Jump Risk Premia in Short-Term Spread Options: Evidence from the German Electricity Market

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Abstract

This paper analyzes the valuation of day-ahead Physical Transmission Rights (PTRs) on the German-Dutch interconnector. From a financial perspective, PTRs are options written on the difference between the German and Dutch hourly electricity prices. We propose a model for the valuation of day-ahead PTR options incorporating the unique characteristics of the underlying spread. We empirically test our model for all PTRs between 2001 and 2008, where we model each hour of the day separately. Overall, especially for calm hours, our approach constitutes an adequate model for the valuation of day-ahead PTR options. Empirical results show a negative or zero market price of jump risk for the turbulent hours 8 to 22. These results correspond to risk-loving or risk-neutral investors and indicate that market participants pay a premium for PTRs during peak hours. This premium is based either on increased hedging demand or on speculation.

JEL classification: C13; G13; Q40.
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1 Introduction

The liberalization of European electricity markets, starting with the adoption of the European Commission's Directive 96/92/EG in 1996, has led to an increase in international cross-border electricity flows. In 2007, 12.6% of all electricity consumption was delivered cross-border across all UCTE countries.[1] In the course of growing international perspectives in power markets, countries strive towards optimizing cross-border interconnections. In 2006, the Belgian, Dutch and French electricity exchanges started a market coupling in order to improve the coordination of their cross-border electricity flows. Just recently, at the end of 2008, Germany and Western Denmark also coupled their electricity markets with the aim of a more efficient usage of available transmission capacity. Along with an improved organization and increasing usage of cross-border electricity connections, prices in the European market continuously converge.[2]

In the course of internationalizing electricity markets and growing importance of cross-border electricity flows, the significance of an adequate risk management capable of adjusting to changing risk factors further increases. The two sources of risk inherent in cross-border electricity markets can be separated, analogous to national markets, into physical and financial risk. Physical risk comprises the risk associated with the proper fulfillment of delivery agreements. Financial risk is concerned with unexpected price variations that lead to substantial financial losses. While the financial risk is inherent in virtually all financial markets, albeit more severe in electricity markets due to higher price variations, the physical risk is unique as electricity exhibits special characteristics compared to other commodities or assets. Two properties mainly describe electricity as a commodity. It is in general not storable at reasonable cost and electricity is grid bound.[3] The first attribute leads to the unique spiky trajectory of electricity prices, since the lack of storability reduces the chance to dampen shocks of supply and demand. This poses a great challenge to the management of financial risk in electricity markets. The latter characteristic increases the risk of physical settlement. Since electricity is transported through a power grid with only limited capacity, a congested cable could hamper delivery. This risk is amplified

[1] See http://www.ucte.org. The Union for the Co-ordination of Transmission of Electricity (UCTE) is the association of continental European transmission system operators (TSOs). UCTE currently comprises 29 TSOs from 24 different countries.

[2] Armstrong and Galli (2005) as well as Zachmann (2008) show an increasing convergence of day-ahead electricity prices in Continental Europe between 2002-2006.

[3] In Europe, Norway with a share of hydropower over 90% might state an exception to the non-storability of electricity. See Von der Fehr et al. (2005) for a discussion of the relevance of hydropower in the Nordic market.

by another property of electricity. According to Kirchhoff's law, electricity spreads across the entire grid as soon as it is injected at any point in the network. In other words, a point to point delivery from the origin to the destination is not feasible. Thus, even a congested cable not originally intended to carry electricity could make timely delivery unfeasible.[4] As congested electricity lines hamper the flow of electricity, diverging electricity prices in neighboring areas result, creating an additional financial risk in cross-border electricity markets.

In order to manage the physical and financial risk in European cross-border electricity markets, Physical Transmission-Rights (PTRs) have been introduced.[5] PTRs are option contracts that allow access to cross-border transmission lines for a specific period of time. Therefore, PTRs offer the opportunity of buying electricity in a specific region A and selling it in another region B. From a financial standpoint, the payoff of this transaction is equal to the difference between both electricity prices. For this reason, PTRs can be seen as exchange options where the electricity price in region B is exchanged for the electricity price in region A. Thus, the payoff of a PTR can be stated as

$$PTR = \max\left[B - A; 0\right]. \tag{1}$$

Margrabe (1978) was the first to discuss the valuation of exchange options. He develops a closed-from solution, based on the Black/Scholes formula, in case both asset prices follow Itô processes. Marckhoff (2009) extends the Margrabe model and proposes, based on Merton (1976), a quasi closed-form solution for exchange options in case of log-normal jumps in both asset prices. Li (2008) shows a more general extension to Margrabe (1978). She uses a multivariate Gram-Chalier approximation to value exchange options in case of non-normal asset distributions. Another approach follow Cherubini and Luciano (2002). They use copula functions to value various options, including exchange options in closed form. Dempster et al. (2008) show, however that in case of co-integrated price processes the spread can be modelled directly, instead of modelling each asset separately. Since electricity prices in neighboring regions are usually co-integrated, modelling the spread directly is a suitable approach for electricity exchange options. Thus, the valuation of electricity exchange options reduces to modelling a single price process, i.e. the spread, and pricing the

[4] A congested cable between Germany and France could, for instance, hamper electricity flows from Germany to the Netherlands even though the cable between the latter two countries is not congested.

[5] In addition, Financial Transmission Rights (FTRs) as well as Contracts for Difference (CfDs) have been introduced in Europe to hedge pure financial risks. See Kristiansen (2004) for an overview of various risk management products in cross-border electricity markets and Marckhoff and Wimschulte (2009) for a detailed discussion of CfDs.

option written on this underlying.

One of the first papers addressing the valuation of electricity derivatives was Lucia and Schwartz (2002). They derive formulas for the valuation of electricity forwards based on one and two factor mean-reversion models. An extension to pure diffusion models propose, among others, Cartea and Figueroa (2005) who include log-normal jumps in their model in order to account for erratic variations observed in electricity prices. However, jumps in electricity prices generally last only a few days. Therefore, in order to mimic the characteristic trajectory of electricity prices, the resulting mean-reversion speeds become unrealistically high for those models. To circumvent this problem, Geman and Roncoroni (2006) propose signed jumps to model negative jumps immediately following positive ones. Recently, Huisman and Mahieu (2003) and Bierbrauer et al. (2007) propose regime switching models in order to match the specific characteristics of electricity prices.

Although modelling the spread between electricity prices directly allows us to refer to standard models for pricing electricity derivatives, the unique characteristics of spread processes need to be incorporated. While Seifert and Uhrig-Homburg (2007) show that spikes in the German electricity market generally last two to three days until prices revert back to their long term mean-reversion level, the duration of spikes in the spread between electricity prices is usually remarkably shorter. Jumps in the day-ahead spread between Dutch and German electricity prices, for example, do not last longer than one day. In order to match these specific spread characteristics, we develop a model based on Simonsen et al. (2004) that produces these pronounced spikes. We separate the diffusion part from observed prices and model the spikes as a normally distributed component that is occasionally added to the underlying diffusion process.[6] The jump times are Bernoulli distributed in order to model that jumps occur instantly and then disappear immediately. The separation of the underlying spread into a diffusion and a spike component further allows us to include only de-spiked prices into the valuation of derivatives. Since spikes only last one day and their occurrence has absolutely no impact on the overall price level, using observed spreads for the valuation of derivatives could lead to large distortions.

In this paper, we use the aforementioned model for the valuation of hourly day-ahead PTR options on the German-Dutch interconnector. We include all hourly PTRs between January 1, 2001 and December 31, 2008, i.e. 2,922 observations for each hour of the day. Our dataset further includes all corresponding hourly day-ahead electric-

[6] Throughout this paper we refer to the diffusion component as the de-spiked process.

ity prices in Germany and the Netherlands. As PTRs and electricity is auctioned explicitly for each hour of the day and each hour has its unique characteristics, we analyze all 24 hours separately. In order to estimate the empirical (or physical) and risk-neutral parameters of our model, we use the method of Markov Chain Monte Carlo (MCMC). MCMC is widely used for estimating equity models and is applied by Eraker et al. (2003) as well as Eraker (2004) and more recently by Rodrigues and Schlag (2009). This efficient and robust estimation procedure does not only allow for the simultaneous estimation of all parameters, but also the estimation of explicit vectors for jump times and jump sizes. Using the latter, we can easily separate the de-spiked price process from observed market prices.

This paper contributes to the current research in various ways. First, to the best of our knowledge, we are the first to comprehensively analyze day-ahead PTR options that are currently the most widely used instruments for managing cross-border electricity flows in the German electricity market.[7] Second, we develop a model capable of incorporating the unique features of hourly electricity spreads. Finally, we estimate our model and analyze the market price of jump risk inherent in day-ahead PTR options. Due to their extremely short time to maturity, jump risk can be considered as the main driver of these contracts. Empirical evidence indicates that our model describes an adequate approach for the valuation of hourly PTR options especially during calm hours. Further, our results show that investors are willing to pay a premium for hourly PTR options for turbulent hours of the day, i.e. hours 8 to 22. This price premium can be explained by increased hedging demand of investors and emphasizes the importance of these contracts and the need for adequate risk managing tools in cross-border electricity markets.

In the course of this paper, we first discuss the national and cross-border electricity markets of Germany and the Netherlands. We then introduce our model for the valuation of PTR options and explain the MCMC estimation of the empirical and risk-neutral parameters. Finally, we discuss the results of our analysis before we shortly sum up our findings.

[7] With the exception of the German and western Denmark interconnector, all German cross-border connections currently use explicit auctions of PTRs to allocate day-ahead cross-border capacity.

2 German-Dutch Electricity Market

2.1 National Electricity Markets

The German and Dutch exchange based electricity markets are very alike. In Germany, electricity is traded on the European Energy Exchange (EEX). The spot market mainly consists of a day-ahead market where every working day, electricity is auctioned for each of the 24 hours of the next day (or days in case of holidays or weekends). The overall trading volume in 2007 in the spot market at EEX was 124 TWh, compared to a total electricity consumption in Germany of 556 TWh. In the Netherlands, electricity is traded on the Amsterdam Power Exchange (APX). Here, the spot market is also a day-ahead market where every day electricity is auctioned analogous to the EEX.[8] The spot market trading volume at APX in 2007 was about 21 TWh compared to a total consumption in the Netherlands of 117 TWh.[9]

Although the market set up in both countries is comparable, their physical electricity markets demonstrate fundamental differences in terms of their power generation mix. These discrepancies are extremely relevant as they economically determine the overall electricity price level and therefore the sign and level of the price spread between these countries. The Netherlands, as one of the largest natural gas producers in the world, generates a major share of electricity from natural gas power plants.[10] Other fuels play only a minor role in the generation of electricity. In Germany, coal fired and nuclear power plants are the most important source of electricity generation, while natural gas and renewables also contribute significantly to the power generation mix. Table 1 provides details on the gross power generation in both countries in 2007.

[Insert Table 1 here]

The power generation mix shown in Table 1 has a significant effect on the spread between German and Dutch electricity prices. Power plants are generally stacked according to their marginal cost of power generation, where the power plant with the lowest marginal cost is used first.[11] In Germany, baseload demand is generally covered

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[8] At EEX and APX, there also exists an intraday market since September 2006, where electricity is traded continuously for certain time blocks.

[9] Exchange based information is available at http://www.eex.de and http://www.apxgroup.com respectively. Consumption figures are obtained from http://www.ucte.org.

[10] In 2007, the Netherlands produced about 76.3 billion cubic meters of natural gas which corresponds to almost 39% of the European Union's natural gas production. See http://www.cia.gov for details.

[11] This stacking is called *merit order* and is also applicable within a power plant concerning different generators. In addition to marginal costs, other factors, e.g. lead time and minimum usage time, need to be considered.

by renewables (without pumped storages), lignite and nuclear power. Natural gas and hard coal power plants are added as demand increases. Peakload is usually covered by gas and oil fired plants or pumped storage facilities. The Netherlands cannot rely on relatively cheap nuclear power or lignite and need to refer to more expensive hard coal and natural gas power plants for base and medium load. Therefore, the usage of efficient powerplants to cover peak demand in the Netherlands is limited and Dutch day-ahead electricity prices are in general higher and more erratic than prices in Germany.

2.2 German-Dutch Cross-Border Market

The German and Dutch power grids are currently connected via three high voltage (380kV) cross-border cables. In Germany, the two southern cables are operated by the transmission system operator (TSO) *RWE TSO* and the northern one by *E.ON Netz*. In the Netherlands, *TenneT* is the sole TSO operating all interconnectors. The capacity of the cross-border connections is auctioned explicitly by *Auction BV*, a 100% subsidiary of *Tennet*. The capacity is auctioned for each direction separately via PTR options.[12]

There are three different types of PTR options auctioned for the German-Dutch interconnector. They differ in the length of the delivery period and are hourly, monthly and yearly PTRs. All PTRs have a volume of 1MW. Hourly PTRs are auctioned day-ahead for every single hour of the following day. Monthly PTRs are auctioned on the 10th working day of the month preceding the delivery month. Yearly PTRs are auctioned on the first working day after September 27th in the year preceding the year of delivery. In case of remaining capacity from the first auction of yearly PTRs, a second auction is held on the first working day after the 27th of November. In general, about one third of the entire available capacity is reserved for each of the three contract types. Investors bid price/volume combinations were several bids per investor are allowed. In case the requested volume is lower than the available capacity, the PTR price is zero. Otherwise, all investors pay the price of the lowest successful bid. If there is more volume requested at the lowest successful bid, the allocation for those bids is partitioned relative to the requested volume.

Although monthly and yearly PTRs are auctioned separately, they actually consist of a portfolio of hourly PTRs for each hour of the respective delivery period. Therefore,

[12] The connections from *RWE TSO* and *E.ON Netz* are auctioned separately. Since the capacity of the *RWE TSO* cables is by far larger, we only refer to these interconnectors and the respective PTRs throughout this paper.

the owner of such a portfolio has the right to exercise each hourly PTR separately. In case of monthly or yearly contracts, investors need to nominate the PTR for each hour three days prior to execution, i.e. they must state whether they use their PTR or not. Should investors nominate the usage of their PTR, they are obliged to induce the corresponding amount of electricity into the grid. In case PTRs are returned, the capacity is available for the hourly auction and the owner is refunded with the proceeds of the auction. If an investor does not nominate the PTR at all, the capacity is returned to the hourly auction and the owner is not refunded. This approach is called the *use it or lose it principle*. Concerning the fulfillment of the PTRs, these contracts are offered firm, i.e. the TSO has no right to curtail the granted capacity of a PTR holder. However, there are two exceptions to the rule. In case of power system safety requirements or force majeure, the TSO might revoke the right of inducing electricity into the grid. In the first case, the TSO is required to compensate for the losses and has to pay the holder of the PTR 110% of the initially paid price. In case of force majeure, the TSO only refunds 100% of the paid PTR price. Out of the 2,922 days between 2001 and 2008, only for six days there was no PTR price available.

As all three types of PTR options basically refer to hourly contracts, the underlying of the PTR is the hourly spread between German and Dutch day-ahead electricity prices. Figure 1 shows this spread between 2001 and 2008.

[Insert Figure 1 here]

Hourly spreads between Germany and the Netherlands vary significantly across different hours of the day. While spreads usually fluctuate mildly with occasional jumps during off peak hours, spreads are highly erratic during peak hours. Since the merit order leads to a concave marginal cost function, price spikes are more likely when general demand is already high as for the peak hours during the day. Furthermore, price spreads and spikes are mostly positive, i.e. Dutch prices are usually higher than German ones. This is in line with the power generation mix of the Netherlands compared to Germany discussed above. Table 2 provides detailed descriptive statistics for the German and Dutch hourly day-ahead spread, where a positive spread refers to higher prices in the Netherlands and vice versa.

[Insert Table 2 here]

Figures in Table 2 confirm the erratic behavior of the electricity price spread and shows a large dispersion of the descriptive statistics across different hours. The minimum and maximum values indicate the relevance and idiosyncratic occurrence of jumps in national electricity prices. Also, the skewness and kurtosis shows significant values for all hours, although during peak hours these figures are even more severe. The varying characteristics of electricity price spreads across the day motivates the distinct modelling of each hour. Further, the mostly positive results for the mean and skewness in addition to the larger maximum compared to the minimum values (in absolute terms) confirm the generally higher and more erratic electricity prices in the Netherlands.

As stated in equation 1, the PTR is an option on the spread between German and Dutch hourly day-ahead price. Since hourly PTRs are auctioned with only one day to maturity, we expect their prices to closely reflect the price spreads in Table 2. The descriptive statistics of hourly PTRs as well as the average volume auctioned (in MW) are shown in Table 3.

[**Insert Table 3 here**]

Table 3 shows that in general PTR prices reflect the underlying price spread. For peak hours, mean PTR prices as well as their standard deviations are higher than for off-peak hours. Further, skewness and kurtosis are extremely high for all hours but the greatest values are observed for off-peak hours. Traded volumes indicate relatively large amounts of available capacity, considering that these values are given per MWh and that hourly PTRs only constitute about one third of overall capacity auctioned. Further, the figures show lower capacities for peak hours compared to off-peak hours. One reason might be the lower levels of returned capacity from year and month auctions, i.e. more PTR holders exercise their long term options rather than selling it in the hourly auction. Another reason could be the delivery of less electricity from the Netherlands to Germany, compared to off-peak hours, which also leads to lower capacities for delivery from Germany to the Netherlands.[13]

Despite overall similarities, there are significant differences between PTR prices and electricity price spreads, especially when considering the extremely short time to maturity of each PTR. The reason is that price spikes are very hard to predict even with only one day to maturity. Positive (negative) price spikes occur in case the Dutch day-ahead electricity price jumps up (down) compared to the German one or vice versa. Jumps in national electricity prices are idiosyncratic and occur in case of a sudden drop in supply or an unpredicted increase in demand, where the first is usually more common. Figure 2 shows the relation between traded PTR prices and the resulting payoffs, i.e. the maximum of the underlying spread and zero, between

[13] Since opposed currents cancel out, scheduled electricity flows from the Netherlands to Germany increase the available capacity for the opposite direction.

2001 and 2008. A negative value indicates PTR prices above the corresponding payoff. It is evident that there is a large dispersion between paid PTR prices and resulting payoffs. PTR prices considerably above their payoff as well as payoffs significantly in excess of PTR prices are both frequently observed across all hours. This confirms the unpredictability of price spikes.

[Insert Figure 2 here]

In addition to unforeseeable price spikes, the PTR auction process further induces a great amount of uncertainty to investors. As PTRs are physical contracts that securitize the right to deliver electricity via an interconnector, the exercise of these options requires a specific period of lead time.[14] In order to profit from a purchased PTR option, assuming that no prior delivery agreement exists, the investor needs to buy electricity in the German day-ahead market and sell the same amount in the Dutch day-ahead auction. However, while the PTRs are auctioned at 9am the day before maturity, the auctions for German and Dutch day-ahead electricity close at noon and 11am respectively. Auction results are submitted 15 minutes later for German day-ahead electricity and 30 minutes later otherwise. Since there is no obligation to use the purchased PTR, the holder can announce its usage until 2pm. In case the investor bought electricity in the Dutch or German day-ahead auction and the price spread does not allow for a profitable usage of the PTR option, positions need to be closed via the intraday market in order to prevent losses. Since the price at which the positions are closed is also unpredictable, this schedule of auctioning the PTR options offers a great amount of uncertainty to investors.

3 Model

3.1 Underlying Processes

In order to model the hourly spread between German and Dutch electricity prices, we decompose the price spread at time t, P_t, into a diffusion component S_t and a jump component J_t. Further, we can neglect any seasonal component that is common in electricity price models, since the price spread is not subject to any seasonal trend. The price spread P_t can be written as

$$P_t = S_t + J_t. \tag{2}$$

[14] Lead time is required since an instantaneous delivery of electricity is hardly feasable. Further, demand and supply have to be balanced at all time so that the TSO needs to schedule any changes in the supply and demand.

S_t follows an Ornstein-Uhlenbeck process since the price spread is subject to mild variations in the short term but reverts back to the mean-reversion level in the long run. J_t mirrors the fact that the price spread is subject to occasional jumps, which in general only last for one day. Formally, we have

$$\begin{aligned} dS_t &= \kappa\left(\nu - S_t\right)dt + \sigma_D dW_t, \\ J_t &= \begin{cases} N_t, & \text{with probability } p_J \\ 0, & \text{with probability } (1-p_J) \end{cases}, \end{aligned} \tag{3}$$

where $N_t \sim \mathcal{N}\left(\mu_J, \sigma_J^2\right)$. Using this set up, we are able to model price spikes in the underlying process, i.e. extreme jumps that only last for one day. Further, since we separate the jump and diffusion component, the mean-reversion speed κ is not biased by jumps in the underlying and we therefore receive more realistic values when estimating κ. In addition, we are able to use the current de-spiked value S_t when calculating the PTR option price. This is reasonable as the occurrence of a spike should have no influence on the PTR price. Following Mikosch (1999) and Cont and Tankov (2003) we get for the process of P_t

$$P_t = S_0 e^{-\kappa t} + \nu\left(1 - e^{-\kappa t}\right) + \sigma_D e^{-\kappa t}\int_0^t e^{\kappa s} dW_s + J_t. \tag{4}$$

The mean and variance of P_t are

$$\begin{aligned} E\left[P_t\right] &= S_0 e^{-\kappa t} + \nu\left(1 - e^{-\kappa t}\right) + p_J\mu_J, \\ Var\left[P_t\right] &= \left(1 - e^{-2\kappa t}\right)\frac{\sigma_D^2}{2\kappa} + \varsigma_J, \end{aligned} \tag{5}$$

where $\varsigma_J = \sigma_D^2 p_J + \mu_J^2 p_J\left(1 - p_J\right)$ is the variance of the jump component J_t. In order to obtain the SDE under the risk-neutral measure $\mathcal{Q}$, we use the Girsanov transform.[15] In our model, we only consider the effect of the measure change on the jump component. Due to the short time to maturity of only one day, we neglect the impact of the market price of diffusion risk. Therefore, we receive more pronounced results for our estimation of the jump risk premia and all diffusion parameters remain constant, i.e. $\kappa_J^Q = \kappa_J$, $\nu^Q = \nu$ and $\sigma_J^Q = \sigma_J$.[16] Under the risk-neutral measure the jump intensity as well as the mean jump size change whereas the variance of the jump size remains constant, i.e. $\sigma_J^Q = \sigma_J$. Thus, we can characterize S_t and J_t under

[15] See Benth et al. (2008) for a detailed discussion of the measure change for jump processes.

[16] We use the super index $\mathcal{Q}$ to indicate the risk neutral measure. For the empirical measure $\mathcal{P}$, we omit the super index.

the risk-neutral measure as

$$dS_t = \kappa (\nu - S_t) dt + \sigma_D dW_t^Q,$$
$$J_t^Q = \begin{cases} N_t^Q, & \text{with probability } p_J^Q \\ 0, & \text{with probability } \left(1 - p_J^Q\right) \end{cases}, \tag{6}$$

Under the risk-neutral measure, the jump probability is p_J^Q and $N^Q \sim \mathcal{N}\left(\mu^Q, \sigma_J^2\right)$. The corresponding process for P_t follows as

$$P_t = S_0 e^{-\kappa t} + \nu \left(1 - e^{-\kappa t}\right) + \sigma_D e^{-\kappa t} \int_0^t e^{\kappa s} dW_s^Q + J_t^Q. \tag{7}$$

Based on the risk-neutral information of the jump component, we can calculate the market price of jump risk θ_J. Considering the distributional characteristics of the jump, i.e. a Bernoulli distributed jump time and a normally distributed jump size, the market price of jump risk is defined as

$$\theta_J = \frac{\mu^Q - \mu}{\sigma_J^2}. \tag{8}$$

Thus, the market price of jump risk is the change in mean jump size per unit of jump size variance. It is negative for a risk-averse investor and positive for a risk-loving investor. Thus, a risk-averse investor requires a discount in order to invest in PTR options since the investment is risky. A risk-loving investor on the other hand is willing to pay a premium for buying the PTR. A risk-neutral investor does not price any risk and therefore the empirical and risk-neutral mean jump sizes are identical, i.e. $\theta_J = 0$.

3.2 Derivation of Call-Price

We derive the price of a PTR option based on the approach of Merton (1976). When conditioning the spread on the occurrence of a jump at time t, the spread is normally distributed. Given a conditional normal distribution, the option price can be easily derived as the discounted expected value under the risk-neutral measure. Based on the risk-neutral process for P_t from equation 7, we can write the price of a PTR option as

$$PTR = e^{-rt} \sum_{n=0}^{1} \Pr(n\ jumps)\, E^Q \left[(P_t)^+ \middle| \, n\ jumps \right]. \tag{9}$$

Due to the set up of our model, the number of jumps until maturity is of no relevance for the valuation of the PTR option. This is based on the idea that spikes only last one day and vanish without any influence on the general price level. The only relevant jump time for the valuation of a PTR option is at maturity. Therefore, we only need to distinguish two scenarios. Either there is a jump in the underlying spread at maturity or there is no jump. Considering the probability of a jump under the risk-neutral measure, i.e. p_J^Q, we can write the value of the PTR option at time zero and maturity at time t as

$$\begin{aligned} PTR = \sum_{n=0}^{1} \left(p_J^Q\right)^n \left(1 - p_J^Q\right)^{1-n} \Big\{ \Big[& S_0 e^{-\kappa t} + \nu \left(1 - e^{-\kappa t}\right) + n\mu_J^Q \Big] \Phi(d) \\ & + \sqrt{\frac{\sigma_D^2}{2\kappa}\left(1 - e^{-2\kappa t}\right) + n\sigma_J^2} \frac{e^{-\frac{1}{2}d^2}}{\sqrt{2\pi}} \Big\} e^{-rt}, \end{aligned} \tag{10}$$

where $\Phi(\cdot)$ is the cumulative normal distribution function and d is defined as

$$d = \frac{S_0 e^{-\kappa t} + \nu \left(1 - e^{-\kappa t}\right) + n\mu^Q}{\sqrt{\frac{\sigma_D^2}{2\kappa}\left(1 - e^{-2\kappa t}\right) + n\sigma_J^2}}.$$

Equation 10 shows that if a jump occurs, the mean and variance of the spread are adjusted by the mean and variance of the jump component. In case of no jump, only the diffusion mean and variance enter the expected spot price. Since only two scenarios need to be considered, i.e. jump and no jump, the formula is kept easily tractable.

4 Parameter Estimation

4.1 Discrete Process

In order to estimate the empirical and risk-neutral parameters, we first need to discretize the process P_t assuming that $\Delta t = 1$. Special interest in the course of the discretization receives the jump component J_t. We decompose the jump component into the product of a jump time variable q_t and a jump size variable N_t. While q_t is Bernoulli distributed to indicate whether a jump occurs at time t, N_t is normally distributed to determine the jump size. This setup allows us to model positive and negative jumps in the underlying spread.

Given the process for P_t under the empirical measure $\mathcal{P}$ from equation 4, we can write the discrete process of P_t as

$$P_{t+1} \left| S_t \right. = S_t e^{-\kappa} + \nu \left(1 - e^{-\kappa t}\right) + \sqrt{1 - e^{-2\kappa}} \varepsilon_{t+1} + N_{t+1} q_{t+1}, \tag{11}$$

where $\varepsilon_{t+1} \sim \mathcal{N}\left(0, \frac{\sigma_D^2}{2\kappa}\right)$. For simplicity we assume $P_0 = S_0 = 0$. Since a spike today has virtually no impact on tomorrow's price, P_t is conditioned on today's de-spiked price S_t. Given the discrete process of P_t, we can formulate the full information likelihood of $P = \{P_t\}_{t=1}^{T}$ as

$$p\left(P\,|\Theta, q, N\right) = \prod_{t=0}^{T-1} p\left(P_{t+1}\,|S_t, q_{t+1}, N_{t+1}, \Theta\right), \tag{12}$$

where

$$p\left(P_{t+1}\,|S_t, q_{t+1}, N_{t+1}, \Theta\right) \propto \exp\left(-\frac{1}{2}\frac{\left(P_{t+1} - S_t e^{-\kappa} - \nu\left(1 - e^{-\kappa t}\right) - N_{t+1}q_{t+1}\right)^2}{\left(1 - e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}\right).$$

The full information likelihood is an essential part in the parameter estimation. The likelihood for the vector of observed price spreads P is simply the product of the likelihood functions of all of its elements P_t, for all $t = 1, ..., T$. Normality of P_t is guaranteed since it is conditioned on the occurrence of a jump. The risk-neutral discrete process of P_t can be derived identically as

$$P_{t+1}\,|S_t = S_t e^{-\kappa t} + \nu\left(1 - e^{-\kappa t}\right) + \sqrt{1 - e^{-2\kappa}}\varepsilon_{t+1}^{Q} + N_{t+1}^{Q} q_{t+1}^{Q}. \tag{13}$$

The full information likelihood for the risk-neutral process of P_t follows analogous to equation 12.

4.2 Empirical Parameters

In order to estimate the parameters of the discretized process, we use the Markov Chain Monte Carlo (MCMC) method. Via MCMC, we generate random samples from the joint posterior distribution, $p\left(\Theta, X\,|P\right)$, of parameters Θ and latent state variables X. The set of parameters includes all relevant parameters to be estimated, i.e. $\Theta = \{\kappa, \sigma_D^2, \nu, \mu_J, \sigma_J^2, p_J\}$, whereas the latent state variables include jump times and jump sizes, i.e. $X = \{q, N\}$. Since the joint posterior distribution is in general not known, we apply the Clifford-Hemmersley theorem that allows us to draw from the complete conditional distributions, i.e. $p\left(\Theta\,|P, X\right)$ and $p\left(X\,|P, \Theta\right)$, instead. If drawing from the complete conditional distribution is still not feasible, the Clifford-Hemmersley theorem can be reapplied until each parameter, conditional on observed prices, latent state variables and all other parameters, is drawn separately. The same holds for the conditional distribution of the state variables. Drawing all parameters iteratively, we receive a Markov Chain that eventually converges to the target posterior distribution. We finally receive the estimated parameter values as the arithmetic mean of all non discarded Monte Carlo draws. We discard the first drawings of our estimation to let the Markov chain come close to its stationary distribution and therefore, to receive more robust results.[17]

MCMC is based essentially on the theory of Bayes. Using Bayes rule, we are able to state the posterior distribution as the factor of the likelihood (see equation 12), the distribution of the state variables, $p(X|\Theta)$, and the prior distribution of the parameter, $p(\Theta)$. As we only require a proportionality relation, we can express the posterior distribution as

$$p(\Theta, X|P) \propto p(P|\Theta, X)\, p(X|\Theta)\, p(\Theta), \tag{14}$$

In case the above stated product of distributions can directly be drawn from, we can use the so called *Gibbs* algorithm in order to draw a new sample. By choosing appropriate prior distributions for the parameters, we mainly refer to Gibbs sampling in this paper. If the posterior distribution cannot directly be sampled from, we use a Metropolis-Hastings step. Here, a new sample is drawn from a proposed distribution. This sample candidate is then accepted as a drawing from the posterior distribution according to a given acceptance criterion. In case of the Metropolis-Hastings algorithm, the conditional posterior only needs to be evaluated numerically. We use the Metropolis-Hastings method when drawing samples for the mean-reversion speed κ.

For our estimation, we start with the jump times q_t. In each iteration step, we generate a vector $q \in \{q_1, ..., q_T\}$ with length equal to the number of observed market prices. Each element is Bernoulli distributed indicating if a jump occurred at time t or not, i.e. $q_i \in \{0, 1\}$, for all $i = 1, ..., T$. The conditional probability of a jump in the next time step, i.e. $q_{t+1} = 1$, has the following distribution[18]

$$q_{t+1}|\,\Theta, P_{t+1}, S_t, N_{t+1} \sim \mathcal{B}ernoulli\left(\varphi_{t+1}\right). \tag{15}$$

φ_{t+1} is the Bernoulli probability of a jump in the next time step. With the vector of jump times, we draw a new jump probability p_J via Gibbs. We assume that p_J has a beta prior distribution, i.e. $p_J \sim \mathcal{B}eta\left(\alpha_{p_J}, \beta_{p_J}\right)$. A beta prior ensures that the jump probability is bound between zero and one. Further, the beta distribution is

[17] See Gamerman and Lopes (2006) for a textbook treatment of MCMC methods and Bayesian theory. Johannes and Polson (2003) thouroughly discuss MCMC methods and give various examples for financial models.

[18] We refer to the appendix for details on the posterior distributions.

conjugate to the Bernoulli likelihood.[19] The posterior distribution of p_J then follows as

$$p\left(p_J\left|q\right.\right) \propto p\left(q\left|p_J\right.\right) p\left(p_J\right). \tag{16}$$

In case we omit indices of observed prices, jump times or jump sizes, we refer to the entire vector. Indices, in contrast, refer to a specific element of the respective vector. The latter is used when drawing the vector of jump times and jump sizes since each element is drawn individually. The first is applied for the estimation of the parameters.

When drawing the vector of jump sizes we proceed analogous to jump times such that we need to generate a vector of jump sizes $N \in \{N_1, ..., N_T\}$, where each element is normally distributed with mean μ_J, and variance σ_J^2. The posterior distribution of each element can therefore be stated as

$$p\left(N_{t+1}\middle|\,\Theta, q_{t+1}, P_{t+1}, S_t\right) \propto p\left(P_{t+1}\middle|\,S_t, N_{t+1}, q_{t+1}, \Theta\right) p\left(N_{t+1}\left|\Theta\right.\right). \tag{17}$$

After having drawn jump times, jump sizes and the jump probability, we continue by successively drawing all remaining parameters. The mean jump size μ_J has the following posterior distribution, assuming a normal prior, i.e. $\mu_J \sim \mathcal{N}\left(m_J, s_J^2\right)$

$$p\left(\mu_J\middle|\,N, q, \Theta_{-\mu_J}\right) \propto p\left(N\middle|\,\Theta\right) p\left(\mu_J\right). \tag{18}$$

$\Theta_{-\mu_J}$ refers to the vector of parameters without μ_J. For the variance of the jump size σ_J^2 we assume an inverse gamma prior distribution, i.e. $\sigma_J^2 \sim \mathcal{IG}\left(\alpha_{\sigma_J}, \beta_{\sigma_J}\right)$, to insure positivity of the variance. Thus, the posterior follows as

$$p\left(\sigma_J^2\middle|\,N, q, \Theta_{-\sigma_J^2}\right) \propto p\left(N\middle|\,\Theta\right) p\left(\sigma_J^2\right). \tag{19}$$

Afterwards, we draw a sample for the mean-reversion speed κ as well as the variance of the diffusion process σ_D^2. For κ we assume a normal prior, i.e. $\kappa \sim \mathcal{N}\left(m_\kappa, s_\kappa^2\right)$ and therefore the posterior of κ is

$$p\left(\kappa\middle|\,N, q, \Theta_{-\kappa}, P, S\right) \propto p\left(P\middle|\,S, N, q, \Theta\right) p\left(\kappa\right), \tag{20}$$

where $S = \{S_t\}_{t=1}^T$. Since a direct draw from the above mentioned product of the likelihood and the prior of κ is not feasible, we use the Metropolis-Hastings approach

[19] A prior distribution is called conjugate to a likelihood function if the resulting posterior distribution is from the same family as the prior, i.e. the product of Bernoulli likelihood and a beta prior is also beta distributed with altered parameters. With the exception of the mean reversion speed, all prior distributions in this paper are conjugate.

to sample κ. Here, we only need to evaluate $\pi(\kappa_{g+1})/\pi(\kappa_g)$, where κ_g is the g^{th} sample drawing of κ and $\pi(\kappa_g)$ is the posterior distribution of κ_g. Using the random walk Metropolis-Hastings algorithm, we draw a proposed κ_{g+1} as $\kappa_{g+1} = \kappa_g + \varepsilon_\kappa$, where $\varepsilon_\kappa \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. Afterwards, we have to decide whether to accept κ_{g+1} as a potential sample drawing for the above stated posterior distribution (from which we could not draw directly). In case we do not accept κ_{g+1} we set $\kappa_{g+1} = \kappa_g$, where the probability α of the acceptance of κ_{g+1} is calculated as[20]

$$\alpha(\kappa_g, \kappa_{g+1}) = \min\left(\frac{\pi(\kappa_{g+1})\, q(\kappa_g|\, \kappa_{g+1})}{\pi(\kappa_g)\, q(\kappa_{g+1}|\, \kappa_g)}, 1\right).$$

Afterwards we need to draw a new σ_D^2 from its posterior distribution, where we assume, analog to the variance of the jump size, an inverse gamma prior distribution of σ_D^2, i.e. $\sigma_D^2 \sim \mathcal{IG}(\alpha_D, \beta_D)$ .Thus, the posterior of σ_D^2 is

$$p\left(\sigma_D^2\middle|\, N, q, \Theta_{-\sigma_D^2}, P\right) \propto p(P|\, S, N, q, \Theta)\, p\left(\sigma_D^2\right). \tag{21}$$

Finally, we need to draw a new sample for the mean-reversion level ν. Assuming that $\nu \sim \mathcal{N}(m_\nu, s_\nu^2)$, the conditional posterior follows as

$$p(\nu|\, N, q, \Theta_{-\nu}, P) \propto p(P|\, S, N, q, \Theta)\, p(\nu). \tag{22}$$

Repeating the drawing of the above mentioned state variables and parameters, the distribution of the resulting Markov Chains will eventually converge to their target posterior distributions. In order to find starting values for the estimation procedure, we randomly draw parameter values from their prior distributions. The starting vector of jump sizes is constructed from observed market data. We discard the first 10,000 iterations and use additional 10,000 drawings as a basis for our parameter estimation. Appropriate choices of prior distributions and their parameters (called hyperparameters) improve fast convergence and are generally used to induce exogenous information into the estimation procedure. However, in our estimation, the results of the parameter estimates are quite insensitive towards changes in hyperparameters.[21]

Besides receiving point estimates for parameter values, MCMC additionally allows

[20] $q(\kappa_{g+1}|\, \kappa_g)$ is the proposal density of κ_{g+1}. In case of symetrical proposal densities, these cancel out simplifying the acceptance criterion to the fraction of posterior distributions.

[21] The mean jump size as well as the mean-reversion level are normally distributed with mean 0.0 and variance 2.25. The mean reversion speed is also normally distributed but with a mean of 0.3 and a variance of 0.01. The variances of the jump size and the diffusion variance are inverse-gamma distributed with an alpha of 10.0 and a beta of 2.0. The jump probability is beta distributed with an alpha and beta equal to 2.0.

us to separate the jump and the diffusion part of the underlying spread. Since we receive a vector of jump times and a vector of jump sizes in each iteration step, we can calculate the diffusion part of the underlying at time t, i.e. S_t, as the difference between the observed spread and the product of jump time and jump size at time t. After having finished the parameter estimation, we receive the final value for S_t as the arithmetic mean of non discarded iterations of S_t for all $t = 1, ..., T$. The calculation of the final jump times and jump sizes follows analogous to the vector S. Figure 3 shows the underlying spread, the de-spiked process as well as the jump times and jump sizes exemplarily for the eighth hour.

[Insert Figure 3 here]

4.3 Risk-neutral Parameters

In order to estimate the risk-neutral parameters, we need to include option prices into our estimation, since spot prices do not contain information about market prices of risk. For the observed PTR prices C, we assume

$$C_t = PTR_t \left(\Theta^Q, \psi_t\right) + \varepsilon_t^c, \tag{23}$$

where $\varepsilon_t^c \sim N\left(0, \sigma_c^2\right)$. $PTR_t\left(\Theta^Q, \psi_t\right)$ is the model price of C_t, given in equation 10, as a function of the parameters Θ^Q and additional factors ψ_t, i.e. the current value of the de-spiked price S_t, interest rate and time to maturity.[22]

Equation 23 states that PTR prices are observed with an error. This assumption is made in order to avoid a singularity problem. Thus, the observed option prices are normally distributed around their theoretical value, i.e. $C_t \sim \mathcal{N}\left(PTR_t\left(\Theta^Q, \psi_t\right), \sigma_c^2\right)$. Given the empirical parameters, we now only need to estimate those parameters that change when changing to the risk-neutral measure. The set of parameters therefore is $\Theta^Q = \left\{\kappa, \sigma_D^2, \nu, \mu_J^Q, \sigma_J^2, p_J^Q, \sigma_c^2\right\}$. Since we do not want to induce any information on how the risk-neutral parameters change when changing to the risk-neutral measure $\mathcal{Q}$, we use the same prior distributions and hyperparameters as before. The full information likelihood of C follows as

$$p\left(C \left| \Theta^Q, S, \sigma_c^2\right.\right) = \prod_{t=1}^{T} p\left(C_t \left| \Theta^Q, S_t, \sigma_c^2\right.\right). \tag{24}$$

We point out that the conditional distribution of C_t does not depend on the jump

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[22] Throughout this paper, we assume a risk free rate of interest of 2% p.a. As we only use PTR options with one day to maturity, the interest rate has virtually no effect on our results.

times and jump sizes. Therefore, PTR prices contain no information on jump times and jump sizes. This does of course not mean that jump times and jump sizes are identical across measures. But in order to generate the vectors q and N, we refer to the same posterior distributions as for the empirical estimation given in equations 15 and 17 and we refrain from stating them again.

The first parameter whose conditional distribution changes is the jump probability p_J^Q. Given the beta prior distribution stated above, the posterior for the jump probability follows as

$$p\left(p_J^Q\left|q^Q, C\right.\right) \propto p\left(q^Q\left|p_J^Q\right.\right) p\left(C_t\left|\Theta^Q, S_t, \sigma_c^2\right.\right) p\left(p_J^Q\right). \tag{25}$$

Since we cannot draw from this distribution directly, we apply Metropolis-Hastings algorithm with a $\mathcal{B}eta$ proposal density. For the mean jump size μ_J^Q the posterior is

$$p\left(\mu_J\left|N^Q, q^Q, \Theta^Q_{-\mu_J^Q}, C\right.\right) \propto p\left(N^Q\left|\Theta^Q\right.\right) p\left(C_t\left|\Theta^Q, S_t, \sigma_c^2\right.\right) p\left(\mu_J^Q\right). \tag{26}$$

As we can also not draw from this distribution directly, we use again a Metropolis-Hastings algorithm with a normal proposal density.[23] Finally, we need to draw the variance of the error term from the observed option prices where we assume, in line with before estimated variances, an inverse gamma prior, i.e. $\sigma_c^2 \sim \mathcal{IG}\left(\alpha_c, \beta_c\right)$, with $\alpha_c = 10$ and $\beta_c = 2$. The posterior distribution is

$$p\left(\sigma_c^2\right| N^Q, q^Q, \Theta^Q_{-\sigma_c^2}, C, P\left.\right) \propto p\left(C\right| P, S, N^Q, q^Q, \Theta^Q\left.\right) p\left(\sigma_c^2\right), \tag{27}$$

from which we can sample directly. Thus, we apply the Gibbs algorithm. As the rest of the parameters is identical across measures, we refrain form drawing them again and use their values estimated before.

5 Empirical Results

In Table 4, the estimation results for the empirical as well as risk-neutral parameters are given for all 24 hours. As mentioned above, the presented values are calculated as the arithmetic mean of the 10,000 non discarded Monte Carlo iterations. Further, below each parameter, the standard error of the estimation is given. The low standard errors in relation to the parameter values confirm a fast convergence of our MCMC

[23] For the proposal density of the jump probability we use the parameters $\alpha = 2, \beta = 2$. The normal proposal density for the mean jump size has mean 0 and variance 100. Both proposal densities are symmetric wich simplifies the acceptance criteria in the Metropolis-Hastings step.

algorithm.

[Insert Table 4 here]

For the empirical parameters, our initial expectations for the underlying spread are confirmed by the parameter estimates. First of all, figures in Table 4 show a relatively large mean-reversion speed κ compared to values observed in national electricity markets. This confirms the observed oscillatory behavior of the diffusion process as presented in Figure 3.[24] Second, the estimation results of the remaining parameters support the time-series properties of the spread. The values of the mean-reversion levels are in line with the median price spreads in Table 2 in terms of sign as well as level. While hours 4 to 8 have negative mean-reversion levels, those for the other hours are all positive and generally higher for peak hours. The mean jump sizes and jump probabilities show no clear pattern across different hours. However, the variance of the jump sizes vary significantly during the day. For morning hours and late at night, the variance of the jump sizes are rather moderate. During peak hours, in contrast, jump variances increase extremely reaching their peak for hour 18 with a variance of over 46,000. Comparing the figures with the time-series results in Table 2 as well as the trajectories in Figure 1, these values are in line with intuition. With minimum and maximum price ranges of over 3,500 and kurtoses of up to 600, these jump size variances are also in line with observed prices. Overall, the empirical parameter estimates in Table 4 confirm the erratic and extremely spiky behavior of daily price spreads from Table 2 and Figure 1 and supports our approach of modelling each hour separately.

The result for the risk-neutral parameters is dichotomous. For hours early in the morning or late at night, the price of jump risk is negative. These are hours 1 to 7 as well as 23 and 24. For the other hours, i.e. 8 to 22, the price of jump risk is either positive or zero. This distinction coincides with the volatility of the respective hourly spreads, especially the jump size variances. While during relatively calm hours market participants are risk-averse, PTRs are priced risk-neutral or even risk-loving during turbulent hours. In order to clarify the results in Table 4, Figure 4 shows the difference between empirical and risk-neutral densities of the underlying spread. A positive value refers to a higher probability under the risk-neutral measure.

[Insert Figure 4 here]

[24] Note that the mean-reversion speed only refers to the diffusion component. Therefore, the spikes in the underlying spread do not affect the estimated values for κ.

Figure 4 confirms the results of Table 4. For hours 4 and 24, negative (positive) outcomes of the spread become more (less) likely under the risk-neutral measure. This shift if distinctive for risk-averse market participants and in line with negative θ_J values in Table 4. Although the is smaller for hour 24, note the different scale for hours 12 to 24 compared to hours 4 and 8, the left shift of the distribution is still significant. For the remaining hours, the differences between the empirical and risk-neutral densities are also clearly visible, although only minimal for hour 12 since both densities are dominated by the extreme jump size variance. However, in contrast to hours 4 and 24, the difference between empirical and risk-neutral densities is symmetric, i.e. there is no shift in the probability of positive and negative outcomes. This confirms the risk-neutral or marginally risk-loving behavior of market participants during turbulent hours indicated by the results for the market price of jump risk in Table 4.

In order to explain our results, two aspects are of relevance. First of all, PTR options are physically settled in contrast to the widely used financially settled options. In case of cross-border supply agreements, investors might need to purchase PTR options at short notice. Since the fulfillment of such an agreement is generally of utmost priority, investors might be willing to pay a premium for PTR options. As the PTR auction is held before the day-ahead auction in the Dutch electricity market, investors are in general not able to purchase any electricity in the Dutch market at all or at the desired price in order to fulfill a potential supply agreement. Thus, the auction set up increases the hedging demand of market participants. This demand to hedge the delivery risk is amplified by the lower amount of available capacity during turbulent hours, inducing an insurance premium in PTR options. The other reason might be the usage of PTR options as purely speculative contracts. As PTR prices are generally small with regards to the extreme price spikes inherent in the underlying price process, PTRs can be thought of as a bet on the occurrence of a jump. As the forecast of jumps in the underlying spread, even with only one day to maturity, is extremely difficult as shown in Figure 2, investors are willing to pay a premium for PTRs in order to benefit from occasional but highly profitable jumps.

Besides the dichotomy in the behavior of market participants investing in PTR options, the pricing performance of our model also differs depending on the volatility in the market. The model fit is indicated by the variance of the residuals between market and model prices, i.e. σ_c^2. While σ_c^2 is rather low for calm hours with only moderate jump size variances, it significantly increases for turbulent hours. Our model therefore describes an adequate approach for the valuation of hourly PTR options during calm hours while in times of extreme volatility its pricing performance decreases.

6 Conclusion

Cross-border electricity flows become increasingly relevant due to connecting pan-European electricity markets. Further, shifts in the power generation mix, i.e. a growing share of renewables along with a decrease in fossil fuel and nuclear power plants, results in additional cross-border electricity flows and increases the physical and financial risk inherent in cross-border electricity markets. Although TSOs continue to invest in the expansion of their intra- and international powergrids, congestion will continue to prevail in the European electricity markets.

Physical Transmission Rights (PTRs) are physical products for managing cross-border electricity flows. These contracts are currently the only used products in the cross-border electricity markets in Germany. Further, all cross border connections, with the exception of the German and Western Denmark interconnection use explicit auctions for their PTR contracts. In this paper, we analyze hourly PTR prices for the German-Dutch interconnector between 2001 and 2008. PTRs are option like contracts written on the difference between hourly day-ahead electricity prices between Germany and the Netherlands. We model this price spread directly considering the unique features of the underlying, especially the extremely short-term price spikes. Due to the diverse characteristics of price spreads across hours, modelling each hour separately is essential. We find that investors are willing to pay a premium for hourly PTR options for turbulent hours of the day, i.e. hour 8 to 22. This price premium can be explained by increased hedging demand or a speculation premium from investors in the German-Dutch corss-border electricity market. The extensive demand for PTRs emphasizes the importance of these contracts and the need for adequate risk management tools in cross-border electricity markets.

Future research could analyze variations in the parameters of our model over time. Moreover, the adoption of a time-varying jump-intensity, as used by Seifert and Uhrig-Homburg (2007), could improve the pricing performance and shed more light on the behavior of jumps in this market. In addition, applying the currently famous regime-switching models could also be promising for modelling price spreads. These models have proven to adequately mirror electricity prices and are, amongst others, used by Haldrup and Nielsen (2006). Finally, testing our model for other underlyings, such as cross-commodity spreads, seems an interesting field of research.

Appendix

Full Conditionals for Estimated Parameters

The Bernoulli probability of a jump is needed in order to generate a vector of jump. Each probability is calculated as

$$\begin{aligned}\varphi_{t+1} &= \Pr\left(q_{t+1}=1\right|\Theta, P_{t+1}, S_t, N_{t+1}) \\ &= \left(1+\frac{1-p_J}{p_J}\exp\left(-\frac{2N_{t+1}\left(P_{t+1}-S_te^{-\kappa}-\nu\left(1-e^{-\kappa t}\right)\right)-N_{t+1}^2}{2\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}\right)\right)^{-1}.\end{aligned}$$

The full posterior distribution for the jump probability is

$$\begin{aligned}p\left(p_J\left|q\right.\right) &\propto p\left(q\left|p_J\right.\right)p\left(p_J\right) \\ &\propto p_J^{\sum_{t=1}^T q_t}\left(1-p_J\right)^{T-\sum_{t=1}^T q_t}p_J^{\alpha_J-1}\left(1-p_J\right)^{\beta_J-1} \\ &\propto p_J^{\alpha_J+\sum_{t=1}^T q_t-1}\left(1-p_J\right)^{\beta_J+T-\sum_{t=1}^T q_t-1}.\end{aligned}$$

The full posterior distribution for the jump size is

$$\begin{aligned}p\left(N_{t+1}\right|\Theta, q_{t+1}, P_{t+1}, S_t) &\propto p\left(P_{t+1}\right|S_t, N_{t+1}, q_{t+1}, \Theta)\,p\left(N_{t+1}\left|\Theta\right.\right) \\ &\propto \exp\left(-\frac{1}{2}\frac{\left(N_{t+1}-\frac{\sigma_J^2 q_{t+1}\left(P_{t+1}-S_te^{-\kappa}-\nu\left(1-e^{-\kappa t}\right)\right)+\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}\mu_J}{\sigma_J^2q_{t+1}^2+\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}\right)^2}{\frac{\sigma_J^2\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}{\sigma_J^2q_{t+1}^2+\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}}\right).\end{aligned}$$

The full posterior distribution for the mean jump size is

$$\begin{aligned}p\left(\mu_J\right|N, q, \Theta_{-\mu_J}, P\Big) &\propto p\left(N\right|\Theta)\,p\left(\mu_J\right) \\ &\propto \exp\left(-\frac{1}{2}\frac{\left(\mu_J-\frac{s_\mu^2\sum_{t=0}^{T-1}N_{t+1}+\sigma_J^2m_\mu}{Ts_\mu^2+\sigma_J^2}\right)^2}{\frac{\sigma_J^2s_\mu^2}{Ts_\mu^2+\sigma_J^2}}\right).\end{aligned}$$

The full posterior distribution for the variance of the jump size is

$$\begin{aligned}p\left(\sigma_J^2\right|N, q, \Theta_{-\sigma_J^2}, P\Big) &\propto p\left(N\right|\Theta)\,p\left(\sigma_J^2\right) \\ &\propto \left(\frac{1}{\sigma_J^2}\right)^{\widetilde{\alpha}_J+1}\exp\left(-\frac{\widetilde{\beta}_J}{\sigma_J^2}\right)\frac{\left(\widetilde{\beta}_J\right)^{\widetilde{\alpha}_J}}{\Gamma\left(\widetilde{\alpha}_J\right)}.\end{aligned}$$

Therefore, the posterior distribution of σ_J^2 is also inverse gamma with parameters $\widetilde{\alpha}_J$

and $\widetilde{\beta}_J$, which follow from the prior parameters as

$$\begin{aligned}\widetilde{\alpha}_J &= \frac{1}{2}T + \alpha_J, \\ \widetilde{\beta}_J &= \frac{1}{2}\sum\nolimits_{t=0}^{T-1} \left(N_{t+1} - \mu_J\right)^2 + \beta_J.\end{aligned}$$

The full posterior distribution for the variance is

$$\begin{aligned}p\left(\sigma_D^2 \middle| N, q, \Theta_{-\sigma_D^2}, P\right) &\propto p\left(P \middle| S, N, q, \Theta\right) p\left(\sigma_D^2\right) \\ &\propto \left(\frac{1}{\sigma_D^2}\right)^{\widetilde{\alpha}_D+1} \exp\left(-\frac{\widetilde{\beta}_D}{\sigma_D^2}\right) \frac{\left(\widetilde{\beta}_D\right)^{\widetilde{\alpha}_D}}{\Gamma\left(\widetilde{\alpha}_D\right)}.\end{aligned}$$

The full posterior distribution for the mean-reversion level is

$$\begin{aligned}p\left(\nu \middle| N, q, \Theta_{-\nu}, P\right) &\propto p\left(P \middle| S, N, q, \Theta\right) p\left(\nu\right) \\ &\propto \exp\left(-\frac{1}{2}\frac{\left(\nu - \frac{\left(1-e^{-\kappa}\right)s_\nu^2 \sum_{t=0}^{T-1}\left(P_{t+1}-S_t e^{-\kappa}-N_{t+1}q_{t+1}\right)+m_\nu\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}{\left(1-e^{-\kappa}\right)^2 s_\nu^2 T+\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}\right)^2}{\frac{\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}s_\nu^2}{\left(1-e^{-\kappa}\right)^2 s_\nu^2 T+\left(1-e^{-2\kappa}\right)\frac{\sigma_D^2}{2\kappa}}}\right).\end{aligned}$$

The full posterior distribution for the variance of observed PTR prices is

$$\begin{aligned}p\left(\sigma_c^2 \middle| N^Q, q^Q, \Theta^Q_{-\sigma_c^2}, C, P\right) &\propto p\left(C \middle| P, S, N^Q, q^Q, \Theta^Q\right) p\left(\sigma_c^2\right) \\ &\propto \left(\frac{1}{\sigma_c^2}\right)^{\widetilde{\alpha}_D+1} \exp\left(-\frac{\widetilde{\beta}_c}{\sigma_c^2}\right) \frac{\left(\widetilde{\beta}_c\right)^{\widetilde{\alpha}_c}}{\Gamma\left(\widetilde{\alpha}_c\right)}.\end{aligned}$$

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Table 1
Gross Electricity Generation in 2007

This table shows the gross electricity generation in Germany and the Netherlands in 2007 in TWh. Relative values are given in brackets in percent of total power generation. Source: Eurostat.

Country	Coal	Natural Gas	Oil	Nuclear	Pumped Storage	Renewables	Other	Total
Germany	267.0	77.5	9.5	167.3	7.4	74.1	33.8	636.6
	(41.9)	(12.3)	(1.5)	(26.2)	(1.2)	(11.6)	(5.3)	(100.0)
Netherlands	23.6	59.4	2.1	3.5	-	9.5	0.1	98.4
	(24.0)	(60.4)	(2.1)	(3.6)	(-)	(9.7)	(0.1)	(100.0)

Table 2
Descriptive Statistics of German-Dutch Day-Ahead Spreads between 2001 and 2008

This table shows descriptive statistics of hourly differences between German and Dutch electricity prices. Basis are the 2,922 hourly price spreads for all days between January 1, 2001 and December 31, 2008. A positive spread corresponds to higher prices in the Netherlands and vice versa.

Hour	Mean	Median	Minimum	Maximum	Std.dev.	Skewness	Kurtosis
1	1.93	1.25	-28.31	137.02	6.55	4.13	66.69
2	1.05	0.50	-21.47	47.69	5.81	1.68	9.56
3	0.65	0.05	-27.00	110.70	6.08	3.15	43.49
4	0.18	-0.14	-28.91	110.61	5.83	3.22	50.74
5	-0.52	-0.56	-31.18	110.46	5.79	2.78	50.94
6	-0.92	-1.02	-43.11	49.31	5.52	0.53	9.07
7	-0.16	-0.73	-35.24	61.62	6.91	1.32	8.61
8	-0.20	-0.64	-210.92	150.08	11.9	-4.01	118.14
9	3.55	1.07	-256.84	467.42	23.32	6.36	131.26
10	13.60	3.24	-301.67	1,945.94	62.78	16.69	434.86
11	17.11	4.24	-546.78	1,544.92	67.31	10.77	198.79
12	19.56	4.98	-1,640.15	1,943.90	87.09	6.03	178.58
13	12.08	6.69	-273.58	1,752.34	58.12	16.58	409.82
14	16.09	4.04	-315.05	1,751.58	65.54	13.25	279.94
15	13.04	3.57	-430.09	1,524.92	55.78	11.98	240.40
16	11.12	2.95	-317.34	1,754.91	67.56	16.92	366.55
17	10.30	2.78	-564.99	1,736.99	59.37	17.64	441.38
18	24.82	3.28	-412.31	1,952.97	102.13	8.77	111.75
19	10.59	2.03	-2,186.62	742.92	69.19	-10.98	407.37
20	7.18	2.00	-407.21	450.04	26.50	3.75	82.93
21	5.25	1.90	-178.53	376.49	18.60	8.38	137.50
22	3.37	1.42	-26.08	468.56	12.65	18.55	633.82
23	2.13	0.92	-29.24	74.43	7.93	2.60	15.08
24	4.55	2.82	-17.73	114.98	7.98	3.22	23.58

Table 3
Descriptive Statistics of German-Dutch PTR Prices between 2001 and 2008

This table shows descriptive statistics of hourly PTR prices for delivery from Germany to the Netherlands between January 1, 2001 and December 31, 2008 based 2,916 PTR prices. For six days, PTR prices are not available.

Hour	Mean	Median	Minimum	Maximum	Std.dev.	Skewness	Kurtosis	Avg. MW
1	0.99	0.22	0.00	25.00	2.07	4.36	26.58	777.78
2	0.57	0.10	0.00	25.01	1.51	6.21	55.57	782.61
3	0.50	0.06	0.00	25.01	1.48	7.09	71.20	783.24
4	0.44	0.06	0.00	20.01	1.29	7.24	73.09	789.76
5	0.39	0.05	0.00	20.01	1.20	7.73	84.87	791.68
6	0.40	0.05	0.00	20.01	1.28	7.88	83.62	792.45
7	0.86	0.09	0.00	350.00	8.88	34.52	1,267.75	762.00
8	1.33	0.25	0.00	350.00	9.09	32.15	1,147.80	693.83
9	3.90	1.00	0.00	350.00	13.27	13.76	278.31	661.55
10	9.56	2.25	0.00	399.00	24.74	6.98	68.64	642.18
11	11.91	3.11	0.00	399.00	27.47	5.95	50.41	636.36
12	14.16	3.74	0.00	579.12	32.97	6.89	74.61	635.48
13	8.48	2.58	0.00	399.00	23.63	7.92	91.15	637.53
14	10.46	2.51	0.00	399.00	26.34	6.84	67.16	638.55
15	8.46	2.06	0.00	399.00	23.32	7.79	87.34	643.43
16	6.61	1.64	0.00	399.00	20.53	9.68	131.10	648.86
17	6.95	1.66	0.00	370.77	20.92	8.81	105.79	648.60
18	15.37	2.01	0.00	648.99	48.61	6.93	60.53	647.36
19	10.03	1.51	0.00	500.01	31.03	7.22	67.44	655.28
20	6.19	1.35	0.00	350.00	17.71	8.92	118.46	659.68
21	4.08	1.06	0.00	350.00	10.66	14.57	399.11	671.94
22	2.64	0.60	0.00	350.00	10.39	22.32	667.68	698.67
23	2.06	0.50	0.00	350.00	9.85	25.85	829.02	722.80
24	2.48	0.75	0.00	350.00	9.63	27.09	898.59	765.20

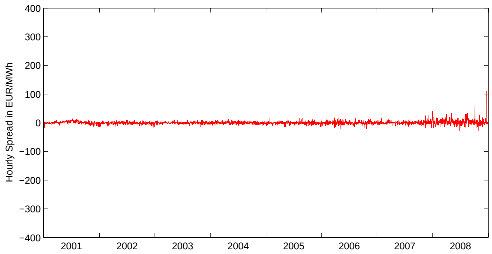
Table 4
Estimated Empirical and risk-neutral Parameters

This table shows all empirical as well as the risk-neutral parameters for the spread between German and Dutch day-ahead electricity prices as well as the respective PTR prices. Further, the market price of jump risk, given in equation 8, is also shown. Parameter values are calculated as the arithmetic mean of all non discarded samples drawn during the estimation. Standard errors of the estimation are given in brackets below each parameter.

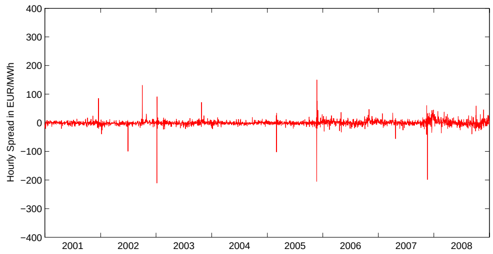
	Empirical Parameters						Risk Neutral Parameters			
Hour	κ	σ_D^2	ν	μ_J	σ_J^2	p_J	μ_J^Q	p_J^Q	σ_c^2	θ_J
1	1.4486	56.19	1.3127	4.5063	204.88	0.1053	-1.6473	0.0659	4.49	-0.0300
	(0.0006)	(0.0341)	(0.0012)	(0.0079)	(0.2473)	(0.0001)	(0.0069)	(0.0001)	(0.0012)	(0.0002)
2	1.2784	36.88	0.4322	3.4572	106.29	0.1650	-2.6264	0.0920	3.31	-0.0572
	(0.0005)	(0.0219)	(0.0011)	(0.0059)	(0.1041)	(0.0002)	(0.0028)	(0.0002)	(0.0007)	(0.0004)
3	1.2955	41.59	0.1346	3.3801	145.93	0.1312	-2.5935	0.0702	3.78	-0.0409
	(0.0006)	(0.0224)	(0.0011)	(0.0071)	(0.1642)	(0.0002)	(0.0023)	(0.0002)	(0.0007)	(0.0003)
4	1.3654	53.08	-0.0461	2.2074	197.75	0.0732	-2.9084	0.0317	3.84	-0.0259
	(0.0006)	(0.0264)	(0.0011)	(0.0089)	(0.3141)	(0.0001)	(0.0073)	(0.0002)	(0.0006)	(0.0003)
5	1.5132	63.37	-0.5918	0.8022	203.53	0.0674	-2.8833	0.0267	4.39	-0.0181
	(0.0006)	(0.0348)	(0.0011)	(0.0094)	(0.3263)	(0.0001)	(0.0006)	(0.0009)	(0.0009)	(0.0002)
6	1.6314	63.44	-1.0007	0.7735	131.93	0.0974	-3.1499	0.0491	4.82	-0.0297
	(0.0006)	(0.0364)	(0.0011)	(0.0075)	(0.1921)	(0.0002)	(0.0043)	(0.0002)	(0.0006)	(0.0003)
7	1.5528	75.97	-0.6849	3.2404	165.80	0.1395	1.9713	0.1166	81.97	-0.0077
	(0.0007)	(0.0449)	(0.0013)	(0.0073)	(0.1753)	(0.0002)	(0.0064)	(0.0001)	(0.0213)	(0.0001)
8	1.4206	113.12	-0.4416	0.8069	1,622.60	0.0566	1.2225	0.0418	85.81	0.0003
	(0.0006)	(0.0552)	(0.0016)	(0.0134)	(2.2065)	(0.0001)	(0.0054)	(0.0002)	(0.0220)	(0.0000)
9	1.3511	128.56	1.3972	2.4266	5,156.84	0.0895	3.0785	0.0800	174.27	0.0001
	(0.0006)	(0.0666)	(0.0018)	(0.0142)	(5.3434)	(0.0001)	(0.0036)	(0.0002)	(0.0453)	(0.0000)
10	1.7476	335.33	3.9393	3.9461	12,936.33	0.1290	4.7790	0.1331	606.56	0.0001
	(0.0007)	(0.1894)	(0.0024)	(0.0148)	(10.6475)	(0.0001)	(0.0133)	(0.0001)	(0.1576)	(0.0000)
11	1.8726	638.14	5.8196	2.7146	23,143.02	0.1161	4.2312	0.1235	746.88	0.0001
	(0.0007)	(0.3761)	(0.0032)	(0.0149)	(19.8108)	(0.0001)	(0.0153)	(0.0001)	(0.1969)	(0.0000)
12	1.8348	810.10	6.2835	2.4282	32,081.58	0.1383	3.8410	0.1395	1,075.12	0.0000
	(0.0006)	(0.4475)	(0.0035)	(0.0149)	(25.0247)	(0.0001)	(0.0160)	(0.0001)	(0.2786)	(0.0000)
13	1.8551	403.06	4.6101	2.4941	15,852.15	0.0916	4.2589	0.1021	55.42	0.0001
	(0.0006)	(0.2257)	(0.0025)	(0.0148)	(15.1873)	(0.0001)	(0.0149)	(0.0001)	(0.1464)	(0.0000)
14	1.9521	459.85	5.0874	2.9444	20,728.57	0.1083	4.3394	0.1155	688.53	0.0001
	(0.0008)	(0.2517)	(0.0025)	(0.0149)	(17.3976)	(0.0001)	(0.0145)	(0.0001)	(0.1803)	(0.0000)
15	1.9533	347.13	4.4949	2.2971	21,650.97	0.0975	3.7397	0.1002	538.46	0.0001
	(0.0007)	(0.1758)	(0.0022)	(0.0148)	(19.0381)	(0.0001)	(0.0156)	(0.0001)	(0.1392)	(0.0000)
16	1.8549	268.06	3.8224	1.0518	38,264.94	0.0632	2.9409	0.0604	417.05	0.0000
	(0.0007)	(0.1364)	(0.0019)	(0.0150)	(40.3128)	(0.0001)	(0.0094)	(0.0001)	(0.0225)	(0.0000)
17	1.8386	220.12	3.4703	1.7472	20,305.41	0.0744	3.5207	0.0786	433.57	0.0001
	(0.0006)	(0.1033)	(0.0018)	(0.0149)	(20.3079)	(0.0001)	(0.0163)	(0.0001)	(0.1124)	(0.0000)
18	1.8040	262.60	3.6699	2.6711	46,001.99	0.1479	3.8846	0.1473	2,345.89	0.0000
	(0.0007)	(0.1501)	(0.0021)	(0.0149)	(31.7184)	(0.0001)	(0.0159)	(0.0001)	(0.6180)	(0.0000)
19	1.4990	188.97	2.5456	3.0567	18,822.19	0.1287	4.0312	0.1505	953.62	0.0001
	(0.0006)	(0.1027)	(0.0021)	(0.0148)	(14.7670)	(0.0001)	(0.0161)	(0.0001)	(0.2471)	(0.0000)
20	1.4863	166.56	2.4207	5.5006	4,909.00	0.1212	5.4211	0.1233	307.91	0.0000
	(0.0006)	(0.0992)	(0.0020)	(0.0147)	(4.4557)	(0.0001)	(0.0123)	(0.0001)	(0.0800)	(0.0000)
21	1.4791	159.85	2.5815	4.6026	3,282.18	0.0797	4.8525	0.0792	110.80	0.0001
	(0.0006)	(0.0901)	(0.0019)	(0.0147)	(3.5864)	(0.0001)	(0.0135)	(0.0001)	(0.0292)	(0.0000)
22	1.4997	118.16	2.1252	3.1520	2,318.22	0.0454	3.9104	0.0437	106.27	0.0003
	(0.0006)	(0.0600)	(0.0016)	(0.0147)	(3.6696)	(0.0001)	(0.0146)	(0.0000)	(0.0279)	(0.0000)
23	1.2860	53.22	0.8971	6.5293	217.47	0.1528	5.5574	0.1427	96.38	-0.0045
	(0.0006)	(0.0339)	(0.0013)	(0.0075)	(0.1950)	(0.0001)	(0.0094)	(0.0001)	(0.0225)	(0.0001)
24	2.0768	182.32	3.5926	8.6124	305.00	0.0620	7.5670	0.0662	91.64	-0.0034
	(0.0008)	(0.1011)	(0.0016)	(0.0126)	(0.5484)	(0.0001)	(0.0021)	(0.0001)	(0.0241)	(0.0001)

Figure 1
Hourly Price Spreads between German and Dutch Day-Ahead Prices

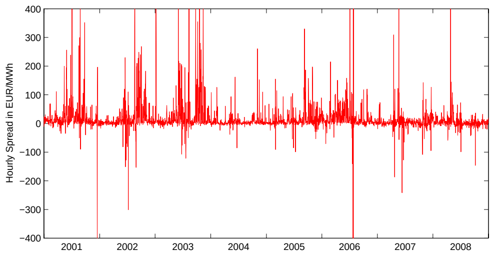
This figure shows the hourly spread of day-ahead prices between German and Dutch day-ahead electricity prices between 2001 and 2008. A positive spread indicates a higher price in the Netherlands and vice versa.



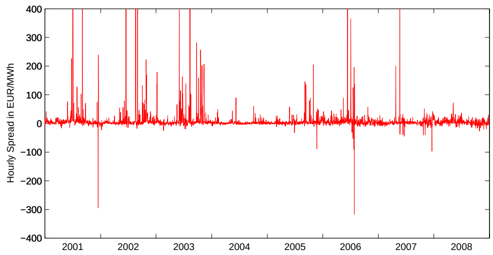
Hour 4



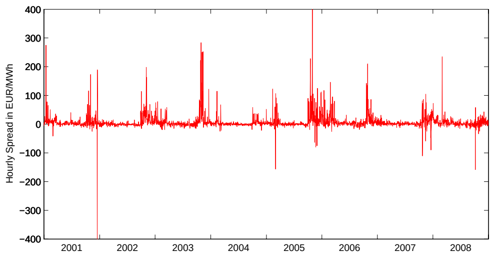
Hour 8



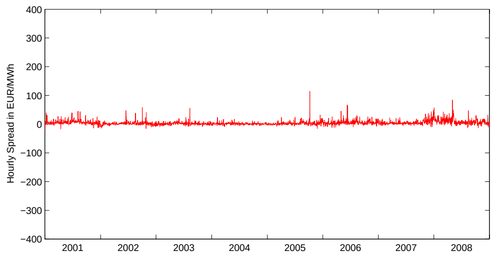
Hour 12



Hour 16



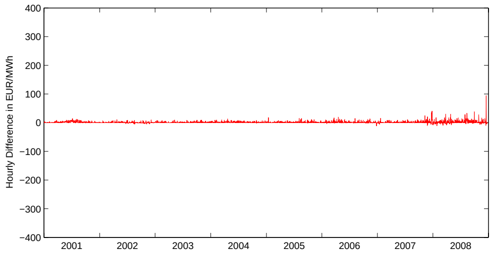
Hour 20



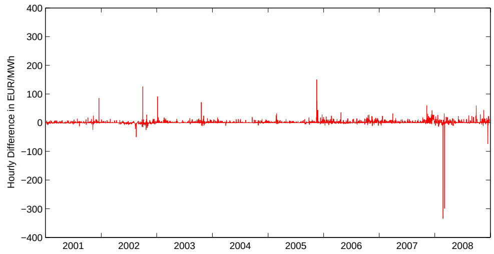
Hour 24

Figure 2
Difference between PTR Prices and Resulting Payoffs

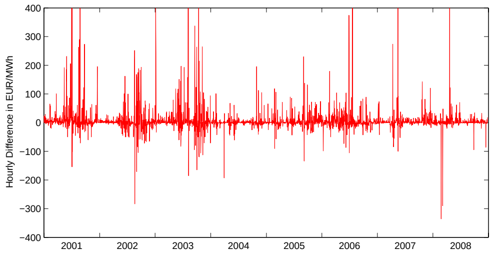
This figure shows the difference between PTR prices and the corresponding payoffs, i.e. the maximum of the resulting spread and zero. A negative difference indicates a PTR price above the resulting payoff and vice versa.



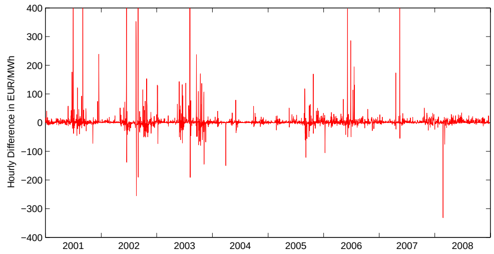
Hour 4



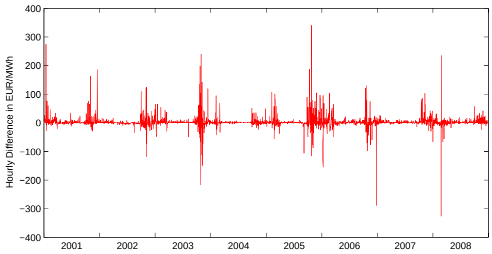
Hour 8



Hour 12



Hour 16



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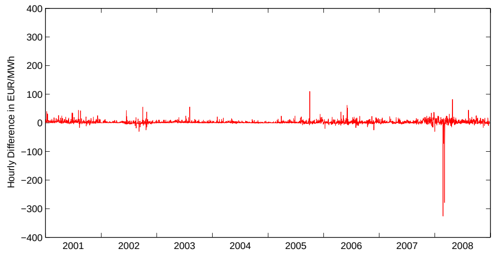
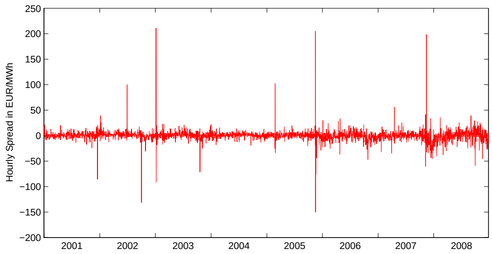


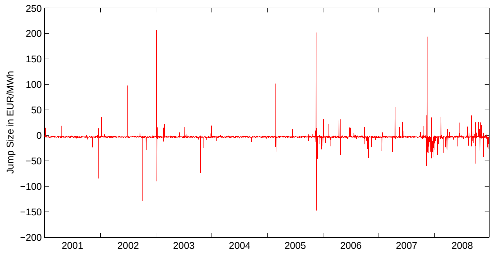
Hour 24

Figure 3
Estimated De-Spiked Process for Hour 8

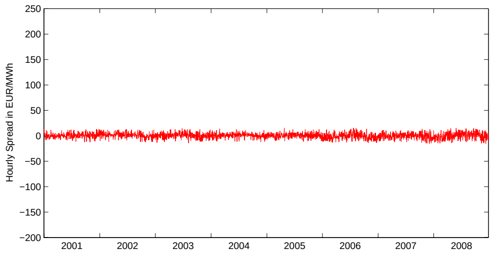
This figure shows the price spread as well as the de-spiked process for the eighth hour between 2001 and 2008. Further, the estimated jump sizes and jump probabilities for each observed price spread are given. The latter two are calculated as the arithmetic mean of all non discarded samples during the estimation.



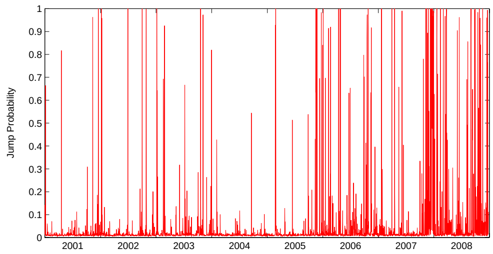
Underlying Spread



Jump Size



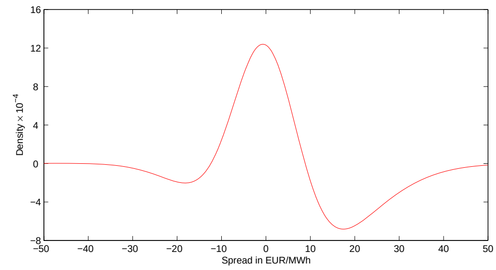
De-Spiked Process



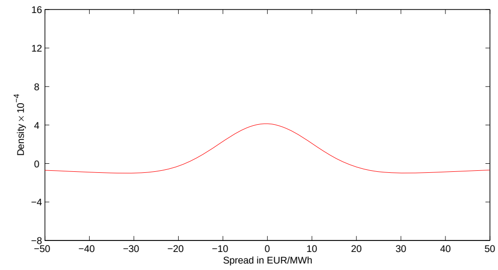
Jump Probability

Figure 4
Difference between Empirical and Risk-neutral Densities

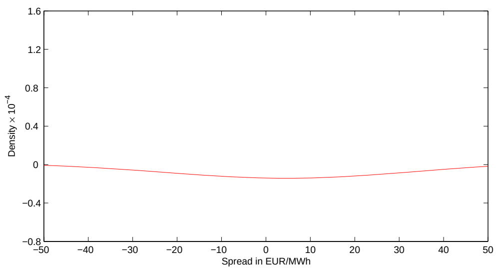
This figure shows the difference between the empirical and risk-neutral densities of the day-ahead German and Dutch electricity spread. A positive value refers to a higher risk-neutral density compared to the empirical counterpart. For simplicity, we assume the current spread to be zero, i.e. $S_0 = 0$. All remaining parameters are shown in Table 4.



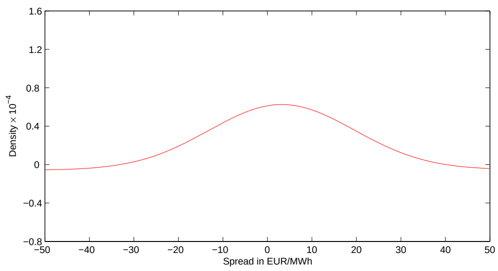
Hour 4



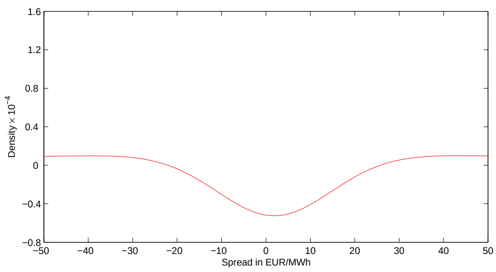
Hour 8



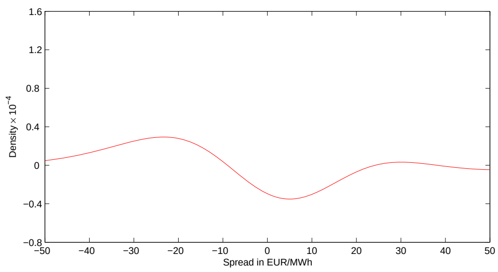
Hour 12



Hour 16



Hour 20



Hour 24