing condition is given as the sum over all transactions in period t

$$m_t d_t^{\mathrm{C}} + (1 - m_t)d_t^{\mathrm{F}} + d_t^{\mathrm{IT}} = 0. \tag{8}$$

Solving (8) for the exchange rate yields the solution of the model

$$S_t = \frac{E_t[S_{t+1}]}{1 - (m_t d_t^{\mathrm{C}} + d_t^{\mathrm{IT}})/(1 - m_t)}, \tag{9}$$

which is a four-dimensional stochastic difference equation system. As can be inferred from (9), larger price reactions occur whenever a low proportion of fundamentalists is confronted with a huge demand of chartists. Since (9) precludes closed analysis, simulations are performed to demonstrate that the underlying structure gives rise to complex exchange rate motion as it is observed empirically.[2]

[2] The mechanics of the model are best described by a stretching and folding of the exchange rate around its perceived fundamental value. In the absence of noise, such a mechanism has the potential to generate chaotic motion.

Table 1
The basic parameter setting for the simulations

Description of parameter	Symbol	Value
Reaction coefficient, Eq. (1)	α^{C}	1
Reaction coefficient, Eq. (1)	$\alpha^{C,1}$	0.225
Reaction coefficient, Eq. (1)	$\alpha^{C,2}$	0.008
Reaction coefficient, Eq. (2)	α^{F}	1
Reaction coefficient, Eq. (7)	α^{IT}	1
Reaction coefficient, Eq. (7)	$\alpha^{IT,1}$	0.0014
Reaction coefficient, Eq. (7)	$\alpha^{IT,2}$	0.001
Random demand of traders, Eq. (1)	δ	$N(0, 1)$
Random demand of firms, Eq. (7)	χ	$N(0, 1)$
Basic level of fundamentalists, Eq. (6)	β^1	0.177
Popularity of fundamentalists, Eq. (6)	β^2	Prob $(\beta_t^2 \sim U(8, 100)) = 2/52$, else $\beta_t^2 = \beta_{t-1}^2$
Distribution of news, Eq. (5)	η	Prob $(\eta_t \sim N(0, 0.0075)) = 1/5$, else $\eta_t = 0$
Misperception, Eq. (4)	λ	$N(0, 0.001)$
Expected adjustment, Eq. (3)	γ	Prob $(\gamma_t \sim U(0, 0.5)) = 1/6$, else $\gamma_t = \gamma_{t-1}$

2.2. *Calibration*

Before we discuss the simulation results, the model has to be calibrated. Unfortunately, only some of the parameters are empirically observable. The parameter setting is displayed in Table 1. With the help of the reaction coefficients, the demand of the traders and the firms is controlled. The relation between the systematic and unsystematic demand of the technical trading rule is around 1–6. In comparison, the demand of the firms is about one third of the total (see BIS).[3] Taylor and Allen show that roughly 15 percent of the market participants always rely on fundamental analysis, i.e. $\beta^1 = 0.177$. The coefficient β^2 is chosen so that the variance of the exchange rate time series matches the variance of daily exchange rate movements. Note that the popularity of being a fundamentalist is not constant, but changes randomly from time to time. This might be justified by changing economic conditions which result in periods of higher and lower uncertainty about the fundamental exchange rate. The development of the fundamental exchange rate takes into account that fundamental shocks do not occur in every period. In contrast to the news arrival process, the misperception of the fundamental exchange rate is rather small. Finally, the agents expect an adjustment of the exchange rate towards its fundamental between 0 and 50 percent. On average, γ changes every six trading periods.

2.3. *Snapshot of the dynamics*

This section demonstrates that simulations of the model are able to replicate three important univariate stylized facts of exchange rate fluctuations: (i) unit roots in the exchange

[3] The firms' demand is mainly random; only 0.3 percent of total transactions are due to the current account relationship. However, this changes in Section 3. Lasting bubbles induce an increase of the current account demand up to 1.7 percent of total transactions, which is consistent with BIS estimates.

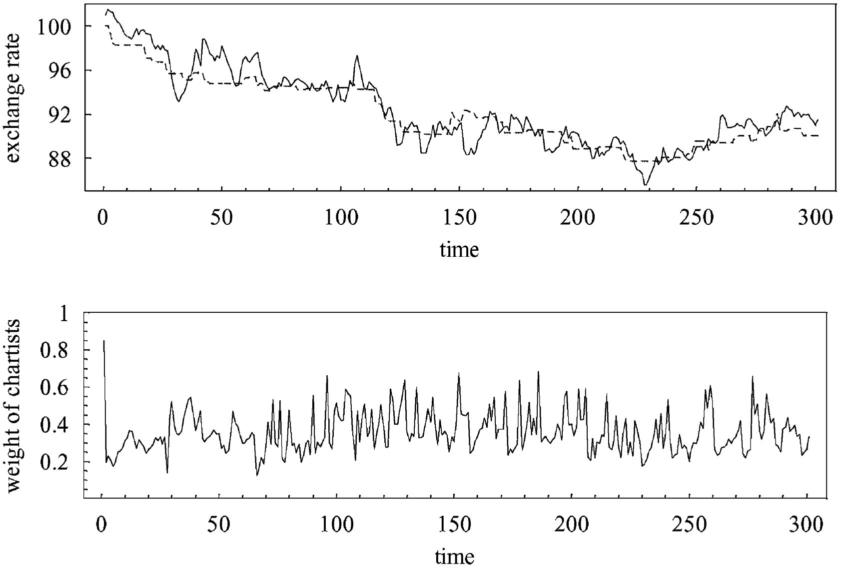

Fig. 1. Exchange rate dynamics and weight of chartists in the time domain. The solid line is the exchange rate, the dashed line its fundamental value. Parameters as in Table 1.

rates, (ii) fat tails for returns, and (iii) volatility clustering (for a general survey, compare Guillaume et al., 1997).

Fig. 1 gives a first impression of the dynamics. The top contains the exchange rate (solid line) and the development of its fundamental (dashed line) for 300 periods; the bottom displays the corresponding weights of the chartists. Even a low probability of fundamental shocks suffices to generate realistic exchange rate movements where the spot rate circles around its fundamental in a complex fashion. The volatility of the exchange rate is clearly higher than its fundamental value. The weight of the technical traders is mostly concentrated in the range from 30 to 60 percent with some peaks going down to 15 or up to 75 percent. Hence, the agents rely on both kinds of investment strategies.

Fig. 1 may help to explain the fuzzy relationship between news and exchange rate movements. Goodhart (1988) reports both systematic underreaction and overreaction to news. Even large price movements unrelated to news are apparent. Visual inspection of Fig. 1 reveals similar findings. Our model suggests that the dynamics are partially caused endogenously through the interactions between the traders. For instance, after a shock has hit the market, the exchange rate reaction may be amplified and prolonged by the positive feedback trading of the technicians. Black (1986) has called this behavior noise trading. According to Black, noise, which can be described as a large number of small events, is essential to the existence of liquid markets. He argues that a person who wants to trade needs another person with opposite beliefs. To explain the high trading volume, it is not reasonable to assume that differences in beliefs are merely the outcome of different information. In this model, the traders may even generate their own trading signals in periods with no new information at all.

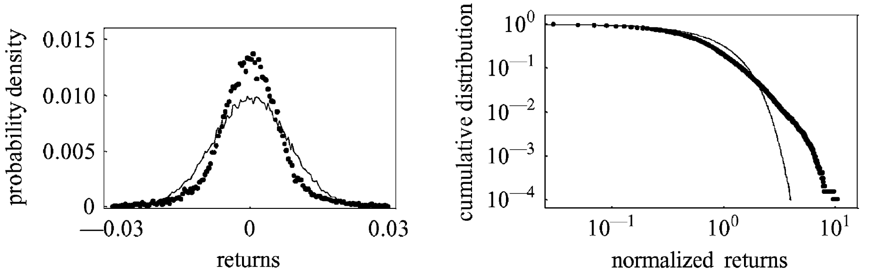

Fig. 2. The dotted lines show the distribution of the returns and the scaling behavior of the cumulative distribution of the positive and negative tails for normalized log-returns (parameters as in Table 1, 20,000 observations), the solid lines the same but for a Normal distribution with identical variance.

One stylized fact of the empirical literature is that exchange rate time series display unit roots (Goodhart et al., 1993). We have tested the null hypothesis that any shock to the exchange rate is permanent against the alternative hypothesis that a shock is only temporary with the augmented Dickey-Fuller test (ADF). Using, for instance, a four-lag specification without intercept delivers

$$\Delta S_t = a_0 S_{t-1} + a_1 \Delta S_{t-1} + a_2 \Delta S_{t-2} + a_3 \Delta S_{t-3} + a_4 \Delta S_{t-4}. \tag{10}$$

Since a_0 is not significant for the computed time series the null is not rejectable. This holds for various lag settings of (10) and values of the coefficients (compare also Table 4). Often, this result is interpreted as evidence for a random walk behavior of the exchange rate. Looking at Fig. 1, the reason for the strong support of the unit root hypothesis for our model becomes obvious: The exchange rate fluctuates around its fundamental, which itself follows a stochastic process. Since the exchange rate path never gets strongly disconnected from its fundamental value for a longer period, it also appears to be a random process. Inspecting the first 10,000 periods, one finds that the difference between both time series stays mainly in the region of ±5 percent. In the most extreme cases, the exchange rate diverges up to 15 percent from its fundamental. However, such deviations are very short-lived. Exactly speaking, the exchange rate and its fundamental are highly cointegrated. Relaxing this property is the aim of Section 3.

Another stylized fact states that the distribution of the returns has fat tails (Guillaume et al.). Returns are defined as log-price changes ($r_t = \text{Log}\, S_t – \text{Log}\, S_{t-1}$). Relative to a Normal distribution with identical variance, one finds a stronger concentration around the mean, thinner shoulders, and more probability mass in the tails of the distribution. Fig. 2 compares the distribution of the returns and its scaling behavior for the simulated returns and for normally distributed returns. Besides visual examination, the estimates of the kurtosis reveal fat tails. Table 2 summarizes some descriptive statistics of a simulation run over 10,000 periods. Since the kurtosis is higher than 3 (the theoretical value of a Normal distribution), the computed time series possesses fat tails. Note that the largest exchange rate movements are not unrealistically high, but they match empirical observations. As already mentioned, the model is calibrated so that the variance fits with daily exchange rate

Table 2
Some descriptive statistics of the returns[a]

Min	Median	Max	Variance	Skewness	Kurtosis
6.49 percent	0.00 percent	6.63 percent	0.000061	0.1136	9.73

[a] Parameters as in Table 1, 10,000 observations.

movements of major currencies. Compared to the variance of $r_t^{\mathrm{F}} = \mathrm{Log}\, S_t^{\mathrm{F}} - \mathrm{Log}\, S_{t-1}^{\mathrm{F}}$, the variance of the exchange rate returns is six times higher. This clearly reflects strong excess volatility.

Fat tails may also be detected by determining the tail index. The tail index α, given as $F(|\mathrm{return}| > x) \approx cx^{-\alpha}$, is estimated from the cumulative distribution of the positive and negative tails for normalized log-returns. The returns are normalized by dividing by the standard deviation. Fig. 2 illustrates that the distribution of the returns roughly follows a power law. A regression on the largest 30 percent of the observations delivers a significant tail index of 3.64, which is consistent with results obtained from empirical data (Guillaume et al.). But what causes fat tails? The model does not only produce stronger outliers as a result of strong random demand shocks, but also if a medium demand has to be absorbed by a low weight of the fundamentalists. The tail index for a Normal distribution (see Fig. 2) is clearly higher and less significant.

The third stylized fact highlights that periods of low volatility alternate with periods of high volatility (Mandelbrot, 1963). Although almost no autocorrelation exists for raw returns, a different picture emerges if one uses absolute returns as a volatility measure (Fig. 3).[4] The autocorrelation for absolute returns is clearly significant and slowly decaying. The reason for the short-term volatility clustering lies in the feedback trading of the agents. A strong exchange rate movement in period t indicates a strong trading signal for period $t + 1$. The long-run autocorrelation stems from different degrees of the popularity of the fundamental trading rules.[5] Note that the trading volume also tends to cluster. However, the cross-correlation of volatility is positive only with current volumes but almost zero for past and future volumes (Brock and LeBaron, 1996).

So far, the model is able to replicate some important stylized facts. Only the strong connection between the exchange rate and its fundamental seems not to be very realistic. For instance, in this case the path of the US dollar in the 1980s would be fundamentally justified. In addition, it would imply that the foreign exchange market is more or less efficient (besides some excess volatility). Next, we discuss a simple extension of the model where the exchange rate may disconnect from its fundamental value.

[4] Fig. 3 reveals a weak tendency of short-run mean-reversion which is also observable in the empirical data (Cutler et al., 1990).

[5] This is also reflected in the famous GARCH models. Using a GARCH (1,1) specification, the ARCH term is estimated as 0.058 and the GARCH term as 0.908. Hence, the conditional variance depends slightly on most recent shocks (ARCH effect) and strongly on the temporary volatility (GARCH effect). Since the sum of both terms is nearly one, the volatility shocks are quite persistent.

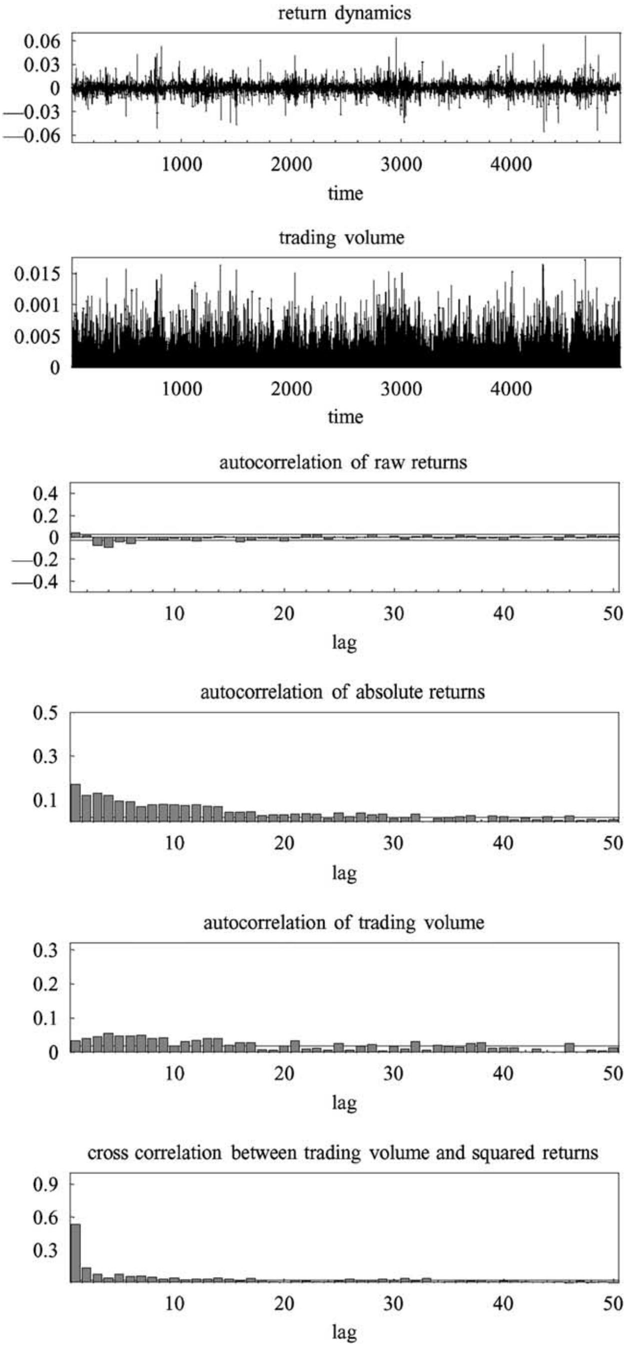

Fig. 3. The dynamic behavior of the returns and of the trading volume. Parameters as in Table 1, 5000 observations. Ninety-five percent confidence intervals are plotted as $\pm 2/\sqrt{T}$ (assumption of white noise).

3. The perception of the fundamental exchange rate

This section explores how the agents perceive the fundamental exchange rate. Psychological evidence indicates that expectations are heavily influenced by the anchor and adjustment heuristic. Tversky and Kahneman report that in many situations, people make estimates by starting from an initial value that is adjusted to yield the final answer. The initial value, or starting point, may be suggested by the formulation of the problem, or it may be the result of a partial computation. In either case, adjustments are typically insufficient, implying biased estimates toward initial values.

Two questions arise. What is the relevant anchor of the agents and how do they carry out the adjustment? It seems natural that the anchor depends on the previously perceived fundamental exchange rate. In addition, there exists evidence that the exchange rate itself may be part of the anchor. We call this self-confirmation, and we include it because of the way technical analysis is conducted. In the words of Murphy, a technician believes that anything that can possibly affect the price—fundamentally, politically, psychologically, or otherwise—is actually reflected in the price of that market. Therefore, a technician indirectly studies fundamentals: If prices are rising, fundamentals must be bullish. A study of price action is all that is required. According to Murphy, agents believe that the exchange rate itself reflects relevant information.

The adjustment procedure consists of two steps. First, agents react, of course, to the arrival of new information. However, the exact meaning of new information is not clear. Experiments even indicate that agents tend to systematically misperceive news. Due to the conservatism heuristic, individuals are slow to change their beliefs in the face of new evidence. Individuals update their posteriors in the right direction, but by too little in magnitude (Edwards, 1968). This may lead to an underreaction to news. On the contrary, the representativeness heuristic highlights the phenomenon that people tend to view events as typical (or representative) of some specific class and to ignore the laws of probability in the process (Tversky and Kahneman). For instance, investors tend to see patterns in truly random sequences. After a consistent series of good news for an asset, they may conclude that the past history is representative of an underlying growth potential. While a consistent pattern of high growth may be nothing more than a random draw for the asset, investors see order among chaos and infer from the in-sample growth path that the asset just keeps growing. This may cause an overreaction to news.

The second adjustment step covers experience-based feedback learning. Although such a procedure may yield a partial error correction, the adjustment is typically quite slow in time and small in magnitude. Psychologists give various reasons for the incomplete changes such as overestimation and overconfidence of the agents, avoidance of cognitive dissonance, or simply a lack of cognitive skills (Kahneman et al., 1986).

To summarize, the agents use a mixture of the previously perceived fundamental exchange rate and the exchange rate itself as an anchor. The adjustment takes part in two steps, first, by an update according to new information, and second, by an error correction in form of feedback learning. The remainder of this section discusses the implications of the anchor and adjustment heuristic for the exchange rate dynamics. Afterwards, a more detailed analysis of what influences the distortion is given. To clarify the forces at work, the discussion of the perception process is divided into two sections: without and with learning adjustment.

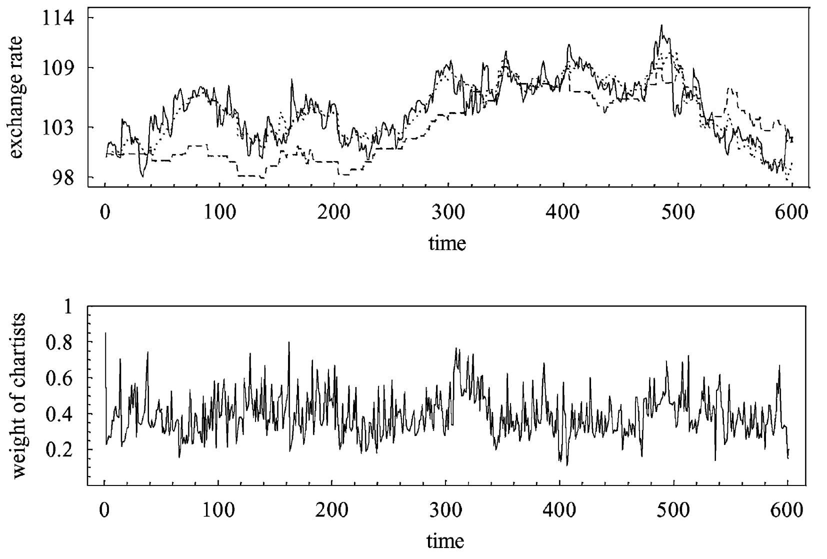

Fig. 4. Exchange rate dynamics and weight of chartists in the time domain (without learning). The solid line is the exchange rate, the dashed line its fundamental, and the dotted line the perceived value of the fundamental. Parameters as in Table 1, additional $\varepsilon = 0.9$, $\kappa \sim U$ (0.25, 1.75).

3.1. Perception without learning

Our first version of the perception of the fundamental exchange rate is

$$\text{Log } S_t^{\text{FP}} = \varepsilon \text{ Log } S_{t-1}^{\text{FP}} + (1-\varepsilon) \text{ Log } S_{t-1} + \kappa_t \eta_t, \tag{4a}$$

where ε indicates the impact of the previously perceived fundamental exchange rate and the exchange rate for the anchor. We assume a self-confirmation of 10 percent ($\varepsilon = 0.9$). The misperception of the news is expressed through κ, where $\kappa \sim U$ (0.25, 1.75). Clearly, on average the news is perceived correctly, but mistakes occur every period.[6]

Fig. 4 shows a typical simulation run for the modified solution. Now, the exchange rate (solid line) fluctuates around its perceived fundamental value (dotted line). Because of self-confirmation, this value slightly changes in every period. Moreover, the exchange rate has the potential to move away from its equilibrium value (dashed line). The selection outcome for the trading rules is not affected through (4a).

This is a main difference to the work of Frankel and Froot. Their dynamics are as follows: caused by an initial shock, the exchange rate starts to shift away from its fundamental. Via self-fulfilling expectations, chartists are gaining prominence so that the bubble path is supported. However, when almost all agents are chartists the bubble dynamics automatically die out. Afterwards, there occurs a fundamentalists revival. The weight of the fundamentalists increases as the exchange rate converges towards its long-term equilibrium. With this

[6] Since $E[\kappa] = 1$, the news is not systematically misinterpreted. For this, compare Section 3.3.

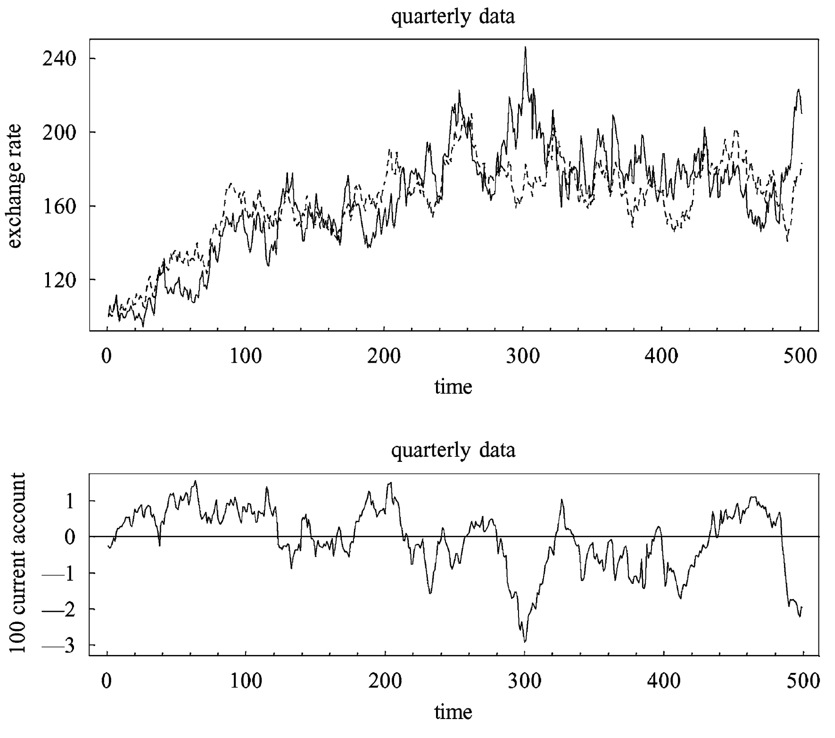

Fig. 5. Exchange rate dynamics and current account reaction in the time domain (without learning). The solid line is the exchange rate, the dashed line its fundamental. Parameters as in Table 1, additional $\varepsilon = 0.9$, $\kappa \sim U$ (0.25, 1.75), 40,000 observations, plotted every 80 periods.

approach, Frankel and Froot explain the bubble path of the US dollar in the 1980s. Note that their weighting scheme changes slowly and smoothly during a (lasting) bubble, whereas in our setup the importance of the trading rules varies more rapidly and irregularly. Since most traders find technical and fundamental analysis equally important for short-term predictions (Taylor and Allen), we find our scenario more realistic.

Fig. 5 displays a simulation run for the exchange rate and its fundamental over 40,000 periods, where the data is plotted every 80 periods. If one assumes that the model is based on daily observations, the entries (roughly) represent quarterly data. The model produces both periods where the exchange rate is close to its fundamental (around quarter 150) and periods of large bubbles (between quarter 280 and 320). In the latter case, the exchange rate is more than 30 percent overvalued. The patterns of the distortions can be quite different. For instance, around quarter 260 one finds a typical bubble. During a series of good news, the exchange rate overshoots its fundamental. Between quarter 90 and 110, the exchange rate is relatively stable whereas the fundamental shifts away. Finally, at quarter 230 the exchange rate and its fundamental even move in opposite directions. In the bottom part of Fig. 5, the current account reaction is given. Naturally, bubbles give rise to current account imbalances.

Fig. 6 is designed to provide a first answer to the question of how distortions may evolve. We compare four different simulations. The top right shows a simulation run where firms' transactions and the self-confirmation are both zero (compare Eqs. (7) and (4a), respectively). Due to the misperception of news, the exchange rate diverges from its fundamental value. In the bottom right, the self-confirmation is included again. As can be seen, for instance in the first 100 quarters, self-confirmation has the power to destabilize the market even further. In the bottom left, the firms' transactions are included, yet there is no self-confirmation. To visualize the impact of the demand of the firms, it has been multiplied by a factor of 40. In contrast to the top right, the exchange rate is pushed closer to its fundamental value but still does not track it. The top left shows the base run again (i.e. Fig. 4). Including both self-confirmation and firms' transactions leads to a closer relationship between the exchange rate and its fundamental value. The reason is that the exchange rate contains relevant information for the perception of the fundamental value. Hence, self-confirmation behavior is not totally irrational. The explanation is as follows. Whenever the difference between the exchange rate and its fundamental value is small, self-confirmation increases the distortion. For example, misperceptions of news or positive feedback trading of the speculators may push the exchange rate away from its fundamental value. These misalignments are settled via self-confirmation. However, with increasing misalignments, current account imbalances also rise. Period for period trade transactions induce some pressure on the exchange rate into the direction of its fundamental value. Due to self-confirmation, this drives the perceived fundamental exchange rate closer to the true fundamental exchange rate so that the bubble path is limited or even brought to an end.

3.2. Perception with learning

So far, the agents adjust their anchor with respect to the arrival of new information. However, the dynamics are able to generate huge current account imbalances. It seems natural that the agents try to learn from these macroeconomic disequilibriums. The final version of the perception of the fundamental exchange rate is

$$\mathrm{Log}\, S_t^{\mathrm{FP}} = \varepsilon\, \mathrm{Log}\, S_{t-1}^{\mathrm{FP}} + (1-\varepsilon)\, \mathrm{Log}\, S_{t-1} + \kappa_t \eta_t + \mathrm{Log}\,[1 + \omega d_{t-1}^{\mathrm{IT}}], \tag{4b}$$

where ω indicates the degree of feedback learning. Note that the agents do not infer the true fundamental exchange rate out of the current account data, but simply adjust their anchor to some extent in the right direction.

Figs. 7 and 8 display the short-run and long-run dynamics of the final system. The simulated time series resembles the previous ones. The exchange rate circles around its perceived equilibrium value and the weights of chartists fluctuate up and down in a range from 15 to 80 percent. Furthermore, the exchange rate has the power to drift away from its equilibrium value for some time, but eventually a mean-reversion reaction sets in. As a consequence of the feedback learning, the distortion is smaller.

To many economists, the reliance on technical trading rules appears to be irrational. However, Huang and Day state that one must note that during bear and bull markets, chartists are correct except at turning points. In this sense, they are right more often then they are wrong. This may be one of the reasons for the existence of chartists even if on average they

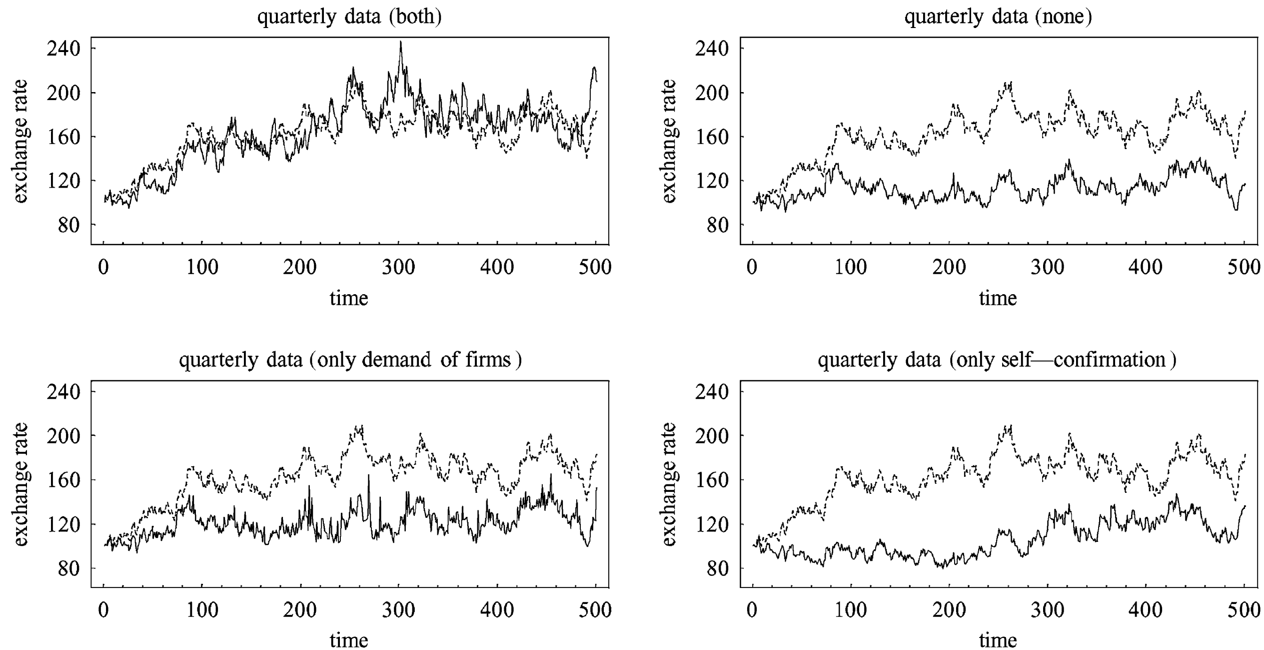

Fig. 6. The evolution of the exchange rate (solid line) and of its fundamental value (dashed line) for four different simulations. The top left contains a simulation with both firms' transactions and self-confirmation, the top right without both of them, the bottom left only demand from the firms (40 times higher than before), and the bottom right only self-confirmation. Parameters as in Table 1, additional $\varepsilon = 0.9$, $\kappa \sim U\ (0.25, 1.75)$, 40,000 observations, plotted every 80 periods.

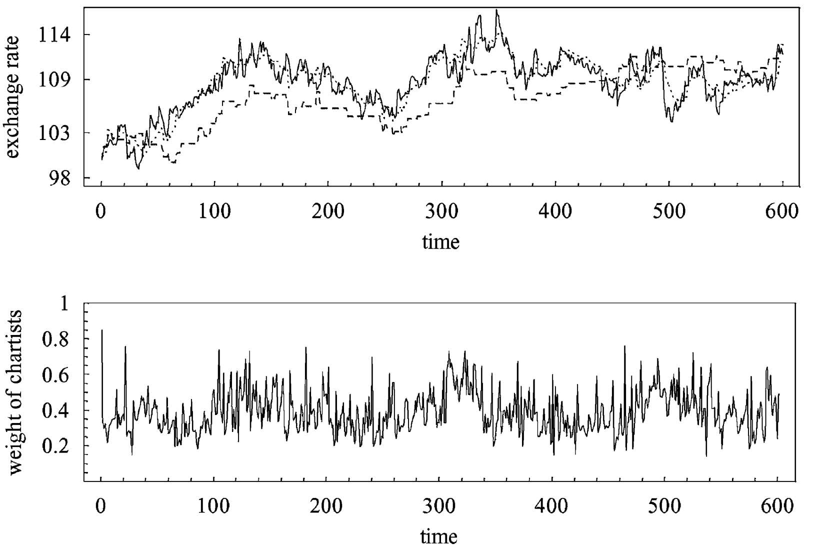

Fig. 7. Exchange rate dynamics and weight of chartists in the time domain (with learning). The solid line is the exchange rate, the dashed line its fundamental, and the dotted line the perceived value of the fundamental. Parameters as in Table 1, additional $\varepsilon = 0.9$, $\kappa \sim U\ (0.25, 1.75)$, $\omega = 0.25$.

lose wealth in the long-run. Next, we try to test this presumption with our data set. Suppose a small individual investor enters the market in period $t = a$ and uses only the technical trading rule as specified by Eq. (1). Then, after T periods his profits are

$$P_T = S_T \sum_{t=a}^{T} d_t^{\mathrm{C}} - \sum_{t=a}^{T} S_{t-1} d_t^{\mathrm{C}}, \tag{11}$$

where the first term stands for the final revenue from clearing the position and the second term for the permanent revenue from building up the position. Fig. 9 shows the evolution of the profits for the case in which the agent enters the market in period $a = 1$ (top) and $a = 20{,}001$ (bottom). In the long-run, technical analysis does not seem to be profitable. Nevertheless, one also finds longer periods where the agent earns money by simply applying his rule. For instance, if the agent starts to trade in period 20,001, then his profits are positive even after 20,000 periods. One reason for this phenomenon is that technical trading rules are especially successful in bubble times. In these periods, they tend to build up a larger position which increases in value as the bubble moves on (compare the last 1000 periods). Thus, we stress that the use of simple technical trading rules as an adaptive scheme of behavior need not be totally irrational per se. Moreover, a number of empirical studies impressively demonstrate the success of a broad range of technical trading rules (Brock et al., 1992).

In addition, Fig. 8 seems to indicate that the foreign exchange market is rather inefficient. A cointegration analysis allows us to check this presumption. Variables are cointegrated if there exists a linear combination that is stationary. If the exchange rate tracks its fundamental, then the difference between the exchange rate and its fundamental should be stationary. In a

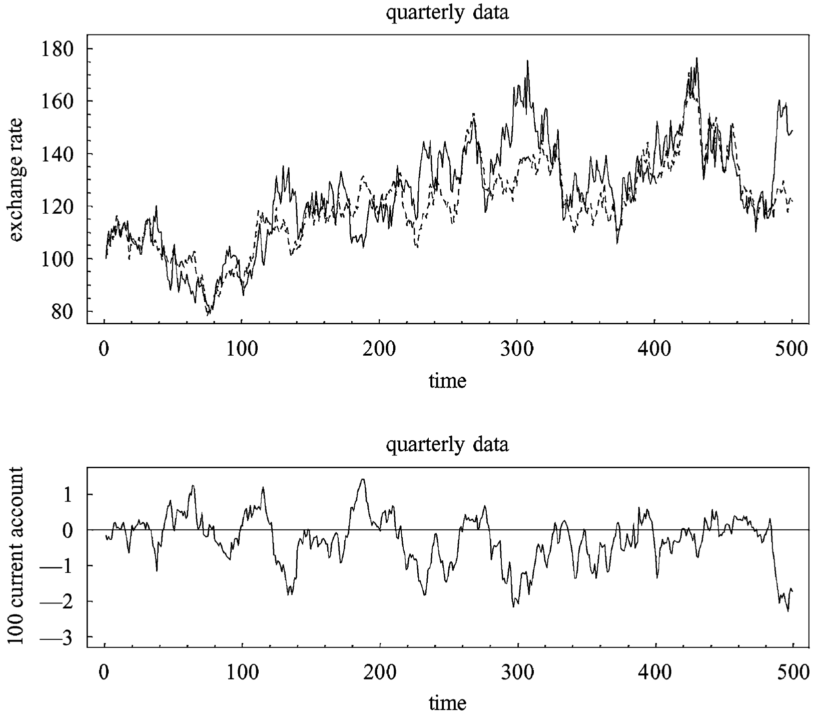

Fig. 8. Exchange rate dynamics and current account reaction in the time domain (with learning). The solid line is the exchange rate, the dashed line its fundamental. Parameters as in Table 1, additional $\varepsilon = 0.9$, $\kappa \sim U$ (0.25,1.75), $\omega = 0.25$, 40,000 observations, plotted every 80 periods.

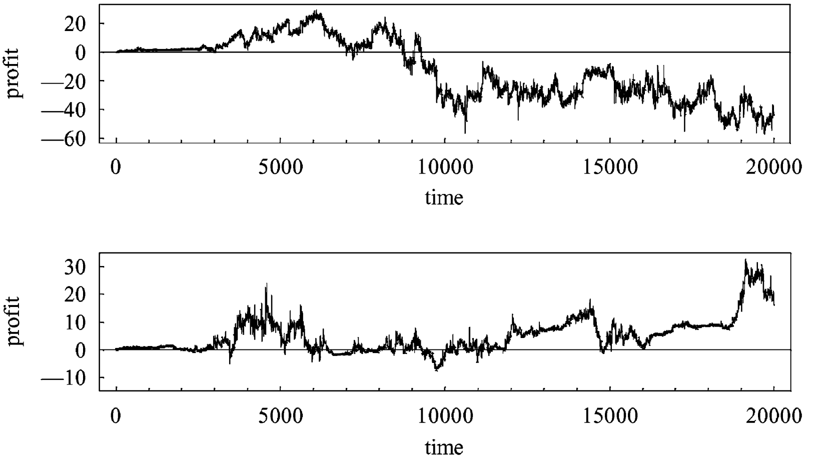

Fig. 9. The profitability of technical trading rules for two different periods. The top shows the profits from periods 1–20,000 and the bottom from periods 20,001–40,000. Parameters as in Table 1, additional $\varepsilon = 0.9$, $\kappa \sim U$ (0.25, 1.75), $\omega = 0.25$.

Table 3
Cointegration analysis between the exchange rate and its fundamental value[a]

Time horizon	40,000	20,000	10,000	5000	4000	2000	1000	500	250
Number of subsamples	1	2	4	8	10	20	40	80	160
Number of unit roots in S	1	2	4	8	10	20	40	80	159
Number of cointegration	1	2	4	5	5	8	13	28	39
Percentage of cointegration	100	100	100	62.5	50	40	32.5	35	24.4

[a] Parameters as in Table 1, additional $\varepsilon = 0.9$, $\kappa \sim U\ (0.25, 1.75)$, $\omega = 0.25$, 40,000 observations. The significance level is 5 percent.

broader sense, an equilibrium exists if the difference between the variables does not become too large.

Cointegration may be tested according to the following procedure. The first step consists of testing the order of integration. By definition, cointegration necessitates that the variables to be integrated are of the same order. For this pre-test, the ADF test can be applied. If both variables are integrated of the order 1, the next step is to estimate the long-run equilibrium relationship between S^{F} and S.[7] For this, one has to regress

$$S_t^{\mathrm{F}} = aS_t + u_t, \tag{12}$$

and then apply the ADF test on the residuals u. Using a four-lag specification delivers

$$\Delta u_t = a_0 u_{t-1} + a_1 \Delta u_{t-1} + a_2 \Delta u_{t-2} + a_3 \Delta u_{t-3} + a_4 \Delta u_{t-4}. \tag{13}$$

If it is not possible to reject the null hypothesis $a_0 = 0$, the hypothesis that the variables are not cointegrated cannot be rejected. In simpler terms, if S and S^{F} were found to be integrated of order 1 and the residuals are stationary, one can conclude that the series are cointegrated.

Table 3 reports the cointegration results for the following simulation design. Subsamples are obtained by dividing a total number of 40,000 observations by a maximal time period. For instance, using a maximum time period of 1000, one obtains 40 subsamples. For each of them, a unit root and cointegration test is carried out with a significance level of 5 percent. Table 3 indicates that the price tracks its fundamental value only in the long-run. For shorter time periods, there seems to be no cointegration. For instance, if the maximum number of observations is 500, only 35 percent of the subsamples show an equilibrium relationship between the exchange rate and its fundamental value. In other words, in 65 percent of the cases, the exchange rate moves away from its long-term equilibrium, which should be interpreted as a clear sign of market inefficiency.

The main characteristic time series properties are, in general, not affected by the modifications. Table 4 summarizes some descriptive statistics of the returns for a simulation run over 10,000 periods (compare with Table 2). In particular, the most extreme exchange rate movements and the variance of the returns behaved well. The kurtosis is clearly higher than 3.

Furthermore, the overall shape of the distribution of the returns, as visible from Fig. 10, matches the empirical one (compare with Fig. 2). For example, taking the largest 30 percent

[7] Variables are integrated of the order 1 if the first differences of the variables are stationary. The level of the exchange rates is, in general, not stationary (unit roots), else the integration order would be 0.

Table 4
Some descriptive statistics of the returns[a]

Min	Median	Max	Variance	Skewness	Kurtosis
7.56 percent	0.00 percent	7.56 percent	0.000068	0.052	11.01

[a] Parameters as in Table 1, additional $\varepsilon = 0.9$, $\omega = 0.25$, $\kappa \sim U$ (0.25, 1.75), 10,000 observations.

of the observations into account, the tail index is estimated as 3.54. Again, this indicates fat tails.

Fig. 11 displays the evolution of the returns and the trading volume together with some autocorrelation functions (compare with Fig. 3). One finds almost no autocorrelation for raw returns, but persistent volatility clustering for absolute returns and trading volume. Finally, the cross-correlation between volatility and total transactions is high for current volumes and low for past volumes.

3.3. Analysis of distortion

In this model, the distortions are caused by the way the speculators perceive the fundamental exchange rate. This phenomenon is robust in the sense that it is qualitatively not affected by different values of the parameters. However, by varying a single coefficient, quantitative changes emerge. Focusing on these changes, this section tries to develop a deeper and more systematic understanding of the driving forces of the bubble processes. To be more accurate, a distortion measure is defined as

$$D = \frac{100}{T} \sum_{t=1}^{T} \left| \frac{S_t - S_t^{\mathrm{F}}}{S_t^{\mathrm{F}}} \right|, \tag{14}$$

where the distortion D is computed as the average relative distance between the exchange rate and its fundamental.

Fig. 12 presents some results for the following simulation exercise. The distortion is calculated out of 40,000 data points for each entry in one part of the figure. The time series

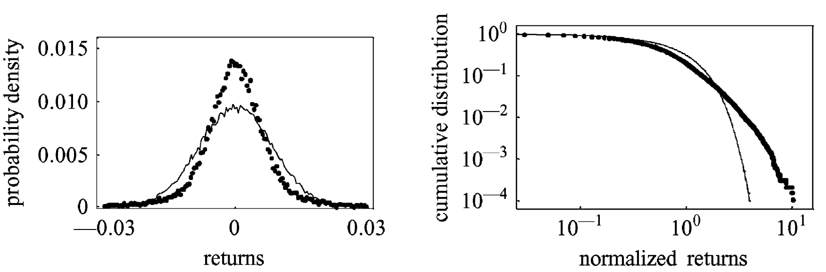

Fig. 10. The dotted lines shows the distribution of the returns and the scaling behavior of the cumulative distribution of the positive and negative tails for normalized log-returns (parameters as in Table 1, additional $\varepsilon = 0.9$, $\kappa \sim U$ (0.25, 1.75), $\omega = 0.25$, 20,000 observations), the solid lines the same but for a Normal distribution with identical variance.

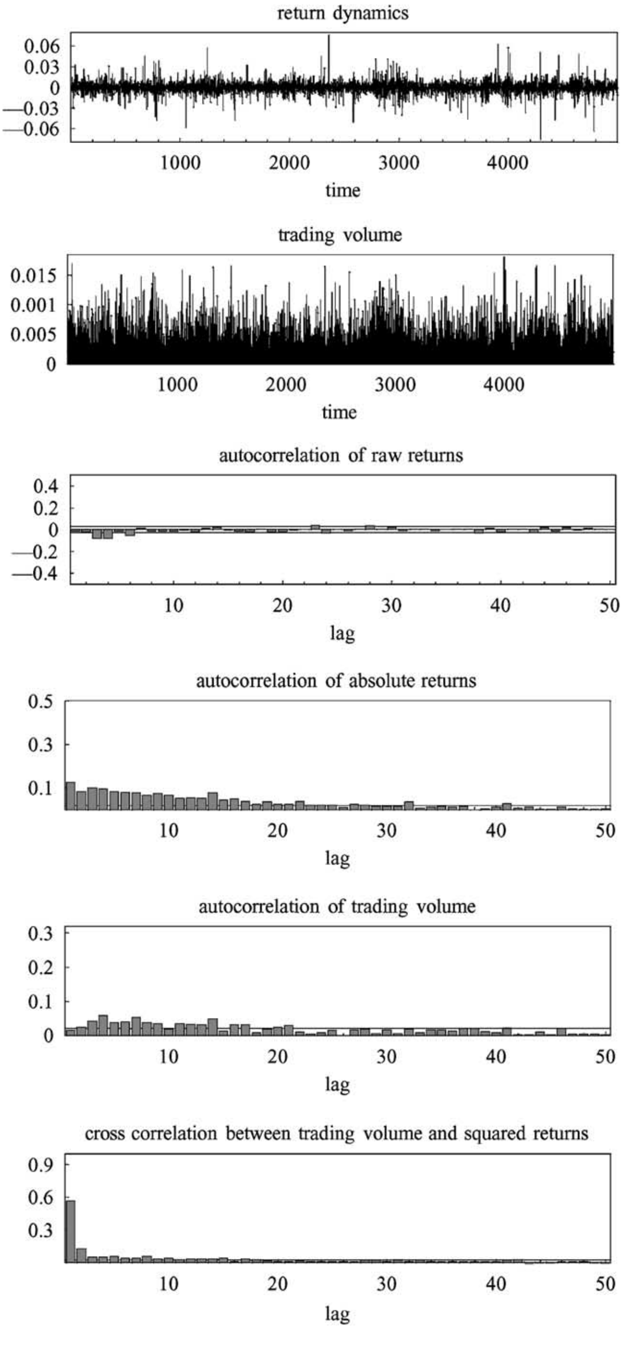

Fig. 11. The dynamic behavior of the returns and of the trading volume. Parameters as in Table 1, additional $\varepsilon = 0.9$, $\kappa \sim U$ (0.25, 1.75), $\omega = 0.25$, 5000 observations. Ninety-five percent confidence intervals are plotted as $\pm 2/\sqrt{T}$ (assumption of white noise).

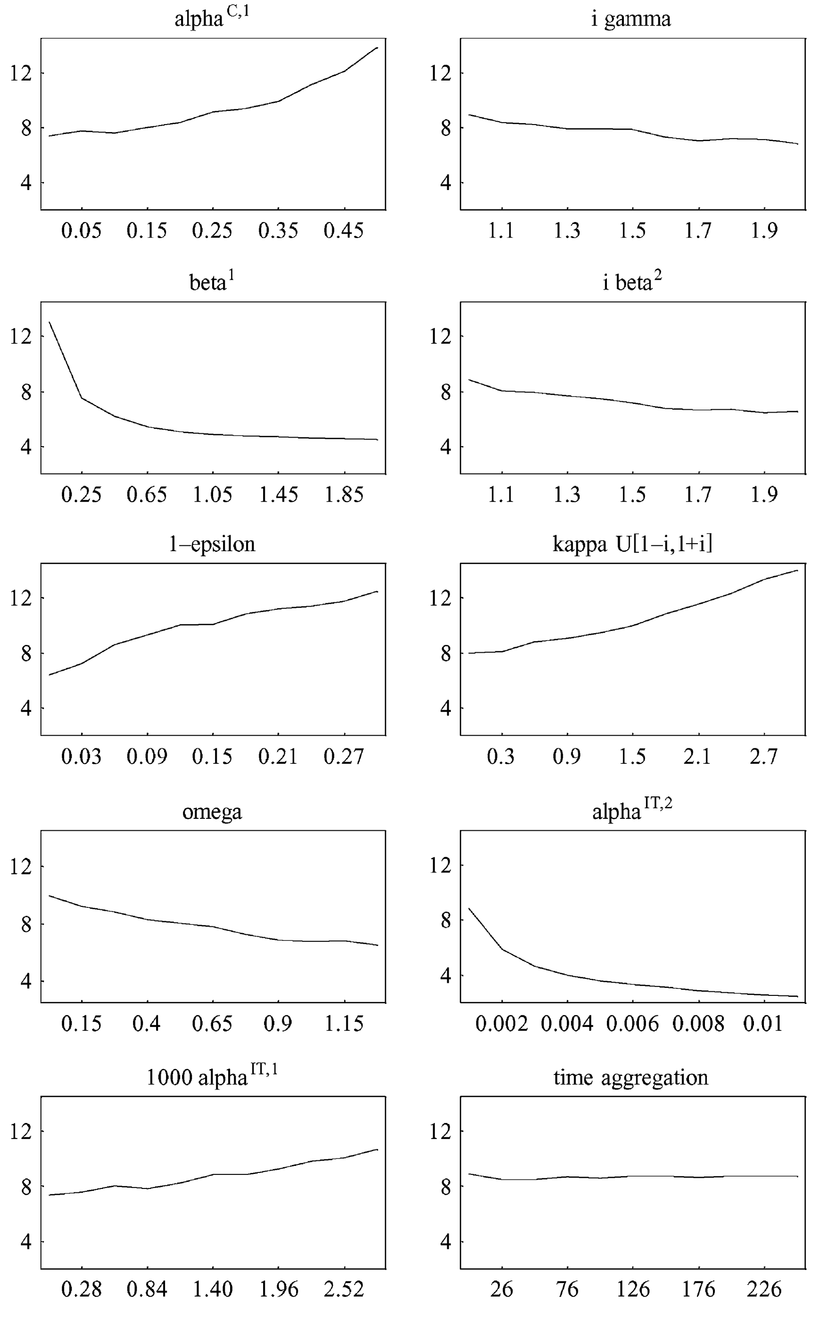

Fig. 12. The evolution of the distortion for certain parameter changes. Parameters as in Table 1, additional $\varepsilon = 0.9$, $\kappa \sim U$ (0.25, 1.75), $\omega = 0.25$, 40,000 observations. Parameters are varied as marked on the axis. The distortion is calculated as given by Eq. (14).

are computed with the same parameter setting as before. Moreover, the same seed for the random variables is used. The parameter under consideration is varied as marked on the axis.

What are the results? A high systematic reaction of the technical trading rule, $\alpha^{C,1}$, goes along with an increase in the distortion. The more systematically the technicians trade, the more often they induce a temporary trend into the time series. Shocks, such as a strong misperception of new information or a high random demand, yield an exchange rate movement which is amplified and prolonged into the next periods.

The expected adjustment speed of the exchange rate towards its perceived fundamental, γ, is randomly drawn from a Uniform distribution within the bounds of 0 to 0.5. In the top right of Fig. 12, γ is multiplied by a factor ranging from 1 to 2. One sees that the higher the adjustment speed, the lower the distortion. The explanation is that for a low γ, the expected future exchange rate is not influenced strongly through its fundamental, so that the mean-reversion is weak. Moreover, if the probability of random changes of γ increases, the distortion shrinks further (not displayed). Lasting sequences of low and high γ are more destabilizing than a stronger mix of γ.

The next two parts of Fig. 12 demonstrate that an increase in the basic fraction of fundamentalists, β^1, or an increase in the popularity of the fundamental trading rule, β^2, reduces the distortion. The higher the degree of fundamentalism, the lower the exchange rate movements. This reduces the demand from technical traders, since their trading signals are less pronounced. As a consequence, their behavior becomes less trend oriented. In contrast to the technical traders, the fundamentalists stabilize the foreign exchange market.

As discussed in Section 3.1 (Fig. 6), the implications of self-confirmation for the distortion appear to be ambiguous (without learning). On the one hand, including the spot rate into the anchor destabilizes the market since misalignments are settled into the course. On the other hand, if the current account is not balanced, the exchange rate may transport relevant information so that self-confirmation can stop bubbles. The overall effect of self-confirmation is revealed in Fig. 12. The distortion is positively correlated with self-confirmation.

The case of misperception is straightforward. The higher the misperception, the higher the distortion. Naturally, the distortion increases further if the perception of the news is biased (not displayed). Periods of overreaction to news lead to traditional bubbles (overshooting), whereas in periods of underreaction to news the exchange rate does not follow the development of its fundamental.

A stronger (error correction) learning on the part of the agents yields a lower distortion. However, the effectiveness of the learning behavior also depends on the clarity of the signal they have. A higher current account reaction to misalignments, $\alpha^{IT,2}$, diminishes the distortion. First, it induces a stronger permanent pressure on the exchange rate. Second, the exchange rate is driven back to its fundamental via self-confirmation. Third, the higher $\alpha^{IT,2}$, the more often the agents receive the true learning signal. The opposite occurs if the random component of the firms' demand, $\alpha^{IT,1}$, rises.

Finally, in our model the agents use daily trade transactions of the firms as the learning signal. The last part of the figure shows the result when the signal is given as an aggregate over some time. A time aggregation of 76 means that the agents only have access to firms' demand quarterly (the average over 76 trading days). Surprisingly, inferring macroeconomic imbalances from different time horizons do not seem to be that relevant. Although time aggregation makes the signal less noisy (since the random components cancel out), the signal arrives with a time lag. One observes a slightly higher distortion only for very long periods, say over 300 trading periods.

4. Conclusion

To sum up, the aim of this paper is threefold: to develop a model that gives rise to bubbles but also delivers realistic exchange rate movements, to explore how the agents perceive the fundamental exchange rate, and to unravel the forces behind the distortions. The trading activity of the agents is described with the aid of an empirical micro-foundation. The interaction between the traders generates complex dynamics which display unit roots in the exchange rates, fat tails for returns and volatility clustering.

Fundamental to this paper is the perception of the fundamental exchange rate. Psychologists claim that the agents behave according to the anchor and adjustment heuristic. Since the adjustment is typically incomplete and mistakes are propagated over time, the perceived value of the fundamental can strongly deviate from its true value even for longer periods. The degree of distortions depends on several forces. For instance, it increases with a higher systematic behavior of the chartists or decreases with a higher popularity of the fundamentalists.

This paper is built on empirical evidence. However, we understand this paper only as one step to achieving a better linking between economic modeling and psychological evidence. What is needed is a better understanding of the human cognition process. How do the agents perceive and process information, how do they select an action, and how do they learn? Recent experiments along the lines of Hommes et al. (1999) hopefully provide better and more detailed empirical grounds for future research.

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